When More is Less: Limited Consideration*

Juan Sebastián Lleras† Yusufcan Masatlioglu‡
Daisuke Nakajima§ Erkut Y. Ozbay¶

August 26, 2015

Abstract

There is well-established evidence that decision makers consistently fail to consider all available options. Instead, they restrict their attention to only a subset of alternatives and then undertake a more detailed analysis of this reduced set. This systematic lack of consideration of available options can lead to a "more is less" effect, where excess of options can be welfare-reducing for a decision-maker (DM). Building on this idea, we model individuals who might pay attention to only a subset of the choice problem presented to them. Within this smaller set, a DM is rational in the standard sense, and she chooses the maximal element with respect to her preference. We provide testable characterization results for choice behavior under different consideration structures. In addition, we show to which options the decision makers must pay attention to at each set, which elements are revealed preferred to which, and discuss welfare implications.

JEL Classification: D11, D81.

Keywords: Revealed Preferences, Attention, Consideration Set, Preference Reversal

*We would like to thank David Ahn, Tilman Börgers, Emel Filiz-Ozbay, Botond Kőszegi, Stephan Lauermann, Paola Manzini, Marco Mariotti, Chris Shannon, Neslihan Uler, and participants of the Berkeley Theory Brunch for their helpful comments. Masatlioglu and Nakajima thank the National Science Foundation for financial support under Grant SES-1024544. Nakajima appreciates the additional support provided by Japan Society for the Promotion of Science (JSPS KAKENHI Grant Number 26780113). Parts of this research were conducted while Masatlioglu was visiting the Institute for Advanced Studies at Hebrew University.

†Email: jslleras@gmail.com

‡University of Michigan. Email: yusufcan@umich.edu

§Otaru University of Commerce. Email: nakajima@@res.otaru-uc.ac.jp

¶University of Maryland. Email: ozbay@umd.edu

1 Introduction

According to Food Marketing Institute, in the USA, an average supermarket carries more than 40,000 products, this is a reflection of the ongoing tendency of increasing the number of available options offered to consumers.[1] According to the classical economic theory models, this abundance of variety is beneficial for consumers; but in reality, too many options tend to overwhelm consumers, and thus lead them to neglect some products that are available.[2] This phenomenon is known as "choice overload" in psychology literature. In this paper, we show how to make welfare analysis under the possibility of choice overload, as well as provide choice theoretical foundation for this phenomenon.

When consumers are overwhelmed by the abundance of options, every product competes for consumers' attention; and as the number of the alternatives increases, the competition gets more severe. The marketing literature calls the set of alternatives that prevails on the competition for the consumers' attention the consideration set (Wright and Barbour 1977). To deal with choice overloads, consumers uses many heuristics to generate consideration sets; we list some of these heuristics to illustrate that the formation of consideration sets varies significantly.

- **Top N**: The decision-maker (DM) pays attention to the top N elements according to some ranking such as the amount of advertisement or the order of internet searches (see Rubinstein and Salant (2006) and Rubinstein and Salant (2012)). For example, a DM considers all the items appearing in the first page of search results and overlooks the rest.

- **Shortlisting**: (Manzini and Mariotti 2007) From every choice problem, the DM creates a shortlist of alternatives that are undominated according to a an asymmetric (possibly incomplete and/or cyclic) binary relation. Any alternative outside of the shortlist will be disregarded.

- **Top on Each**: There are several rankings, and the DM considers only the top N elements in each ranking. For instance, one may consider *only* the cheapest car, the most fuel efficient car, and the most advertised car in the market.[3]

[1]For instance, nowadays a shopper in a supermarket needs to select from 285 varieties of cookies, 85 flavors and brands of juices, 230 different soups, and 275 varieties of boxed cereal (Schwartz 2005).

[2]In financial economics, (see e.g. Huberman and Regev (2001)), it is known that investors make investing decision based on a limited number of all the available options, possibly good ones. Similar examples can be found in job search (Richards et al. 1975), university choice (Laroche et al. 1984, Rosen et al. 1998), and airport choice (Basar and Bhat 2004).

[3]This behavior is often called "all or nothing" or "extreme seeking" behavior (Gourville and Soman 2005).

Another example is that we only consider the top N job candidates in each field to hire an assistant professor.

- **Categorization**: (Manzini and Mariotti 2012) The DM categorizes all alternatives on the basis of some criterion, for example "similarity".[4] These categories can be (partially) compared. These comparisons are summarized by an asymmetric (possibly incomplete) binary relation. The DM considers only those belonging to an undominated category of options.

- **Rationalization**: (Cherepanov et al. 2013) Rationalization is the necessity to provide an logical explanation, avoiding the true reasons for the behavior. The DM only considers alternatives she can rationalize to choose. To do so, she finds one of the subjectively appealing rationales[5] to herself (and/or her family, society) that ranks that alternative as the best course of action given the set of alternatives. For example, a consumer considers a Subaru which is the best car among symmetrical all-wheel drive cars even though she does not care this feature.

- **Narrowing Down**: The DM narrows down the size of the considered alternatives by considering only options that meet certain criteria. For example, the DM considers all products appearing in the search result if the total number is n or less. Otherwise, the DM adds another keyword to narrow down her search.

In this paper, we assume that once the consideration set is formed, consumers are able to maximize their well-defined preference within their consideration set. Reutskaja et al. (2011), a recent experimental paper, provides evidence for this assumption by utilizing both eye tracking and choice data. They find that subjects are quite adept at optimizing within the set of items that they see (they call it as "seen set").

Our aim is to uncover preferences from solely from observed choices. Given examples above, there are many ways of constructing consideration sets. One can commit a particular consideration set formation and study the revealed preference implications of such model. However, this approach is not going to be fruitful when we do not directly observe the way that consideration sets are constructed. Instead, here we impose a property on consideration set to capture the idea of competition among products. All the examples above satisfy this property. Hence a revealed preference result based on this property will be applicable to all the examples above.

Our property, *Competition Filter*, formalizes the idea that if a product grabs the consumer's consideration in a large supermarket, then it will grab her attention in

[4]See Smith et al. (1998) and Pothos (2005).

[5]As opposed to Top N, a rationale could be incomplete.

a small convenience store with fewer rivals.[6] This property intuitively captures the idea of competition, and that competition is fiercer with more rivals. For instance, the eye-tracking study of Reutskaja et al. (2011) provides supporting evidence for our property by showing that as the number of alternatives increase, the frequencies of the least looked parts, such as the right bottom corner, decrease.

Our approach is similar to the one on Masatlioglu et al. (2012). Similar to ours, they also impose a property on consideration sets rather than focusing a particular formation of consideration sets. Their property is based on the idea of unawareness: if a consumer is not only unaware of a particular product but she is also unaware that she overlooks that product, then, her consideration set stays same if that product is removed. While their property is appealing in terms of unawareness, it will be violated by examples such as Categorization, Rationalization and Narrowing Down. Hence, their revealed preference result is *not* applicable for these examples. In addition, their property allows for a DM that considers *everything* in a bigger set but not in a smaller subset of it. Hence, their property is orthogonal to the idea of choice overload. Given that two properties are distinct, our paper complements their paper.

