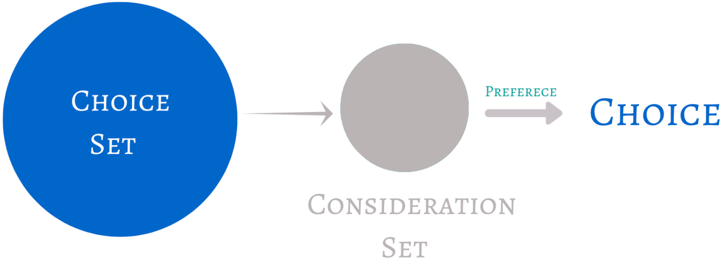
Disentangling Attention and Memory

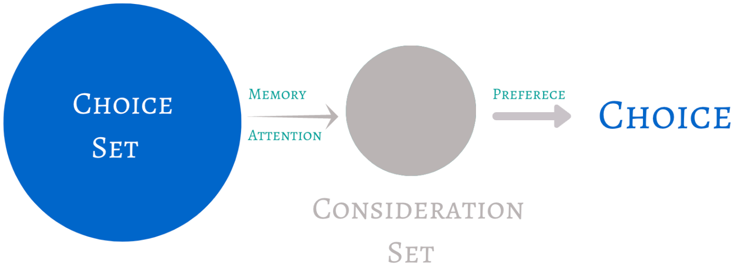
Yusufcan Masatlioglu & Ece Yegane

BRIC, June 12, 2024

Limited Consideration Models



Attention + Memory



Purchase Funnel

Marketing, Finance and Psychology Literatures: Howard and Sheth 1969, Narayana and Markin 1975, Wright and Barbour 1977, Bettman 1979, Hauser and Wernerfelt 1990, Nedungadi 1990, Alba et al 1991, Roberts and Lattin 1991, Shocker et al. 1991, Roberts and Nedungadi 1995, Chiang et al 1999, Punj and Brookes 2001, Swait et al. 2002, Goeree 2008, Seiler and Yao 2017...

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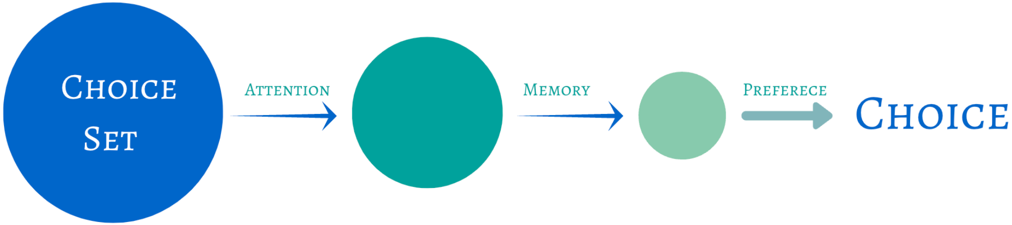

Our Aim

- Disentangle awareness and recall from observed choices?

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- Can we measured them?

Our Aim

- Disentangle awareness and recall from observed choices?
- Can we measured them?
- A simple parametric model of attention and memory

Memory

- **Limited Memory** : Yegane (2022)

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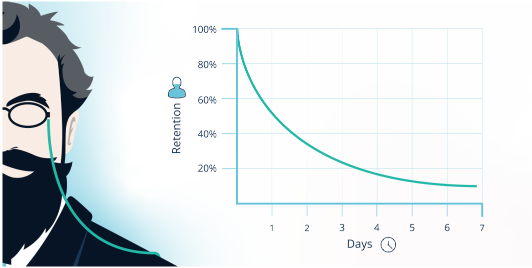
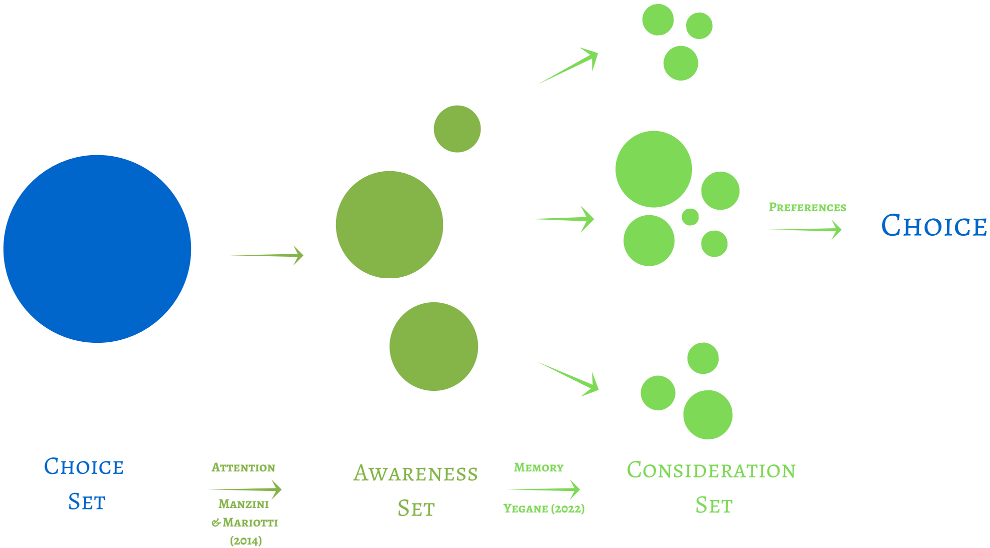
- German psychologist Hermann Ebbinghaus



