Growing Attention

Yusufcan Masatlioglu and Tri Phu Vu
University of Maryland

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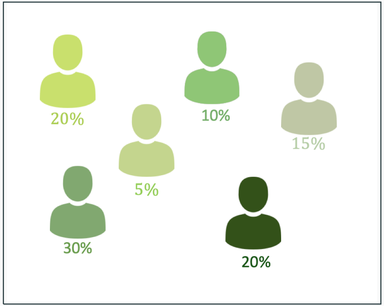
Random Choice

Think of a stochastic choice coming from repeated choices of a single person or from a heterogeneous population.

S $\dashrightarrow$

Decision Problem



Unobservable

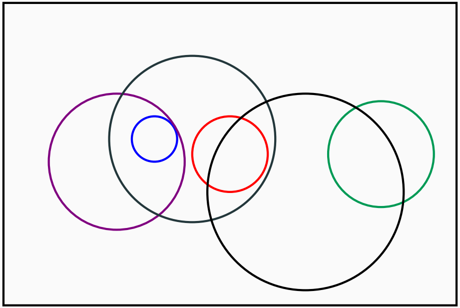
$\dashrightarrow \pi(\cdot, S)$

Choice

$\pi(x, S) =$ frequency of types choosing x from S

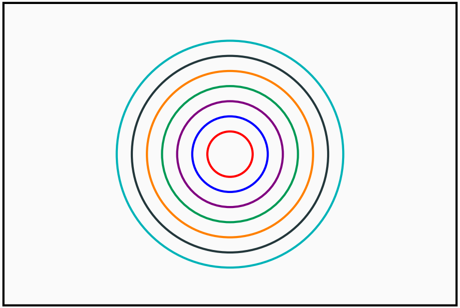
Limited and heterogeneous attention

Each type has **limited attention** and attention is **heterogeneous.**



Growing Attention Property

Put **a structure (an order)** on heterogeneous attention



$$\Gamma_1(S) \subseteq \Gamma_2(S) \subseteq \cdots \subseteq \Gamma_m(S)$$

Growing Attention: A classic example

More sleep, better attention: Attention is ordered by the number of hours of sleep

(Durmer and Dinges, 2005)

CNN.com
https://www.cnn.com › sleep-deficit-recovery-wellness

Recovering from a lack of sleep takes longer than you ...

9 thg 9, 2021 — A chronic lack of sleep therefore **impacts your ability to pay attention, learn new things**, be creative, solve problems and make decisions. Even ...

At its most basic, insufficient sleep results in reduced attention and impaired memory, hindering student progress and lowering grades. More alarmingly, sleep deprivation is likely to lead to mood and emotional problems, increasing the risk of mental illness. 20 thg 9, 2018

The New York Times
https://www.nytimes.com › opinion › sunday › sleep-sch...

Opinion | Let Teenagers Sleep In - The New York Times

Growing Attention Property

- **Satisficing** (Simon 1955; Aguiar et al., 2016)

  $$\Gamma_i(S) = \{x \in S : v(x) \geq v_i\}, \text{ with } v_i \in \mathbb{R} \text{ is a crude measure of values (salience, utilities, etc.)}$$

  If $v_1 \geq v_2 \geq \cdots \geq v_m$ then $\Gamma_1(S) \subseteq \Gamma_2(S) \subseteq ... \subseteq \Gamma_m(S)$.

- **Rationalization** (Cherepanov et al., 2013)

  $$\Gamma(S) = \{x \in S : \exists i \text{ s.t. } xP_iy \ \forall y \neq x, y \in S\}$$

  - Suppose each rationale is a linear order.
  - Let R_i be the set of rationales used by type i. Suppose higher-indexed types use more rationales: $R_i \subseteq R_j$ when $i < j$,
  - Then $\Gamma_1(S) \subseteq \Gamma_2(S) \subseteq ... \subseteq \Gamma_m(S)$ for all $S \in \mathcal{X}$.

Growing Attention Property

Generally, expansion of consideration sets can result from:

1. Inner motivation of strategic consideration (sequential search model of Fershtman and Pavan, 2023; rational inattention, Caplin et al., 2019); or

2. Interactions with external contents like advertising (competitive marketing, Eliaz and Spiegler, 2011) or recommendation (Cheung and Masatlioglu, 2023).