We call our choice model as *Choice with Limited Consideration (CLC).* In our model, the DM has a well-defined preference and is maximizing her preference within her consideration set; where the formation of consideration sets satisfies the Competition Filter property. It is natural to ask for the falsifiability of our model. Our characterization result provides necessary and sufficient conditions on the observed choices such that even if the consideration set is not observable or the heuristic is not known, it is still possible to conclude as an analyst that observed choices are consistent with CLC. Surprisingly, it turns out that our model is characterized through one testable property of choice, which is a version of the weak axiom of revealed preference (WARP) in the classical choice setting.

As opposed the WARP, our condition distinguishes between "being feasible" and "being considered," which is a key difference when making behavioral inference on observed choices. Our axiom relaxes the WARP by replacing being-feasible with being-considered in the WARP. According to the WARP (without indifference), if x is chosen while y is available, x is revealed preferred to y so y should not be chosen in any other circumstance where x is available. This is not necessarily true under limited consideration as y may be chosen over x if x is not considered. Instead, our axiom understands x is preferred to y only when x is chosen while it is certain that y is considered. Then, the axiom requires that y should never be chosen when we are sure that x is considered.

[6]This *consistency* property is the same as Sen's α property. But the important difference is that Sen's property is on the choices while ours is on the consideration set.

Although our characterization theorem shows how to test the CLC model, it does not directly deliver information about preferences unlike the classical choice theory (a la Samuelson).[7] In the classical theory, the chosen alternative is (weakly) revealed to be preferred to any other available alternative. In CLC, being chosen is not enough to make that inference since in our model there is a distinction between "being feasible" and "being considered," as previously mentioned. Nevertheless, we are able to show how to make welfare inferences (and also derive information about preferences) when limited consideration is present. Having information about which elements are being considered is vital to make inferences about preferences; it turns out that "choice reversals" is the criterion required to make inferences about preference, as it reveals consideration of certain elements, which is necessary to derive any conclusion about preferences. Choice reversals refer to the situations where choices from a small set and a larger set are "inconsistent" in the classical sense (i.e. some element is chosen from a set, and then not chosen from a subset of it still containing the choice from the superset).

To illustrate this point, consider two nested menus, $T \subset S$, both including x and y. Assume that x is the chosen alternative from T, but from the larger set, S, the chosen one is y. Firstly, x should be considered in T, and y should be considered in S since they are chosen in those sets. Competition property implies that since y is considered in S, then it must be considered in its subsets, in particular in T, i.e. both x and y should be considered in T. Hence, the choice of x in T reveals that x is better than y (revealed preference). Since in the larger set, S, the better alternative x is not chosen, we can infer that x is not considered in S. Hence, having more option leads to not considering the better option, and choosing the suboptimal one. Although this example illustrates, the necessity of the choice reversal, we prove that choice reversal is both necessary and sufficient condition for revealed preference. Hence, observing choice reversal will allow us to conclude the negative consequences of having more alternatives.

The remainder of the paper will be structured as follows Section 2 formally defines and discusses the relevant competition filters. We also provide a characterization the choice with limited consideration model for functions and linear orders and discuss the revealed preference implications on this framework. In section 3, we investigate what are the additional restriction on choice if consideration sets also satisfy the condition of Masatlioglu et al. (2012). We establish both characterization and reveled preferences result. Section 4 discusses the related literature, and finally section 5 concludes.

[7] See Samuelson (1938).

2 Choice with Limited Consideration

Let X be an arbitrary non-empty set and $\mathcal{X}$ be the set of all non-empty subsets of X. $\Gamma(S)$ denotes the consideration set under $S \in \mathcal{X}$; that is, the set of alternatives is considered when the DM is facing feasible set S. We assume that $\Gamma(S)$ is always a subset of S, as the DM can only consider options that are available. Note that we do not assume any prior knowledge about how consumers form their consideration set since there are many ways to construct consideration sets, as we discuss with the examples in the Introduction. That is why we do not commit to a specific formation of consideration sets. Instead, we directly impose a condition on the structure of consideration sets that not only captures the idea of more is less, but it is also is satisfied by many different intuitive consideration decisions (see examples in the Introduction).

Definition. *A function $\Gamma : \mathcal{X} \to \mathcal{X}$ is called a **competition filter** if for all $x \in S \subset T$ and $x \in \Gamma(T)$ then $x \in \Gamma(S)$.*

Competition filters capture systematic failures to consider all available options based on the "more is less" phenomenon. In a nutshell, when the size of the opportunity set gets larger, consumers tend to overlook more options. This idea is consistent with the Miller (1956) findings of the limited amount of information that DMs can process, and with the empirical evidence which shows the complexity of a decision process as a function of size of the menu. In addition, analyzing consideration as a function of the size of the menu is common on the marketing literature in the works of Shugan (1983) or Hauser and Wernerfelt (1990), where as both the number of options and the information about options increase, DMs consider fewer alternatives and process a smaller fraction of the overall information available regarding each alternative.

We now describe the behavior of our DM: she picks her most preferred alternative within her consideration set, not the entire feasible set. Formally, let c be a choice function: $c : \mathcal{X} \to X$ and $c(S) \in S$ for all $S \in \mathcal{X}$. Let $\succ$ be a complete, transitive and antisymmetric binary relation (a linear order) over X and denote the best element in S with respect to $\succ$ by $\max(\succ, S)$. Given this, we define our model: *Choice with Limited Consideration (CLC).*[8]

Definition. *A choice function c is a Choice with Limited Consideration (CLC) if*

[8] We consider strict preferences in the body of the paper and show on the Appendix that the analysis can be extended to the weak order case where we allow for indifference.

there exists a strict linear order $\succ$ *and a competition filter* Γ *such that*

$$c(S) = \max(\succ, \Gamma(S))$$

Occasionally, we will say that $(\Gamma, \succ)$ represents c. We also mention that $\succ$ represents c, which means that there exists some competition filter Γ such that $(\Gamma, \succ)$ represents c.

Our characterization result is concerned with finding necessary and sufficient conditions for the type of choice behavior is consistent with our model; in other words we answer the question on how one could test whether choice data is consistent with CLC. It turns out that CLC can be simply characterized through one observable property of choice, just like WARP in the classical choice model.

Before we state our behavioral postulate, recall the standard Weak Axiom of Revealed Preference (WARP). WARP requires that every set S has the "best" alternative b^* (for choice functions), that is, b^* must be chosen from a budget set T whenever b^* is available and the choice from T lies in S. Formally,

> **WARP.** For any nonempty S, there exists $b^* \in S$ such that for any T including b^*,
>
> $$\text{if } c(T) \in S \text{ then } c(T) = b^*.$$

Unlike the standard theory, we must distinguish between "being feasible" and "being considered." Therefore, we cannot conclude that b^* is chosen from T without confirming that b^* is considered. So, when can we infer that b^* is considered? Since Γ is a competition filter, in order for b^* to be considered at T, b^* must chosen from a superset of T. That is, if $b^* = c(T')$ for some $T' \supset T$, then $b^* \in \Gamma(T')$ since a necessary condition for choice is that the b^* is considered. Since Γ is a competition filter, $b^* \in \Gamma(T)$. This discussion suggests Limited Consideration WARP (LC-WARP), which is a weakening of WARP:

> **(A1) LC-WARP.** For any nonempty S, there exists $b^* \in S$ such that for any T including b^*,
>
> $$\text{if} \quad \begin{array}{l} (i)\ c(T) \in S,\ \textit{and} \\ (ii)\ b^* = c(T') \text{ for some } T' \supset T \end{array} \quad \text{then } c(T) = b^*$$