How do we do it

- Attention and memory are stochastic.
- Three sets of unobservables govern choice:
  - Preferences
  - Awareness sets
  - Consideration sets
- Simple model with two sets of parameters


Choice
Set
Attention
Manzini
& Mariotti
(2014)
Awareness
Set
Memory
Yegane (2022)
Consideration
Set
Preferences
Choice

Summary of results

- We can distinguish between preferences, awareness sets and consideration sets from choices.

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- We can distinguish between preferences, awareness sets and consideration sets from choices.
- We provide behavioral foundations for the attention and memory model.
- The model is conceptually and observationally distinct from related models of
    - Block and Marschak (1960), Rubinstein and Salant (2006), Manzini and Mariotti (2014), Aguiar, Boccardi and Dean (2016), Brady and Rehbeck (2016), Yildiz (2016), Cattaneo, Ma, Masatlioglu and Suleymanov (2020), Kovach and Ulku (2020)

Stochastic Choice

- $X = \{x, y, z, ...\}$

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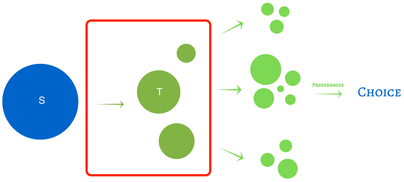
- $X = \{x, y, z, ...\}$
- $S \subseteq X$ - a choice set
- Outside option o
- $p(x, S) > 0$ - probability that x is chosen from $S \cup \{o\}$
- Stochastic choice is a *consequence* of stochastic attention and memory.

Choice procedure

First stage: Awareness (Manzini & Mariotti, 2014)



- $\gamma(x)$ is the probability of paying attention to the product x.

Choice procedure

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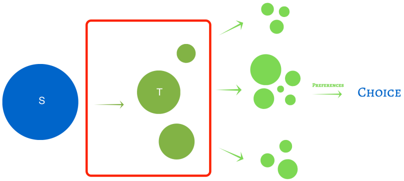
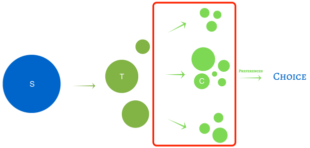


- $\gamma(x)$ is the probability of paying attention to the product x.

$$\Gamma(T, S) = \prod_{x \in T} \gamma(x) \prod_{y \in S \setminus T} (1 - \gamma(y))$$

Choice procedure

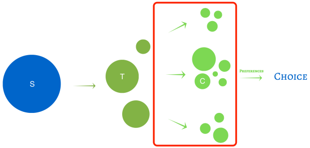
Second stage: Memory



- Forgetting is modeled after the observation that memories fade with time.

Choice procedure

Second stage: Memory



- Forgetting is modeled after the observation that memories fade with time.
- Alternatives are investigated at different times - more recently observed alternatives are more likely to be recalled at the time of choice.