Main Contribution

We develop a model that allows for

- limited attention of each type.
- unobserved heterogeneous attention of types.
- unobserved types distribution.
- (possibly) unobserved endogenous preferences.

where we can

1. uniquely identify consideration sets and types distribution.
2. (partly) identify unobserved preferences.
3. test implications of the model by simple predictions.

Structure of the presentation

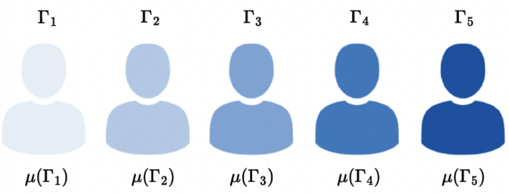
1. Model
2. Behavioral characterization
3. Endogenous preference
4. Identification of consideration sets and type distributions
5. An application to optimal list design

Notations

- X is a finite set of alternatives.
- $\mathcal{X}$ is the set of non-empty subsets of X.
- $S \in \mathcal{X}$ is a menu or a choice set.
- Random choice function (RCF) $\pi : X \times \mathcal{X} \to [0,1]$ such that $\sum_{x:\, x \in S} \pi(x, S) = 1$ and $\pi(x, S) = 0$ if $x \notin S$.
- Consideration set: $\Gamma_i : \mathcal{X} \to \mathcal{X}$ with $\Gamma_i(S) \subseteq S$ for all $S \in \mathcal{X}$.
- Limited attention: A consideration set is an attention filter if (Masatlioglu et al., 2012)

$$\Gamma_i(S) = \Gamma_i(S \setminus x), \quad \forall x \in S \text{ but } x \notin \Gamma_i(S)$$

Model



Definition

RCF π has a GAM($\succ$) if $\exists$ a collection of consideration sets $\Gamma = \{\Gamma_1, \Gamma_2, \dots, \Gamma_m\}$ and a probability measure μ over Γ such that

$$\pi(x, S) = \sum_{i:\, x = \max(\succ, \Gamma_i(S))} \mu(\Gamma_i), \quad \forall x \in S \text{ and } S \in \mathcal{X}$$

where each Γ_i is an attention filter and collection Γ has the growing attention property $\Gamma_1(S) \subseteq \Gamma_2(S) \subseteq \dots \subseteq \Gamma_m(S), \quad \forall S \in \mathcal{X}$.

Behavioral Characterization

Classic Monotonicity (Regularity): $\pi(y, S) \leq \pi(y, S \setminus x)$ for all $x, y \in S, x \neq y$.

Axiom (weak-MON):

$\pi(y, S) \leq \pi(y, S \setminus x)$ if $x \succ y$.

GAM can allow for monotonicity violations.

Axiom (Independence):

$\pi(z, S) = \pi(z, S \setminus x)$ if $\{x, y, z\} \subseteq S$ and $\pi(y, S) > 0$ and $x \succ y \succ z$.

- $\pi(y, S) > 0$ implies that y is minimally attractive at S.
- Then y is also attractive at $S \setminus x$.
- Because of $y \succ z$ and y is attractive, y absorbs any changes when removing x from the choice set.
- Probability of choosing z remains unchanged.

Behavioral Characterization (cont.)

Theorem

RCF π has a GAM($\succ$) representation iff π satisfies w-MON and independence.

Sketch of the proof:

1. Identify $\max(\Gamma_i(S), \succ)$ using a technique in Filiz-Ozbay and Masatlioglu (2023).
2. Define consideration sets as:

$$\Gamma_i(S) = \max(\Gamma_i(S), \succ) \cup \{x : x \in S \text{ s.t.} \max(\Gamma_i(S), \succ) \succ x\}.$$

3. Prove by induction that Γ_i is attention filter and $\Gamma_i(S) \subseteq \Gamma_j(S)$ when $i \leq j$.

Endogenous Preference

What happens if preference $\succ$ is unknown?

Definition: Endogenous GAM

An RCF π has an endogenous GAM representation if $\exists$ a preference order $\succ$ such that π has a GAM($\succ$) representation.

Definition [Revealed Preference]

x is revealed to be preferred to y if x is preferred to y in every preference representing π.

w-MON axiom: $\pi(y, S) \leq \pi(y, S \setminus x)$ if $x \succ y$.

Prop (Revealed Preference 1)

Suppose RCF π has an endogenous GAM representation. If $\pi(y, S) > \pi(y, S \setminus x)$ for some $S \supseteq \{x, y\}$ then y is revealed to be preferred to x.