While LC-WARP is much weaker than WARP, only three observations can falsify it. For example, consider the following choice pattern:

$$c(\{x, y, z, t\}) = y, \quad c(\{x, y, z\}) = x \text{ and } c(\{x, y\}) = y.$$

To see how LC-WARP rules out the example above, take $S = \{x, y\}$; we will see that no element satisfies the condition of b^* for S. Either x or y should obey the condition (i.e. have the role of b^*) in the axiom for S. Suppose that $b^* = y$, and consider $T' = \{x, y, z, t\}$, and $T = \{x, y, z\}$. Since $y = c(\{x, y, z, t\})$ and $c(\{x, y, z\}) \in S$, if c satisfies LC-WARP then $y = c(\{x, y, z\})$, which is not the case. If $b^* = x$, then consider $T' = \{x, y, z\}$ and $T = \{x, y\}$. Since $c(\{x, y, z\}) = x$, then LC-WARP would require $x = c(\{x, y\})$, which is not true either. So there is no element in S that satisfies the condition for b^*. Therefore, the above example violates LC-WARP for $S = \{x, y\}$. Note that the axiom applies to any set of alternatives so it rules out more than this example.

To put this example in the context of our model, $c(\{x, y, z, t\}) = y$ reveals y is considered under $\{x, y, z, t\}$; this in turn implies that the DM must consider y in $\{x, y, z\}$ from the competition filter property. Hence $c(\{x, y, z\}) = x$ requires that she prefers x to y ($x \succ y$) since she considers y and chooses x. By a similar argument, $c(\{x, y, z\}) = x$ and $c(\{x, y\}) = y$ imply she prefers y to x ($y \succ x$) since she considers x and chooses y. This is a contradiction, since it would imply a cycle of size 2.[9]

The argument above hints that the choice reversals in our model directly reveals her preference: whenever her choices from a small set and a larger set are inconsistent, the former reflects her true preference under this framework. Formally, for any distinct x and y, define the following binary relation:

$$xPy \text{ if } x = c(S) \text{ and } y = c(T) \text{ such that } \{x, y\} \subseteq S \subset T \tag{1}$$

If we observe y being chosen from a larger menu, we infer that y must be considered in any subset of the menu including y (competition filter). Since x is chosen from a smaller menu containing y, then x must be preferred to y by the DM. This is a direct revelation. In addition, we can also conclude that she prefers x to z if xPy and yPz for some y, even when xPz does not hold (i.e. we do not observe a choice reversal from z to x). This is an indirect revelation. To denote both direct and indirect revelations, we use P_T, which is the transitive closure of P.

The above discussion makes it clear that P_T is necessary condition for the revealed

[9]As we will see in Lemma 1, LC-WARP does not allow cycle of any size.

preference. Before we investigate this relationship, we first identify the link between LC-WARP and P_T, which is the only behavioral postulate needed to characterize our *Choice with Limited Consideration* model.

Lemma 1. *A choice function c satisfies LC-WARP if and only if P_T is acyclical.*

Proof. First, we show that c LC-WARP implies that P (and thus P_T) is acyclical by contraposition. Assume that $x_n P x_{n-1} P \cdots P x_1 P x_n$ occurs. Then there exists no element in $\{x_1, \dots, x_n\}$ serving the role of b^* in the axiom. For example, x_k cannot be b^* since $x_{k+1} P x_k$, i.e. there exist $S_k \subset T_k$ with $\{x_k, x_{k+1}\} \subset S_k \subset T_k$ such that $x_{k+1} = c(S_k)$ and $x_k = c(T_k)$. To show that P_T acyclical implies that c satisfies LC-WARP, take $S \in \mathcal{X}$, since P is acyclical, there exists at least one alternative in S which is undominated with respect to P. Then it is routine to check that any of them serves the role of b^* in the axiom. □

The following theorem shows that a CLC is captured by a single behavioral postulate: LC-WARP.

Theorem 1. *A choice function c satisfies LC-WARP if and only if c is a CLC.*

Proof. The if-part is a direct implication of Lemma 1. If c violates LC-WARP, its revealed preference has a cycle. Let us prove the only-if part. By Lemma 1 and the existence of a linear order that is an extension of a partial order on a nonempty X, there is a preference that includes P_T. Take such a preference arbitrarily and define

$$\Gamma^m(S) = \{x \in S |\ x = c(T) \text{ for some } T \supseteq S\}$$

By construction, if $x \notin \Gamma^m(S)$ then there exists no T such that $x = c(T)$ and $S \subseteq T$. This implies that there exists no T such that $x = c(T)$ and $S \cup \{y\} \subseteq T$. Hence, $x \notin \Gamma^m(S \cup y)$ for all y, therefore Γ^m is a competition filter.

For any S, we need to show that $c(S)$ is the $\succ$-best element in $\Gamma^m(S)$. Note that $c(S) \in \Gamma^m(S)$. Let $x \neq c(S)$ and $x \in \Gamma^m(S)$. Then there exists $T \supseteq S$ such that $x = c(T)$. By construction of $\succ$, it must be $c(S) P x$, so $c(S) \succ x$. Hence $(\Gamma, \succ)$ represents c. □

This theorem states that it is possible to test our model non-parametrically by using a revealed-preference technique even when the consideration sets themselves are unobservable.

As we previously mentioned, while the theorem provides a test for our model, it does not deliver any information about the decision-maker's preferences. As opposed to the classical choice theory, there are multiple possible preference rankings which can rationalize the same choice pattern.[10] Here, we use the notion of the revealed preference introduced by Masatlioglu et al. (2012). They define the revealed preferences as follows. If x is ranked above y in *every* possible representation, x is **revealed preferred** to y.

If one wants to know whether x is revealed preferred to y, it seems to be necessary to check for every possible representation whether it represents her choice or not, which is not practical especially when there are many alternatives. We shall now provide a characterization of her revealed preference.

As we have seen before, it is sufficient to have xP_Ty to conclude that x is revealed preferred to y. A natural follow-up question is whether there is some revealed preference that is not captured by P_T. The next proposition states that the answer is no: P_T is the revealed preference.

Proposition 1. *Let c be a CLC. x is revealed to be preferred to y if and only if xP_Ty.*

Proof. We have already illustrated the if-part. To see the only-if part, take any pair of x and y without xP_Ty. Then there exists a preference $\succ$ including P_T and $y \succ x$ since P_T is transitive. By the proof of Theorem 1, $(\Gamma^m, \succ)$ represents c. Since $y \succ x$, by definition, x cannot be revealed to be preferred to y. □

In the classical theory, "more is always better": a bigger budget set is always welfare enhancing. In our model, we find that "more is sometimes less": a smaller selection is sometimes welfare enhancing. We indeed identified exactly when this happens. A smaller selection S is *welfare enhancing* over T if $c(T) \in S \subset T$ and $c(S) \neq c(T)$.

$$\text{“More is less” if } c(T) \in S \subset T \text{ and } c(S) \neq c(T)$$

However, this does not mean that in our model, the smaller sets is *always* welfare enhancing. For instance, if $c(T)$ does not belong to S, then T could be revealed to be welfare enhancing over S. Especially, this happens when $c(T)$ is preferred over any alternative in S.