Choice procedure

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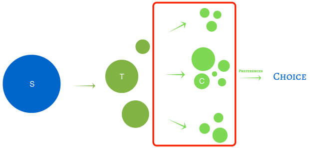

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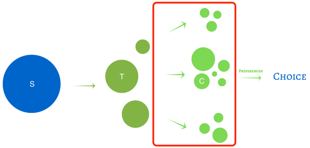

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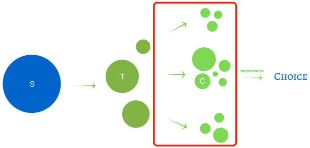

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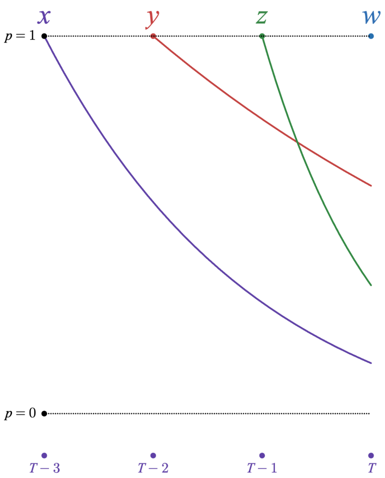
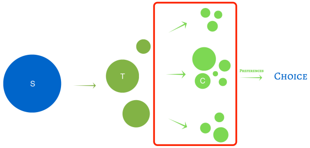
- Example: IKEA

Second stage: Memory



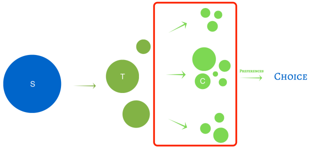
Choice procedure



Second stage: Memory **(Yegane, 2022)**

- The probability of $C \subseteq T$ being the consideration set when the awareness set is T is given by

Choice procedure



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- The probability of $C \subseteq T$ being the consideration set when the awareness set is T is given by

$$\pi_{\triangleright}(C, T) = \prod_{x \in C} q(x)^{\nabla(x,T)} \prod_{y \in T \setminus C} (1 - q(y)^{\nabla(y,T)})$$

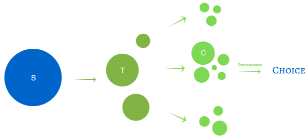
where $\nabla(x, T)$ = number of alternatives that follow x in T, according to list $\triangleright$

Choice procedure

Final stage: Choice

- DM maximizes a strict preference relation $\succ$ over the alternatives she initially paid attention to and she can recall.

Model



Model

A random choice rule p associated with list $\rhd$ has *attention and memory* representation if there exists $\succ$, γ, and q such that

$$p(x, S) = \sum_{T \subseteq S} \sum_{C \subseteq T} \underbrace{\Gamma(T, S)}_{\text{awareness set is } T} \underbrace{\pi_{\rhd}(C, T)}_{\text{consideration set is } C} \mathbb{1}(x \text{ is } \succ\text{-best in } C)$$

Example: Choice Reversals

Regularity: $p(x, S) \geq p(x, S \cup \{y\})$

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- Huber, Payne and Puto (1982): Adding a decoy to the choice set can consistently violate regularity.
- Random utility model cannot accommodate regularity violations.

Example: Choice Reversals

Attention and memory model allows regularity violations.

x $\succ y \succ z,\ x \rhd y \rhd z,\ \gamma(x) = \gamma(y) = \gamma(z) = 0.9,\ q(x) = 0.8,\ q(y) = 0.9$

S	$p(x, S)$	$p(y, S)$	$p(z, S)$	$p(o, S)$
$\{x, y\}$	0.74	**0.25**	-	0.01
$\{x, y, z\}$	0.61	**0.33**	0.06	0

Example: Choice Reversals

Recall probability	x	y
$\{x, y\}$	$q(x)$	1
$\{x, y, z\}$	$q(x)^2$	$q(y)$

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Recall probability	x	y
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$\{x, y, z\}$	$q(x)^2$	$q(y)$

- The effect on the recall probability of x **boosts** the choice probability of y
- The effect on the recall probability of y **decreases** the choice probability of y

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When does the attention and memory model predict a regularity violation in y when z is added to choice set $\{x, y\}$?

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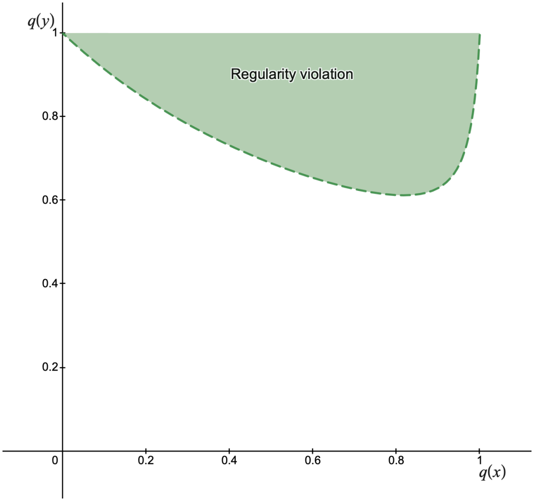
When does the attention and memory model predict a regularity violation in y when z is added to choice set $\{x, y\}$?