Independence axiom: $\pi(z,S) = \pi(z, S \setminus x)$ if $\{x,y,z\} \subseteq S$ and $\pi(y,S) > 0$ and $x \succ y \succ z$.

A positive (or full-support) RCF: $\pi(x,S) > 0$ for all $x \in S$.

Prop (Revealed Preference 2)

Suppose a positive RCF π has an endogenous GAM representation. If $\exists z$ such that one of the following occurs

$$i)\quad \pi(x,\{x,y,z\}) \neq \pi(x,\{x,y\}) \text{and } \pi(x,\{x,y\}) < \pi(x,\{x,z\}) < \pi(y,\{y,z\}); \text{ or}$$

$$ii)\quad \pi(x,\{x,y,z\}) \neq \pi(x,\{x,z\}) \text{and } \pi(x,\{x,z\}) < \min\{\pi(y,\{y,z\}), \pi(x,\{x,y\})\},$$

then x is revealed to be preferred to y.

For $x \neq y$, define:

$$
xPy \text{ if } \quad
\begin{array}{rl}
i) & \exists S \supseteq \{x, y\} \text{ s.t. } \pi(x, S) > \pi(x, S \setminus y) \text{ or;} \\
ii) & \exists z \text{ s.t. } \pi(x, \{x, y, z\}) \neq \pi(x, \{x, y\}) < \pi(x, \{x, z\}) < \pi(y, \{y, z\}) \text{ or;} \\
iii) & \exists z \text{ s.t. } \pi(x, \{x, y, z\}) \neq \pi(x, \{x, z\}) < \min\{\pi(y, \{y, z\}), \pi(x, \{x, y\})\}.
\end{array}
$$

Then xPy implies x is revealed to be preferred to y.

Endogenous Preference

Independence axiom: $\pi(z, S) = \pi(z, S \setminus x)$ if $\{x, y, z\} \subseteq S$ and $\pi(y, S) > 0$ and $x \succ y \succ z$.

Independence axiom says that $x, y \succ z$ imply $\pi(z, \{x, y, z\}) \in \{\pi(z, \{x, z\}), \pi(z, \{y, z\})\}$.

Prop (Revealed Preference 3)

Suppose a positive RCF π has an endogenous GAM representation. Then xPz and $\pi(z, \{x, y, z\}) \notin \{\pi(z, \{x, z\}), \pi(z, \{y, z\})\}$ imply that z is revealed to be preferred to y.

Endogenous Preference: An Example

π_α	$\{x,y,z\}$	$\{x,y\}$	$\{x,z\}$	$\{y,z\}$
x	α	0.30	0.50	–
y	$0.60-\alpha$	0.70	–	0.60
z	0.40	–	0.50	0.40

Table 1: Probabilistic choice functions with $\alpha \in (0, 0.5)$.

	Prop. 1	Prop. 2	Prop. 3	Possible candidates	Identified preferences
$\alpha \in (0.3, 0.5)$	$x \succ z$	$x \succ y$	$y \succ z$	$x \succ y \succ z$	$x \succ y \succ z$
$\alpha \in (0, 0.3)$	-	$x \succ y$	$y \succ z$	$x \succ y \succ z$	$x \succ y \succ z$
$\alpha = 0.3$	-	-	-	All orders	$x \succ y \succ z$
					$z \succ y \succ x$

Identification of consideration sets

- Identifying consideration sets provides valuable insights into decision-making processes.
- Insights are crucial for managerial decisions.

Theorem (Identification)

Suppose a positive RCF π has a GAM($\succ$) representation. Then (Γ, μ) is unique.

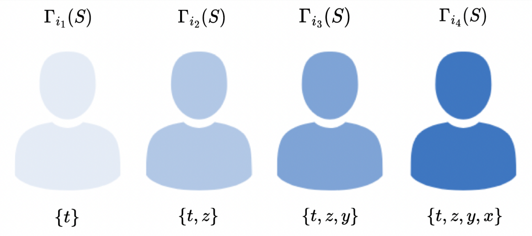
Three sources of variation in GAM ($\succ$):

1. Vary type distribution μ.
2. Vary *observed* characteristics of the collection of consideration sets Γ.
3. Vary *unobserved* characteristics of the collection of consideration sets Γ.

When RCF π has full support, no variation is admissible.