---

[10] To illustrate this, consider the choice function with three elements exhibiting a cycle:

$$c(\{x, y, z\}) = y, \quad c(\{x, y\}) = x, \quad c(\{y, z\}) = y, \quad c(\{x, z\}) = z.$$

One possibility is that her preference is $x \succ y \succ z$ and she overlooks x both at $\{x, y, z\}$ and $\{x, z\}$. Another possibility is that her preference is $z \succ x \succ y$ and she does not pay attention to x only at $\{x, y, z\}$. Consequently, we cannot determine which of them is her true preference from her choice data. On the other hand, both of the preferences rank x above y. Therefore, if these two pairs are the only possibilities, we can unambiguously conclude that she prefers x to y.

3 An Additional Property on Competition Filters

As we mentioned in the Introduction, Masatlioglu et al. (2012) introduce a very intuitive property on consideration sets. If a consumer is not only unaware of a particular product (but she is also unaware that she overlooks that product), then, her consideration set stays same if that product is removed. Since this property is orthogonal to ours, in this section, we investigate the additional implication of this property within the idea of choice overload. We provide a characterization result parallel to Theorem 1. By adding more structure on consideration set we gain predictive power of the CLC model.

3.1 Independence of Overlooked Alternatives

The additional condition on the competition filters is taken from Masatlioglu et al. (2012); this condition requires that a DM who overlooks some feasible alternative, has the same consideration structure if that alternative is removed. This property is called Independence of Overlooked Alternatives (IOA).[11]

$$\textbf{IOA} : \text{If } x \notin \Gamma(S \cup x) \text{ then } \Gamma(S \cup x) = \Gamma(S)$$

Like the intuition for the competition filter property, the motivation for this property is also straightforward. Referring back to the supermarket example, if there is a product that is not considered by a DM, the DM will not notice if it is suddenly removed from the shelves. Indeed, the first three examples previously mentioned in Introduction satisfy IOA.

In section 2 we showed that LC-WARP was a weaker version of WARP that was necessary and sufficient to characterize a CLC choice function. By restricting the competition filter to satisfy IOA, we need to strengthen LC-WARP appropriately.

As we discuss before, the notion of "being considered" is vital for our analysis. Remember that under the competition filter being chosen from a superset reveals consideration. With IOA, we can infer more about whether a certain alternative is considered whenever a choice reversal is observed. For example, assume that the DM chooses x from a particular set of alternatives but removing y form the feasible set changes her choice. Under a CLC this can happen only when her consideration set has changed as y was removed. This would be impossible if she did not consider

[11] Masatlioglu et al. (2012) calls a consideration set satisfying IOA as an Attention filter. We here refer to the property rather than the specific consideration set.

y (IOA); hence, y must have been considered, despite the fact that y might not be chosen at all.

To identify whether an alternative x is considered at T, we must observe either (i) x is chosen at some $T^{'} \supset T$ (like we argue in the previous section); or (ii) removing x from T causes choice reversals (like we argue in the previous paragraph). We can also reach the same conclusion if removing x creates a choice reversal at $T^{'}$, by observing $c(T') \neq c(T' \setminus x^*)$. So it is possible to conclude consideration of an element even if it is not chosen from T' or T by IOA. This discussion suggests a strengthening of LC-WARP that fits these two properties:

(A2) LC-WARP*. For any nonempty S, there exists $b^* \in S$ such that for any T including b^*,

$$\text{if} \quad \begin{array}{l} (i)\ c(T) \in S, \quad \text{and} \\ (ii)\ c(T') \neq c(T' \setminus b^*) \text{ for some } T' \supseteq T \end{array} \qquad \text{then } c(T) = b^*$$

First of all it is clear that LC-WARP is less restrictive than LC-WARP* since whenever $b^* = c(T')$ clearly $b^* \notin c(T' \setminus b^*)$ because b^* is not feasible. As an example of the type of behavior that is ruled out by adding IOA to the competition filter Γ, consider the following choice pattern:

$$c(\{x,y,z\}) = x, \quad c(\{x,y\}) = y, \quad c(\{y,z\}) = y, \quad c(\{x,z\}) = z.$$

First, note that c is a CLC.[12] However, there is no $(\Gamma, \succ)$ pair that can rationalize this data, where Γ satisfies IOA. To see this, notice that x must be the worst element in $\{x,y,z\}$ (due to Γ being a competition filter). So, the fact that x is chosen at $\{x,y,z\}$ has to imply that x must be the only alternative considered at $\{x,y,z\}$. However, since $c(\{x,y,z\}) \neq c(\{x,y\})$, z must have been considered at $\{x,y,z\}$ (IOA), which is a contradiction since x is the worst element from $\{x,y,z\}$.[13]

The next result states that LC-WARP* is the necessary and sufficient condition for CLC with IOA.

Theorem 2. *A choice function c satisfies LC-WARP* if and only if c is a CLC where Γ satisfies IOA.*

Proof. In Appendix. □

[12] The following $(\Gamma, \succ)$ pair represents c: $y \succ z \succ x$ and $\Gamma(\{x,y,z\}) = \{x\}$, otherwise $\Gamma(S) = S$.
[13] Likewise, since $c(\{x,y,z\}) \neq c(\{x,z\})$, y must have been considered at $\{x,y,z\}$.

Here we illustrate that adding IOA provides more information about preferences. To do this, we revisit the cyclical choice behavior in Footnote 10,

$$c(\{x,y,z\}) = y, \quad c(\{x,y\}) = x, \quad c(\{y,z\}) = y, \quad c(\{x,z\}) = z$$

From the previous discussion, we know and c is a CLC and that x is preferred over y but there is no other revealed preference (see Proposition 1). Interestingly, we can uniquely pin down the preference for the cyclical choice example when Γ satisfies IOA as well. By IOA, $c(\{x,y,z\}) \neq c(\{x,y\})$ implies that y is preferred over z. By transitivity, we identified the entire preference.

Theorem 2 characterizes a special of class of CLC choice behavior where the competition filter satisfies IOA as well. Similar to Theorem 1, the characterization involves a single behavioral postulate which is stronger than LC-WARP, but still weaker than WARP. This model has higher predictive power, as we showed in the example of cyclical choice behavior where we can pin down uniquely the order and the consideration set if we focus on competition filters with IOA, which we could not do for the CLC. However, this better predictive power unavoidably comes with diminishing explanatory power as we showed in the earlier example of a choice pattern that is a CLC but does not satisfy IOA.

Now let us characterize the reveled preference when Γ is a competition filter with IOA. When $c(T) \neq c(T \setminus y)$, we conclude that y must be considered at T (IOA). Since Γ is a competition filter, y must be also considered at any decision problem S which includes y and is a subset of T. Therefore, if $c(S) \neq y$, $c(S)$ is revealed to be preferred to y. Hence, it follows that whenever $c(T) \in S \subset T$ and $c(T) \neq c(S)$, we have $c(S) \succ c(T)$. Formally, for any distinct pair of x and y define the following binary relation like in the previous section:

$$xP'y \text{ if there exist } S \text{ and } T \text{ such that } \quad \begin{array}{l} (i) \;\; \{x,y\} \subseteq S \subset T \text{ and } x = c(S) \\ (ii) \; c(T) \neq c(T \setminus y) \end{array} \tag{2}$$

As before, if $xP'y$ and $yP'z$ for some y, we also conclude that she prefers x to z even when $xP'z$ does not hold. Indeed, analogy of Lemma 1, LCA-WARP is equivalent to the acyclicity of the revealed preference, P'_T. P'_T characterizes the entire revealed preference in this model.