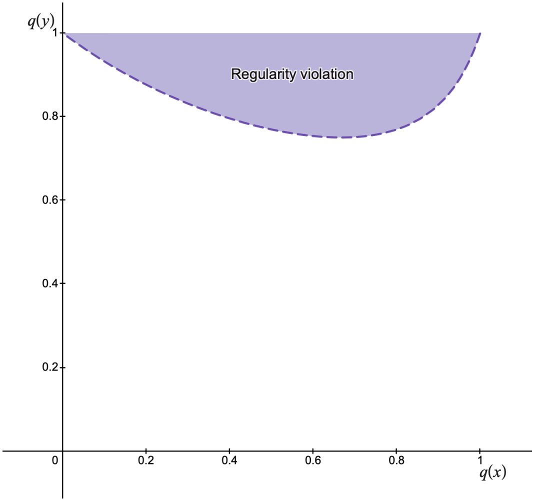
- Relevant parameters: $q(x)$, $q(y)$, $\gamma(x)$

Example: Choice Reversals



$$\gamma(x) = 0.95$$

Example: Choice Reversals



$$\gamma(x) = 0.75$$

Example: Choice Reversals

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Example: Choice Reversals

- The size of the regularity violation is
    - Increasing in $q(y)$, $\gamma(x)$
    - Increasing then decreasing in $q(x)$
    - Conditional on observing a regularity violation, increasing in $\gamma(y)$, $\gamma(z)$

Revealed preference

- Classical revealed preference theory: if x is chosen when y is available, then x is revealed to be preferred to y.

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- Under random attention and memory, an alternative not being chosen can be due to
  1. the presence of a more preferred alternative
  2. the alternative not being considered (initially paid attention to and forgotten, or not paid attention to)

Identification

- We use singleton and binary choices.

Identification

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- No memory effects in singleton choices

$$p(x, \{x\}) = \gamma(x)$$

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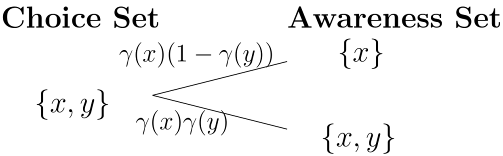

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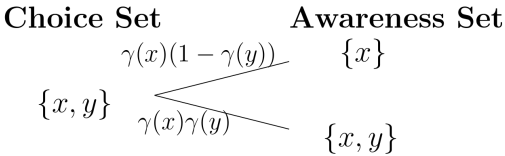

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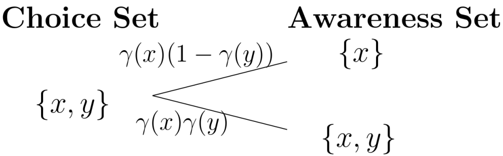

- It must be that x appears in the list after y (no memory effects)
- Otherwise, x is recalled with probability $q(x) < 1$ when the awareness set is $\{x, y\}$

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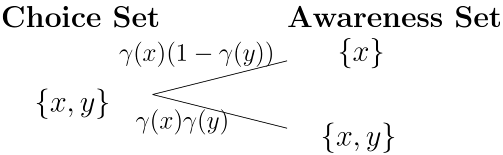

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Revealed preference

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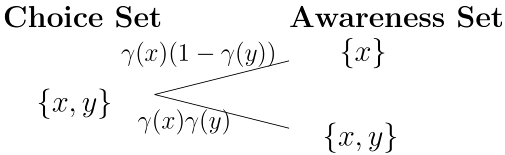

- It must be that x is preferred to y.
- Otherwise, y must be forgotten when the awareness set is $\{x, y\}$ for x to be chosen, which happens with probability $1 - q(y) < 1$

Revealed preference

- What if we observe $p(x, \{x\}) \neq p(x, \{x, y\})$ and $p(y, \{y\}) \neq p(y, \{x, y\})$?