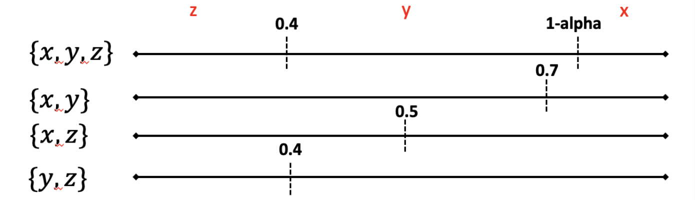
Identification of consideration sets (cont.)

Collection Γ has a special structure: Suppose $x \succ y \succ z \succ t$ and $S = \{x, y, z, t\}$. Then

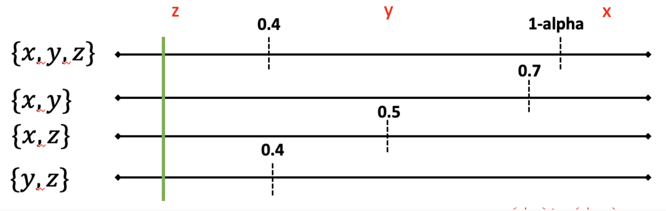


When $\alpha \in (0, 0.3)$: π_α has full support and has an unique GAM($\succ$) representation with $x \succ y \succ z$.

π_α	$\{x, y, z\}$	$\{x, y\}$	$\{x, z\}$	$\{y, z\}$
x	α	0.30	0.50	–
y	$0.60 - \alpha$	0.70	–	0.60
z	0.40	–	0.50	0.40



Identification of consideration sets (cont.)



S	$\Gamma_1(S)$	$\Gamma_2(S)$	$\Gamma_3(S)$	$\Gamma_4(S)$	$\Gamma_5(S)$
$\{x,y,z\}$	$\{z\}$	$\{y,z\}$	$\{y,z\}$	$\{y,z\}$	$\{x,y,z\}$
$\{x,y\}$	$\{y\}$	$\{y\}$	$\{y\}$	$\{x,y\}$	$\{x,y\}$
$\{x,z\}$	$\{z\}$	$\{z\}$	$\{x,z\}$	$\{x,z\}$	$\{x,z\}$
$\{y,z\}$	$\{z\}$	$\{y,z\}$	$\{y,z\}$	$\{y,z\}$	$\{y,z\}$
μ	0.4	0.1	0.2	$0.3-\alpha$	α

An Application: Optimal list design

Purchasing fabric softener on Amazon

Third-party item

Amazon's item

- Online platforms are functioning in a dual role: as a marketplace and as a seller.
- Platforms have incentives to manipulate the list of search results: *"Self-preferencing"* phenomenon (Hagiu et al., 2022; Padilla et al., 2022; Farronato et al., 2023; Motta, 2023)
- Farronato et al. (2023): "Amazon-branded products are ranked higher than observably similar products in consumer search results".
- The phenomenon has attracted growing attention recently: antitrust violation + customer welfare concerns

An Application: Setup

Motivated by the "*self-preferencing*" phenomenon:

- Denote a list L as $L = [x_1, x_2, \ldots, x_n]$.
- A designer wants to construct a list to maximize an objective function.
- The designer has a pairwise distinct weight $w(x) > 0$ to each item $x \in L$ (Manzini et al., 2023).
- Customers have the same preference $\succ$, facing an ordered list of items.
- Customers type i consider the first i options in the list (Honka, 2014; Cattaneo et al., 2023; Manzini et al., 2023):

$$\Gamma_i(S) = \{x_1, x_2, \ldots, x_i\}$$

- All types have positive measures.

The designer's objective function

The designer's objective function $W(L)$ has three properties:

1. $W(L)$ depends on the weights of items and their probabilities of being selected.
2. $W(L)$ does not depend on an unchosen option.
3. *(monotonicity)* For two lists L and L' that only differ in the chosen probabilities of x and y $(\pi(z, L) = \pi(z, L')$ for all $z \neq x, y)$

$$W(L) > W(L') \text{ if } w(x) > w(y) \text{ and } \pi(x, L) > \pi(x, L')$$

Example: The well-known family of functions with constant elasticity of substitution $(\sigma > 1)$

$$W(L) = \left(\sum_{x \in L} \pi(x, L) w(x)^{\frac{\sigma - 1}{\sigma}} \right)^{\frac{\sigma}{\sigma - 1}}$$

L-algorithm

Definition (optimal list)

The list is optimal if it solves $\max_{L \in \mathcal{L}} W(L)$, where $\mathcal{L}$ is the set of all possible lists.