Proposition 2. *Suppose c is a CLC with IOA. Then, x is revealed to be preferred to y if and only if xP'_Ty.*

Proof. The if-part has been already demonstrated. The only-if part can be shown paralleled with Theorem 2, where we shall show that any $\succ$ including P'_T represents c by choosing Γ properly. $\square$

4 Related Literature

Our model is related to a growing body of work on sequential elimination procedure. Manzini and Mariotti (2007) provide a model where a DM sequentially eliminates inferior alternatives according to asymmetric binary relations (rationales) until only one alternative remains as the final choice. The order of rationales are fixed. Manzini and Mariotti (2007) deliver a very simple characterization when there are only two rationales (shortlisting). Apesteguia and Ballester (2009) provide a characterization for any arbitrary number of *acyclical* rationales and link this procedure with other seemingly different procedures. Dutta and Horan study how to identify rationales in shortlisting procedure from observed choice behavior.[14] The differences between our model and shortlisting (two rationales) are two folded: (i) our second stage preference is assumed to be complete and transitive, and (ii) our competition filter allows much richer structure compared to their set of surviving alternatives. If there are more than two rationales, the set of surviving alternatives might violate our property.[15]

Our model is also closely related to *categorization* of Manzini and Mariotti (2012) and *rationalization* of Cherepanov et al. (2013), all of which utilize the idea of limited consideration. Both of them follow the two-stage choice process: in the first stage, a decision maker focuses on a small set of alternatives, and in the second stage she maximizes her preference among the alternatives surviving after the first stage. In the categorization model, a DM considers only alternatives belonging to undominated categories. In the rationalization model, each consideration set consists of alternatives a decision maker can rationalizes among all available alternatives, called the permissible alternatives. Both categorization and rationalization models are characterized by a single axiom, so-called Weak-WARP. Therefore, there is no way to distinguish these two models by using the standard choice data. Having said that, one should note that their motivations and underlying stories are completely different, which makes them conceptually different and distinguishable. The main difference between our model and theirs is again the second stage preferences. That is why our model

[14] Au and Kawai (2011), Bajraj and Ulku (2014), Nicolas (2007), Houy and Tadenuma (2009), Mariotti and Manzini (2012), Mandler et al. (2012), Garcia-Sanz and Alcantud (2010), Tyson (2013) also study variations of the sequential elimination procedure.

[15] There are competition filters that cannot be expressed as the set of surviving alternatives according to a set of rationales.

has a different characterization. While Weak-WARP only guarantees the revealed preference asymmetric, LC-WARP assures that second rationale is also acyclic.

The lack of consideration of some alternatives plays a relevant role in several papers. Masatlioglu and Ok (2014) propose a reference-dependent model where each status quo generates a psychological constraint set of alternatives that the DM is prepared to choose from given that status quo.[16] Masatlioglu and Nakajima (2013) study behavioral search by utilizing the idea of consideration sets. In this model, the consideration set dynamically evolves during the course of search. Caplin and Dean (2011) also study behavioral search by utilizing "choice process data," which include what the decision maker would choose at any given point in time if she were suddenly forced to quit searching. Eliaz et al. (2011) consider a model where the consideration sets of DMs are directly observed. Salant and Rubinstein (2008) propose a model where the DM considers only the top N elements according to some ranking and chooses her most preferred element from that restricted set.

5 Conclusion

Consumers generally do not consider all the available alternatives; they intentionally or unintentionally ignore some of the alternatives and focus on a limited number of alternatives. In this paper, we relax the implicit full consideration assumption of the standard choice theory to allow for the choice with limited consideration, where we allow "feasible alternatives" and "considered alternatives" to differ for a given choice problem.

Marketing and finance literatures argue that the abundance of alternatives is the basic motive for limiting the consideration set. It is well documented that different types of filters have been used by the consumers to limit their consideration sets. Motivated by real-life examples observed different situations, we provide characterization for a general filter that includes many of these examples: if a consumer considers an alternative among a large set of option, she will still continue to considering the same alternative when some alternatives become unavailable. Moreover, we add another general intuitive property that is also consistent with observed behavior on different markets: if an alternative that the consumer does not consider becomes unavailable, his consideration set will not be affected.

Although the consideration sets are not observable, our axiomatic approach enables to distinguish and identify preferences and consideration by observing DM's

[16] A recent paper by Dean et al. (2014) investigate the implication of competition filter on reference-dependent behavior.

choices. We show our choice with limited consideration is capable of explaining behavioral anomalies that look puzzling under standard choice theory. Additionally, we believe these insights may help companies to develop new marketing strategies such that their products will attract attention by the consumers.

References

J. Apesteguia and M. A. Ballester. A theory of reference-dependent behavior. *Economic Theory*, 40(3):427–455, 2009.

P. H. Au and K. Kawai. Sequentially rationalizable choice with transitive rationales. *Games and Economic Behavior*, 73(2):608–614, 2011.

G. Bajraj and L. Ulku. Choosing two finalists and the winner. Working paper, 2014.

G. Basar and C. Bhat. A parameterized consideration set model for airport choice: an application to the san francisco bay area. *Transportation Research Part B: Methodological*, 38(10):889–904, 2004.

A. Caplin and M. Dean. Search, choice, and revealed preference. *Theoretical Economics*, 6(1), January 2011.

V. Cherepanov, T. Feddersen, and A. Sandroni. Rationalization. *Theoretical Economics*, 8(3), September 2013.

M. Dean, O. Kibris, and Y. Masatlioglu. Limited Attention and Status Quo Bias. Working paper, 2014.

R. Dutta and S. Horan. Inferring Rationales from Choice: Identification for Rational Shortlist Methods. *American Economic Journal: Microeconomics*. Forthcoming.

K. Eliaz, M. Richter, and A. Rubinstein. Choosing the two finalists. *Economic Theory*, 46(2):211–219, February 2011.

M. D. Garcia-Sanz and J. C. R. Alcantud. Sequential rationalization of multivalued choice. *Mathematical Social Sciences*, 74:29–33, March 2015.

J. T. Gourville and D. Soman. Overchoice and assortment type: When and why variety backfires. *Marketing Science*, 24(3):382–395, 2005. ISSN 1526-548X.

J. R. Hauser and B. Wernerfelt. An evaluation cost model of evoked sets. *Journal of Consumer Research*, 16:383–408, March 1990.

N. Houy and K. Tadenuma. Lexicographic compositions of multiple criteria for decision making. *Journal of Economic Theory*, 144(4):1770–1782, 2009.

G. Huberman and T. Regev. Contagious speculation and a cure for cancer: A non-event that made stock prices soar. *Journal of Finance*, 56(1):387–396, 02 2001.

M. Laroche, J. Rosenblatt, and I. Sinclair. Brand categorization strategies in an extensive problem solving situation: A study of university choice. *Advances in Consumer Research*, 11:175–179, 1984.

M. Mandler, P. Manzini, and M. Mariotti. A million answers to twenty questions: Choosing by checklist. *Journal of Economic Theory*, 147(1):71–92, 2012.

P. Manzini and M. Mariotti. Sequentially rationalizable choice. *American Economic Review*, 97(5):1824–1839, December 2007.

P. Manzini and M. Mariotti. Categorize Then Choose: Boundedly Rational Choice And Welfare. *Journal of the European Economic Association*, 10(5):1141–1165, October 2012.