Revealed preference

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- Preference and list ranking of x and y must coincide.

Revealed preference

- What if we observe $p(x, \{x\}) \neq p(x, \{x, y\})$ and $p(y, \{y\}) \neq p(y, \{x, y\})$?
- Preference and list ranking of x and y must coincide.
- The order $\triangleright$ is observable.

Revealed preference

Define a binary relation P on X,

$$xPy \text{ if one of the following is observed: } (i)\ p(x,\{x\}) = p(x,\{x,y\}), \text{ or}$$
$$(ii)\ p(y,\{y\}) \neq p(y,\{x,y\}) \text{ and } x \rhd y$$

Identifying the consideration set distribution

Choice probabilities from singleton sets reveal **attention parameters**

$$\gamma(x) = p(x, \{x\})$$

Identifying the consideration set distribution

How to identify **memory parameters**?

If xPy and $x \rhd y$

$$p(x, \{x, y\}) = \underbrace{\gamma(x)\gamma(y)q(x)}_{\text{awareness set is } \{x,y\}} + \underbrace{\gamma(x)(1-\gamma(y))}_{\text{awareness set is } \{x\}}$$

Identifying the consideration set distribution

How to identify **memory parameters**?

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$$q(x) = \frac{p(x, \{x, y\}) - p(x, \{x\})(1 - p(y, \{y\})}{p(x, \{x\})p(y, \{y\})}$$

Characterization

Characterization

Axiom 1

If x appears in the list after y, then $p(y, \{y\}) > p(y, \{x, y\})$.

Characterization

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If x appears in the list after y, then $p(y, \{y\}) > p(y, \{x, y\})$.

- With some probability, x causes y to be forgotten.

Characterization

Axiom 2

The revealed preference relation is transitive.

Characterization

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The revealed preference relation is transitive.

Axiom 1 and 2 $\rightarrow$ revealed preference relation P is complete, transitive, and asymmetric

Characterization

Axiom 3

If x is revealed to be preferred to y and z, and x appears in the list before y and z,

$$\frac{p(x, \{x, y\}) - p(x, \{x\})(1 - p(y, \{y\}))}{p(x, \{x\})p(y, \{y\})} = \frac{p(x, \{x, z\}) - p(x, \{x\})(1 - p(z, \{z\}))}{p(x, \{x\})p(z, \{z\})} > 0$$

Characterization

"Divide-and-conquer"

- Break down a choice problem $\{x, y, z\}$ into smaller problems $\{x, y\}$ and $\{x, z\}$

Characterization

"Divide-and-conquer"

- Break down a choice problem $\{x, y, z\}$ into smaller problems $\{x, y\}$ and $\{x, z\}$
- When can we divide-and-conquer choosing x?

Characterization

- x is the most preferred alternative in $\{x, y, z\}$

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$$p(x, \{x, y, z\}) = \gamma(x)\gamma(y)\gamma(z)\, q(x)^{\nabla(x,\{x,y,z\})} + \gamma(x)(1-\gamma(y))\gamma(z)\, q(x)^{\nabla(x,\{x,z\})} + \gamma(x)\gamma(y)(1-\gamma(z))\, q(x)^{\nabla(x,\{x,y\})}$$
$$+ \gamma(x)(1-\gamma(y))(1-\gamma(z))$$

Characterization

- x is the most preferred alternative in $\{x, y, z\}$

$$\begin{aligned} p(x, \{x, y, z\}) &= \gamma(x)\gamma(y)\gamma(z)\, q(x)^{\nabla(x,\{x,y,z\})} + \gamma(x)(1-\gamma(y))\gamma(z)\, q(x)^{\nabla(x,\{x,z\})} + \gamma(x)\gamma(y)(1-\gamma(z))\, q(x)^{\nabla(x,\{x,y\})} \\ &+ \gamma(x)(1-\gamma(y))(1-\gamma(z)) \\ &= \frac{[\gamma(x)\gamma(y)\, q(x)^{\nabla(x,\{x,y\})} + \gamma(x)(1-\gamma(y))][\gamma(x)\gamma(z)\, q(x)^{\nabla(x,\{x,z\})} + \gamma(x)(1-\gamma(z))]}{\gamma(x)} \end{aligned}$$