- **Step 1:** Start the list from the item with the highest weight: $x_1 = \operatorname*{argmax}_{x \in L} w(x)$.
- **Step 2:** Construct lower contour set of x_1: $L_{\succ}(x_1)$. Then $x_t \in L_{\succ}(x_1)$ for all $t = 2, 3, \ldots, k+1$ with $k = |L_{\succ}(x_1)|$.
- **Step 3:** Choose $x_{k+2} = \operatorname*{argmax}_{x:\, x \notin L_{\succ}(x_1)} w(x)$; x_{k+2} has the highest weight among those in upper contour set of x_1.
- **Step 4:** Follow step 2 but construct the lower contour set of x_{k+2}; restricting to items that have not been positioned.
- **Step 5:** Repeat steps 3-4 until every position in the list is occupied.

L-algorithm: An example

An example: Six items labeled as $z_1, z_2, \dots, z_6$ with weights $w(z_i) = i$ for all i. The customer's preference is $z_4 \succ z_5 \succ z_1 \succ z_6 \succ z_2 \succ z_3$.

- **Step 1:** $L^* = [z_6, ., ., ., ., .]$ because $z_6 = \underset{z \in L}{\operatorname{argmax}}\, w(z)$.
- **Step 2:** $L_{\succ}(z_6) = \{z_2, z_3\}$. Two possible optimal lists: $L^* = [z_6, z_2, z_3, ., ., .]$ and $L^{**} = [z_6, z_3, z_2, ., ., ., .]$.
- **Step 3:** Since $z_5 = \underset{z:\, z \notin L_{\succ}(z_6)}{\operatorname{argmax}} w(z)$, the list is updated with z_5. Hence, $L^* = [z_6, z_2, z_3, z_5, ., .]$ and $L^{**} = [z_6, z_3, z_2, z_5, ., .]$.
- **Step 4:** Among those have not positioned, $L_{\succ}(z_5) = \{z_1\}$. The list is updated with z_1: $L^* = [z_6, z_2, z_3, z_5, z_1, .]$ and $L^{**} = [z_6, z_3, z_2, z_5, z_1, .]$.
- **Step 5:** Position z_4

$$L^* = [z_6, z_2, z_3, z_5, z_1, z_4] \text{ and } L^{**} = [z_6, z_3, z_2, z_5, z_1, z_4].$$

Main result

Theorem (Optimal list)

The list is optimal iff it results from running the *L-algorithm*.

- Simple algorithm to identify all optimal lists.
- The list order may not correspond to the designer's priority order; items with less values to the designer may appear near the top of the list.
- In previous example: $z_1, z_2, \ldots, z_6$ with weights $w(z_i) = i$ for all i, but the optimal lists are

$$L^* = [z_6, z_2, z_3, z_5, z_1, z_4] \text{ and } L^{**} = [z_6, z_3, z_2, z_5, z_1, z_4],$$

Additional results

1. Characterize GAM when introducing a *direction* of consideration sets following a list.
2. Provide simple conditions, supported by empirical evidence, for at most two preferences in identifying endogenous GAM.
3. Characterize GAM with heterogeneous preferences.

Literature Review

- **Probabilistic choice with heterogeneous types:** Apesteguia et al., 2017; Petri, 2023; Apesteguia and Ballester, 2023; Filiz-Ozbay and Masatlioglu, 2023.
- **Identification of types and consideration sets:** Manzini and Mariotti, 2014; Cattaneo et al., 2020; Dardanoni et al., 2020; Abaluck and Adams-Prassl, 2021; Barseghyan et al., 2021; Turansick, 2022.

Relationship to other models

- GAM nests SCRUM of Apesteguia et al. (2017).
- GAM is nested in RAM of Cattaneo et al. (2020).
- GAM is independent of RUM; MM (Manzini and Mariotti, 2014); BR (Brady and Rehbeck, 2016); the additive perturbed utility model of Fudenberg et al. (2015); fixed distribution satisficing model of Aguiar et al. (2016); attribute rule model of Gul et al. (2014); less-is-more PRC of Filiz-Ozbay and Masatlioglu (2023); etc.

Conclusion

- We study probabilistic choice where each type exhibits limited and heterogeneous attention.
- Types have the growing attention structure.
- Consideration sets and type distributions are uniquely identified.

THANK YOU!