M. Mariotti and P. Manzini. Choice by lexicographic semiorders. *Theoretical Economics*, 7(1), January 2012.

Y. Masatlioglu and D. Nakajima. Choice by iterative search. *Theoretical Economics*, 8(3), September 2013.

Y. Masatlioglu and E. A. Ok. A canonical model of choice with initial endowments. *The Review of Economic Studies*, 81(2):851–883, 2014.

Y. Masatlioglu, D. Nakajima, and E. Y. Ozbay. Revealed Attention. *American Economic Review*, 102(5):2183–2205, August 2012.

G. A. Miller. The magic number seven, plus or minus two: Some limits on our capacity for processing information. *The Psychological Review*, 63:81–97, March 1956.

H. Nicolas. Rationality and Order-Dependent Sequential Rationality. *Theory and Decision*, 62(2):119–134, March 2007.

E. M. Pothos. The rules versus similarity distinction. *Behavioral and Brain Sciences*, 28:1–14, 2 2005. ISSN 1469-1825.

E. Reutskaja, R. Nagel, C. F. Camerer, and A. Rangel. Search Dynamics in Consumer Choice under Time Pressure: An Eye-Tracking Study. *American Economic Review*, 101(2):900–926, April 2011.

M. D. Richards, J. E. Sheridan, and J. W. Slocum. Comparative analysis of expectancy and heuristic models of decision behavior. *Journal of Applied Psychology*, 60(3):361–368, 1975.

D. E. Rosen, J. M. Curran, and T. B. Greenlee. College choice in a brand elimination framework: The administrator's perspective. *Journal of Marketing for Higher Education*, 8(4), 1998.

A. Rubinstein and Y. Salant. A model of choice from lists. *Theoretical Economics*, 1 (1):3–17, March 2006.

A. Rubinstein and Y. Salant. Eliciting Welfare Preferences from Behavioural Data Sets. *Review of Economic Studies*, 79(1):375–387, 2012.

Y. Salant and A. Rubinstein. (a, f): Choice with frames. *The Review of Economic Studies*, 75(4):1287–1296, 2008.

P. Samuelson. A note on the pure theory of consumers behavior. *Economica*, 5:61–71, 1938.

B. Schwartz. *The Paradox of Choice : Why More Is Less*. Harper Perennial, January 2005.

S. M. Shugan. Cost of thinking. *Journal of Consumer Research*, 7:99–111, 1983.

E. E. Smith, A. L. Patalano, and J. Jonides. Alternative Strategies of Categorization. *Cognition*, (65):167–196, 1998.

C. Tyson. Behavioral implications of shortlisting procedures. *Social Choice and Welfare*, 41(4):941–963, October 2013.

P. Wright and F. Barbour. Phased decision strategies: Sequels to an initial screening. In M. Starr and M. Zeleny, editors, *Studies in Management Sciences, Multiple Criteria Decision Making*, pages 91–109. North-Holland, Amsterdam, 1977.

A Appendix

A.1 CLC: Choice Correspondences

In the following section we show that the approach can be extended to choice correspondences. Here, we generalize the main characterization and revealed preference results from the paper, the characterization of CLC, by allowing choices to be multi-valued and choice sets be arbitrary.

In this section we consider the general case where X is the (possibly infinite) choice set. $\mathcal{X}$ is the set of all non-empty finite subsets of X. Similarly, a choice correspondence will be given by $C : \mathcal{X} \to X$, such that $C(S) \subseteq S$ for every $S \in \mathcal{X}$. So far, c has been used to represent a choice function; here, we use C to represent a choice correspondence. When choices are multi-valued, instead a linear order we need to consider weak orders to allow for the possibility of an indifference relation. Now we let $\succsim$ be a weak order on X. We show it is possible to characterize a CLC in this (more general) setting with only a few changes that account for the possibility of a "revealed indifference".

In section 2 we showed that the only behavioral postulate that characterizes a CLC with a competition filter was LC-WARP. In this section, we characterize the CLC for choice correspondences by first adding a consistency axiom for multivalued choices (which will characterize the indifference relation with limited consideration); and second by modifying LC-WARP.

The new condition, which guarantees that the indifference relation is identifiable, requires multi-valued choices to also be consistent across choice sets. This new axiom, called Weak Revealed Indifference (WRI), does not allow for choice reversals involving only one element when two elements have been chosen from a larger menu. In other words, if WARP is violated from S to T[17], then the intersection between $C(T)$ and $C(S)$ must be empty.

Referring back to the intuition for LC-WARP, if there is more than one element, x^*, y^* satisfying the property for LC-WARP for a particular set, S, then those two elements must satisfy (or not) LC-WARP together for any set that contains both of them. In other words, once x^* and y^* satisfy the condition from LC-WARP for S, for any other set T such that $\{x^*, y^*\} \subseteq T \subset S$ if x^* satisfies the LC-WARP condition so does y^*.

> **Weak Revealed Indifferences (WRI).** If for $\{x, y\} \subseteq T \subset S$, $x, y \in C(S)$, then $x \in C(T)$ implies $y \in C(T)$.

One consequence of WRI is that we are be able to distinguish between (strict) revealed preference and revealed indifference from choice data, as shown by the following lemma.

[17] I.e $T \subset S$ and $y \in C(S) \cap T$, and $y \in C(T)$, then $C(S) \cap C(T) = \emptyset$.

Lemma 2. *Let C satisfy WRI. Then $x \in C(T)$ and $x \notin C(T')$ for some $S \subseteq T$ implies $C(T') \cap C(T) = \emptyset$.*

Proof. Suppose there exists $y \in C(T) \cap C(T')$, then by WRI we have $x \in C(T')$, since $x, y \in C(T)$ and $y \in C(T')$ which would be a contradiction. □

The second axiom that we introduce to characterize CLC for correspondences is called No Cyclic Choice Reversals. Abusing terminology, we say that there is a choice reversal from x to y whenever there is a violation of WARP, i.e. $x \in C(T)$ and $y \in C(T')$ for $T' \subset T$. No Cyclic Choice Reversals is a stronger condition than LC-WARP, since it guarantees not only that LC-WARP is satisfied, but also, that once there is a choice reversal form x to y, we cannot find a chain of pairwise choice reversals that would indirectly imply the a choice reversal from x to y (i.e. from x to z_0, from z_0 to z_1,..., and from z_n to y).

It is straightforward to see that NCCR implies LC-WARP for choice functions.

> **No Cyclic Choice Reversal (NCCR).** Consider two families of sets $\{S_i\}$ and $\{T_i\}$ such that $T_i \subseteq S_i \subseteq X$ for $i = 1, \ldots, n$. If $C(T_{i+1}) \cap C(S_i) \neq \emptyset$ for all $i \leq n-1$ and $C(T_i) \cap C(S_i) = \emptyset$ for some i then $C(T_1) \cap C(S_n) = \emptyset$

Again, we define two binary relations: P and I. The former will capture the revealed preferences, the later captures the revealed indifference.

Definition. *Given a choice correspondence C define two binary relations, P and I as follows.*

1. *xPy if there exists $\{x, y\} \subseteq S \subset T$ such that $x \in C(S)$ and $y \in C(T) \setminus C(S)$.*
2. *xIy if there exists $\{x, y\} \subseteq S$ such that $\{x, y\} \subseteq C(S)$.*

First of all, note that if a choice correspondence satisfies our two axioms, we have that P and I are disjoint.