Characterization

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$$
\begin{aligned}
p(x, \{x, y, z\}) &= \gamma(x)\gamma(y)\gamma(z)\, q(x)^{\nabla(x,\{x,y,z\})} + \gamma(x)(1-\gamma(y))\gamma(z)\, q(x)^{\nabla(x,\{x,z\})} + \gamma(x)\gamma(y)(1-\gamma(z))\, q(x)^{\nabla(x,\{x,y\})} \\
&+ \gamma(x)(1-\gamma(y))(1-\gamma(z)) \\
&= \frac{[\gamma(x)\gamma(y)\, q(x)^{\nabla(x,\{x,y\})} + \gamma(x)(1-\gamma(y))][\gamma(x)\gamma(z)\, q(x)^{\nabla(x,\{x,z\})} + \gamma(x)(1-\gamma(z))]}{\gamma(x)} \\
&= \frac{p(x, \{x, y\})\, p(x, \{x, z\})}{p(x, \{x\})}
\end{aligned}
$$

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&= \frac{p(x, \{x, y\})\, p(x, \{x, z\})}{p(x, \{x\})}
\end{aligned}
$$

- $p(x, \{x, y, z\})$ is independent from the probability of $y/\, z$ being forgotten

Characterization

- Now suppose yPx

$$p(x,\{x,y,z\}) = \gamma(x)\gamma(y)\gamma(z)\,q(x)^{\nabla(x,\{x,y,z\})}(1-q(y)^{\nabla(y,\{x,y,z\})}) + \gamma(x)(1-\gamma(y))\gamma(z)\,q(x)^{\nabla(x,\{x,z\})}$$
$$+\gamma(x)\gamma(y)(1-\gamma(z))\,q(x)^{\nabla(x,\{x,y\})}(1-q(y)^{\nabla(y,\{x,y\})}) + \gamma(x)(1-\gamma(y))(1-\gamma(z))$$

Characterization

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\begin{aligned}
p(x,\{x,y,z\}) &= \gamma(x)\gamma(y)\gamma(z)\,q(x)^{\nabla(x,\{x,y,z\})}(1-q(y)^{\nabla(y,\{x,y,z\})}) + \gamma(x)(1-\gamma(y))\gamma(z)\,q(x)^{\nabla(x,\{x,z\})} \\
&+ \gamma(x)\gamma(y)(1-\gamma(z))\,q(x)^{\nabla(x,\{x,y\})}(1-q(y)^{\nabla(y,\{x,y\})}) + \gamma(x)(1-\gamma(y))(1-\gamma(z)) \\
&= \frac{[\gamma(x)\gamma(y)\,q(x)^{\nabla(x,\{x,y\})}(1-q(y)^{\nabla(y,\{x,y\})}) + \gamma(x)(1-\gamma(y))][\gamma(x)\gamma(z)\,q(x)^{\nabla(x,\{x,z\})} + \gamma(x)(1-\gamma(z))]}{\gamma(x)} \\
&+ \gamma(x)\gamma(y)\gamma(z)\,q(x)^{\nabla(x,\{x,y,z\})}\left[(1-q(y)^{\nabla(y,\{x,y,z\})}) - (1-q(y)^{\nabla(y,\{x,y\})})\right]
\end{aligned}
$$

Characterization

- Now suppose yPx

$$
\begin{aligned}
p(x,\{x,y,z\}) &= \gamma(x)\gamma(y)\gamma(z)\,q(x)^{\nabla(x,\{x,y,z\})}(1-q(y)^{\nabla(y,\{x,y,z\})}) + \gamma(x)(1-\gamma(y))\gamma(z)\,q(x)^{\nabla(x,\{x,z\})} \\
&+ \gamma(x)\gamma(y)(1-\gamma(z))\,q(x)^{\nabla(x,\{x,y\})}(1-q(y)^{\nabla(y,\{x,y\})}) + \gamma(x)(1-\gamma(y))(1-\gamma(z)) \\
&= \frac{[\gamma(x)\gamma(y)\,q(x)^{\nabla(x,\{x,y\})}(1-q(y)^{\nabla(y,\{x,y\})}) + \gamma(x)(1-\gamma(y))][\gamma(x)\gamma(z)\,q(x)^{\nabla(x,\{x,z\})} + \gamma(x)(1-\gamma(z))]}{\gamma(x)} \\
&+ \gamma(x)\gamma(y)\gamma(z)\,q(x)^{\nabla(x,\{x,y,z\})}\left[(1-q(y)^{\nabla(y,\{x,y,z\})}) - (1-q(y)^{\nabla(y,\{x,y\})})\right] \\
&= \frac{p(x,\{x,y\})p(x,\{x,z\})}{p(x,\{x\})} + \gamma(x)\gamma(y)\gamma(z)\,q(x)^{\nabla(x,\{x,y,z\})}\left[(1-q(y)^{\nabla(y,\{x,y,z\})}) - (1-q(y)^{\nabla(y,\{x,y\})})\right]
\end{aligned}
$$