Proposition 3. *If C satisfies NCCR and WRI, then $P \cap I = \emptyset$.*

Proof. Let xPy, then there exists $\{x, y\} \subseteq S \subset T$ such that $x \in C(S)$ and $y \in C(T) \setminus C(S)$. Suppose there exists $T' \in \mathcal{X}$ such that $x, y \in C(T')$. By lemma 2, $C(T) \cap C(S) = \emptyset$, and $\{x, y\} \subseteq T'$, then $C(\{x, y\}) = \{x, y\}$ by WRI. And by NCCR, since $C(\{x, y\}) \cap C(S) \neq \emptyset$, we have $C(\{x, y\}) \cap C(T) = \emptyset$, which is a contradiction. So there does not exists such a T', and therefore $\neg(xIy)$.

Let xIy, then there exists $S \supseteq \{x, y\}$ such that $x, y \in C(S)$. By WRI we must also have $C(\{x, y\}) = \{x, y\}$. Suppose there exists $\{x, y\} \subseteq S \subset T$ such that $x \in C(S)$ and $y \in C(T) \setminus C(S)$. Then by lemma 2 we must have $C(S) \cap C(T) = \emptyset$. And $C(S) \cap C(\{x, y\}) = x$ implies $C(T) \cap C(\{x, y\}) = \emptyset$, which is a contradiction since $y \in C(T)$ and $C(\{x, y\}) = \{x, y\}$. Therefore no such $S \subset T$ exists, thus $\neg(xPy)$. □

Lemma 3. *Let $R = P \cup I$, then xRy if and only if there exists $\{x, y\} \subseteq S \subseteq T$ such that $x \in C(S)$ and $y \in C(T)$.*

Proof. This follows from the definitions of P and I. $\square$

Hence we can see that the two axioms are equivalent to not being able to find two conflicting choice reversals. In parallel to the function case, this will imply that once we take the transitive closure of R, we will not have cycles.

Proposition 4. *C satisfies NCCR and WRI if and only if for any set of elements in X, $\{x_i\}_{i=1}^n$, such that $x_n R x_{n-1} R \ldots R x_2 R x_1$ imply $\neg(x_1 P x_n)$.*

Proof. ($\Rightarrow$) Consider $x_i \in X$ for $i = 1, ..., n$, such that $x_n R x_{n-1} R \ldots R x_2 R x_1$. Then by the definition of R, there must exists for each $i = 2, \ldots, n$ sets and subsets $S_i \supseteq T_i$ with $x_i \in C(S_i)$ and $x_{i-1} \in C(T_i)$ (see lemma 3).

Suppose $x_1 P x_n$, then there exists $\{x, y\} \subseteq S' \subset T'$ such that $x_1 \in C(S')$ and $x_n \in C(T') \setminus C(S')$. By WRI, $C(S') \cap C(T') = \emptyset$. Let $S_1 = S'$ and $T_1 = T'$, then by NCCR $C(S_n) \cap C(T_1) = \emptyset$, but $x_n \in C(S_n)$ by definition, and $x_n \in C(T_1)$ by $x_1 P x_n$, a contradiction. Therefore $\neg(x_1 P x_n)$.

($\Leftarrow$) Let $x, y \in C(T)$ and $x \in C(S)$ for some $S \supseteq \{x, y\}$. Then we have yIx, which by definition of R implies yRx. If $y \notin C(S)$ then by definition of P we have xPy, but this is a contradiction since yRx implies $\neg(xPy)$ by the condition. So C satisfies WRI.

Let $S_i \subseteq T_i$ such that $x_i \in C(S_i)$ and $x_{i-1} \in C(T_i)$ for $i = 2, \ldots, n$ and $x_1 \in C(S_1)$. So we have $C(S_i) \cap C(T_{i+1}) \neq \emptyset$ for all $i = 2, \ldots, n$. This implies $x_n R x_{n-1} R \ldots R x_2 R x_1$. Now we prove the contrapositive, let C fail NCCR. WLOG let $x_n \in C(T_1)$ and $C(T_1) \cap (S_1) = \emptyset$, so $C(S_n) \cap C(T_1) \neq \emptyset$, and for one of the S_i, T_i, $C(S_i) \cap C(T_i) = \emptyset$. Then we have $x_n R x_{n-1} R \ldots R x_2 R x_1$, and since $x_n \in C(T_1)$ and $x_n \notin C(S_1)$, and $x_1 \in C(S_1)$, by definition of P $x_1 P x_n$. This fails the condition that $x_n R x_{n-1} R \ldots R x_2 R x_1$ implies $\neg(x_1 P x_n)$. $\square$

Proposition 4 implies that we can take the transitive closure of R, R_T, without creating any conflict. We then show that any completion of R_T will represent C by choosing an appropriate Γ satisfying the Competition Filter property. The following theorem shows that CLC behavior when allowing for choice correspondences is completely characterized by the two axioms NCCR and WRI.

Theorem 3. *A choice correspondence C is a CLC if and only if C satisfies NCCR and WRI.*

Proof. ($\Rightarrow$) First we show necessity of the two axioms. Let C be a CLC represented by $(\succsim, \Gamma)$, where Γ is a competition filter.

To prove NCCR, let $T_i \subseteq S_i$ be a set of menus such that $C(S_i) \cap C(T_{i+1}) \neq \emptyset$. Without loss let $C(S_1) \cap C(T_1) = \emptyset$. Let $x_i \in C(S_i)$ and $y_i \in C(T_i)$ be elements of the respective choice sets.

Since C is a CLC, the information $C(S_1) \cap C(T_1) = \emptyset$, $C(S_i) \cap C(T_{i+1}) \neq \emptyset$, and $C(S_i) \cap C(T_{i+1}) \neq \emptyset$ tells us $x_i, y_i \in \Gamma(T_i)$ for all i and therefore we can conclude

$$\begin{aligned} y_1 &\succ x_1, \\ y_i &\succsim x_i \ \forall i \\ x_{i+1} &\sim y_i \ \forall i \end{aligned}$$

Therefore we have $y_n \succsim x_n \sim y_{n-1} \succsim \cdots \succsim y_2 \succsim x_2 \sim y_1 \succ x_1$. Since $\succsim$ is a weak order, we must have $y_n \succ x_1$. For any $z \in C(S_n)$, $z \sim y_n$ since C is a CLC. So $z \succ x_1$ and $z \succ w$ for any $w \in C(T_1)$. This implies that for all $w \in C(T_1)$, $w \notin C(S_n)$. Similarly for any $z \in C(T_1)$, $z \sim x_1$ and since C is a CLC represented by $(\succsim, \Gamma)$, $w \succ z$ for all $w \in C(S_n)$, and we get $z \notin C(T_1)$ since c is a CLC. Therefore $C(S_n) \cap C(T_1) = \emptyset$.

Now we prove the necessity of WRI. Let c be a CLC represented by $(\succsim, \Gamma)$. Suppose $x, y \in C(S)$ for some $\{x, y\} \subseteq T \subset S$. Then $x, y \in \Gamma(S)$ and since Γ is a competition filter, $x, y \in \Gamma(T)$. Given that $x, y \in \Gamma(S) \cap \mathcal{C}(S)$, and there is a weak order $\succsim$ such that $C(S) = \max_{\succsim} \Gamma(S)$ for all S, we must have $x \sim y$. Let $x \in C(T)$, then for all $z \in \Gamma(T)$, $x \succsim z$. By transitivity $y \succsim z$ for all $z \in \Gamma(T)$, since $x \sim y$ and $y \in \Gamma(T)$. Therefore $y \in C(T)$. Which means that c satisfies **WRI**.