$$p(x, \{x, y, z\}) - \frac{p(x, \{x, y\})p(x, \{x, z\})}{p(x, \{x\})}$$
$$= \gamma(x)\gamma(y)\gamma(z)q(x)^{\nabla(x,\{x,y,z\})}\left[(1 - q(y)^{\nabla(y,\{x,y,z\})}) - (1 - q(y)^{\nabla(y,\{x,y\})})\right]$$

Characterization

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- Divide-and-conquer if $\nabla(y, \{x, y, z\}) = \nabla(y, \{x, y\})$
- if z appears in the list before y

Characterization

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Characterization

- The next axiom formalizes this observation and extends to sets with $|S| > 3$.
- How to divide S when $|S| > 3$?
- Let $\rhd_1$ and $\rhd_2$ denote the first and second alternative in list $\rhd$, in choice set S respectively.
- Partition S as $\{x, \rhd_i\}$ and $S \setminus \{\rhd_i\}$ where

$$\rhd_i = \begin{cases} \rhd_1 \text{ if } x \neq \rhd_1 \\ \rhd_2 \text{ if } x = \rhd_1 \end{cases}$$

Axiom 5

If x is revealed to be preferred to $\triangleright_i$
then we can divide-and-conquer choosing x in S as $S \setminus \{\triangleright_i\}$ and $\{x, \triangleright_i\}$.

Axiom 6

In any choice set, we can divide-and-conquer choosing the outside option into singleton sets.

Characterization

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Characterization

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- We define a function $f_x(S)$ that is closely related to $p(x, S) - \frac{p(x,\{\triangleright_i, x\})p(x, S\setminus\{\triangleright_i\})}{p(x,\{x\})}$

Characterization

- What about alternatives that do not satisfy divide-and-conquer?
- We define a function $f_x(S)$ that is closely related to $p(x, S) - \frac{p(x,\{\rhd_i, x\})p(x, S \setminus \{\rhd_i\})}{p(x,\{x\})}$
- The idea behind $f_x(S)$ - isolate the choice probability of x when the awareness set is S.

Characterization

Axiom 7

If $\triangleright_1$ is revealed to be preferred to x

$$\frac{f_x(S)}{\sum_{y \in L(x,S)} f_y(S)} = \mathsf{q}(x)^{\nabla(x,S)}$$

where $L(x, S)$ is the lower counter set of x in S with respect to P and $\mathsf{q}(x)$ is the observable counterpart of $q(x)$.

Characterization

- Recall the case with three alternatives and yPx

$$p(x,\{x,y,z\}) - \frac{p(x,\{x,y\})p(x,\{x,z\})}{p(x,\{x\})}$$
$$= \gamma(x)\gamma(y)\gamma(z)\,q(x)^{\nabla(x,\{x,y,z\})}\left[(1-q(y)^{\nabla(y,\{x,y,z\})}) - (1-q(y)^{\nabla(y,\{x,y\})})\right]$$

Characterization

Axiom 7

If $x = \rhd_1$ and $\rhd_2$ is revealed to be preferred to x

$$\frac{f_x(S)}{\sum_{y \in L(x,S)} f_y(S)} = \mathsf{q}(x)^{\nabla(x,S)-1}$$

where $L(x, S)$ is the lower counter set of x in S with respect to P and $\mathsf{q}(x)$ is the observable counterpart of $q(x)$.

Characterization

Theorem

A random choice rule p associated with list $\triangleright$ has an attention and memory representation if and only if $(p, \triangleright)$ satisfies Axiom 1-7.

Wrap-up

- A parametric model to study how the allocation of attention to different options and the accessibility of options from memory affect decision making.

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- Three sets of unobservables that govern choice; preferences, awareness sets and consideration sets can be distinguished from each other.
- We provide behavioral foundations for the attention and memory model.
- Future work: application to monopoly pricing and list design