($\Leftarrow$) By Proposition 4, the transitive closure of R_T is well-defined. Let $\succsim$ be any completion of R_T. Define

$$\Gamma^m(S) = \{x \in S |\ x \in C(T) \text{ for some } T \supseteq S\}$$

We now show that $(\Gamma^m, \succsim)$ represents C.

First let $x \in C(S)$, we want to show that x is $\succsim$-maximal in $\Gamma^m(S)$. x belongs to $\Gamma^m(S)$ by construction. Let $y \in \Gamma^m(S)$, then $y \in C(T)$ for some $T \supseteq S$, therefore xRy by definition of R. Since $R_T \subseteq \succsim$, $x \succsim y$ follows by construction. Therefore, x is $\succsim$-maximal in $\Gamma^m(S)$.

Now let $x \notin C(S)$. To obtain a contradiction, assume $x \in \Gamma^m(S)$. This implies $x \in C(T)$ for some $T \supseteq S$. Let z be in $C(S)$. Hence, we have zPx, so $z \succ x$. This means that x is not $\succsim$-maximal in $\Gamma^m(S)$. Therefore, we can conclude that $(\Gamma^m, \succsim)$ represents c. □

Similarly to the definition of revealed preference for choice functions, we now define revealed (strict) preference and revealed indifference for choice correspondences.

Definition. *Let C be a CLC correspondence and that there are k different attention filter, weak orders representing C;*

$$(\Gamma_1, \succsim_1), (\Gamma_2, \succsim_2), \ldots, (\Gamma_k, \succsim_k)$$

1. *x is **revealed preferred** to y if $x \succsim_i y$ for all i.*
    - *x is (strictly) **revealed preferred** to y if $x \succ_i y$ for all i.*

- *x is **revealed indifferent** to y if $x \sim_i y$ for all i.*

The next proposition states the revealed preference result for choice correspondences.

Proposition 5. *Let $(\Gamma, \succsim)$ represent c. Let R_T be the transitive closure of R. Let C be a CLC correspondence. Then x is revealed preferred to y ($x \succsim_R y$), if and only if $x R_T y$.*

Proof. ($\Rightarrow$) See the proof of Theorem 3.

($\Leftarrow$) Let xR_Ty, then $z_1, \ldots, z_n$ such that $x = z_1 R z_2 R \ldots R z_n = y$ (possibly $z_2 = y$, in which case xRy). For any $z_i R z_{i+1}$, there exists $\{z_i, z_{i+1}\} \subseteq S_i \subseteq T_i$ such that $z_i \in C(S_i)$ and $z_{i+1} \in C(T_i)$. Since Γ_i is an attention filter, $z_i, z_{i+1} \in \Gamma(S_i)$; and $z_i \in C(S_i)$ implies that z_i is $\succsim -$maximal in $\Gamma(S')$, i.e. $z_i \succsim z_{i+1}$ because C is a CLC correspondence. Therefore $x = z_1 \succsim z_2 \cdots \succsim z_n = y$, and by transitivity of $\succsim$, $x \succsim y$ follows. Therefore $R_T \subseteq \succsim$. □

Corollary 1. *Let I_T and P_T be the symmetric and asymmetric components of R_T respectively.*

(i) x is revealed indifferent to y if and only if xI_Ty

(ii) x is (strictly) revealed preferred to y if and only if xP_Ty.

A.2 Proofs

The Proof of Theorem 2

Define $xP''y$ if and only if there exist T and T' with $x, y \in T \subset T'$ such that

$$x = c(T) \text{ and } c(T') \neq c(T' \setminus y)$$

Lemma 4. *P'' is acyclic if and only if c satisfies LC-WARP*.*

The proof of Lemma 4 is completely analogous to earlier Lemma, so we skip it.

Let P''_R be the transitive closure of P'' and let $\succ$ be any arbitrary completion of P''_R. For every S, we call $B \subset S$ is a minimum block of S if and only if $c(S) \neq c(S \setminus B)$ but $c(S) = c(S \setminus B')$ for any $B' \subsetneq B$. Given this, define Γ recursively as follows:

1. $\Gamma(X)$ consists of the $\succ$-worst element of each of X's minimum block.
2. Suppose Γ has been already defined for all proper supersets of S. Then, define $\Gamma(S)$

   (a) First, put $x \in S$ into $\Gamma(S)$ if $x \in \Gamma(T)$ for some $T \supsetneq S$.

(b) If there is a minimum block of S that does not have an element in $\Gamma(S)$ according to the above, pick the $\succ$-worst element into $\Gamma(S)$.

Lemma 5. *For any S,*

(i) $\{c(S)\}$ *is a minimum block of S. There is no other minimum block that includes $c(S)$.*

(ii) *If B is a minimum block of S other than $\{c(S)\}$, then $c(S) \succ x$ for all $x \in B$.*

(iii) *If $c(T) \neq c(S)$ and $T \supsetneq S$, then T has a minimum block that is a subset of $T \setminus S$.*

Proof. Part (i) and (iii) are obvious so only prove Part (ii). Let $B' = B \setminus x$ (it may be empty). Then we have

$$c(S) = c(S \setminus B') \neq c((S \setminus B') \setminus x)$$

Therefore, we have $c(S)P''x$ so it must be $c(S) \succ x$. □

Claim 1. Γ *is a competition filter satisfying IOA.*

Proof. Γ is a competition filter by construction so we shall prove that Γ satisfies IOA. Suppose $x, y \in S$, $x, y \notin \Gamma(S)$, but $y \in \Gamma(S \setminus x)$. Then there exists $T \supset S$ such that (i) $T \setminus x$ has a minimum block B and y is the worst element in B and (ii) none of elements in B is included in $\Gamma(T')$ for any $T' \supsetneq T \setminus x$.

Then, we must have $c(T) = c(T \setminus x)$. Otherwise $\{x\}$ is a minimum block of T' so we have $x \in \Gamma(T')$ that implies $x \in \Gamma(S)$. Therefore, we have

$$c(T) = c(T \setminus x) \neq c((T \setminus x) \setminus B) = c(T \setminus (\{x\} \cup B))$$

Therefore, by Lemma 5 (iii), T has a minimum block that is a subset of $x \cup B$ so at least one element in $x \cup B$ must be in $\Gamma(T)$, which is a contradiction. □

Now we want to show that $(\succ, \Gamma)$ represents c. Since Lemma 5 (i) implies that $c(S) \in \Gamma(S)$, all we need to show is that $c(S) \succ y$ for all $y \in \Gamma(S) \setminus c(S)$.

Claim 2. *If $y \in \Gamma(S)$ and $y \neq c(S)$, then $c(S) \succ y$.*

Proof. Since $y \in \Gamma(S)$, there exists $T \supset S$ such that $y \in \Gamma(T)$. Furthermore, T has a minimum block B where y is the worst element and none of elements in B is in $\Gamma(T')$ for any $T' \supsetneq T$. There are three easy cases: (i) if $c(S) = c(T)$ then by Lemma 5 (ii) we have $c(S) = c(T) \succ y$, (ii) if $y = c(T)$ then we have $c(S)P''y$ so it must be $c(S) \succ y$, and finally (iii) if $c(S) \in B$, then $c(S) \succ y$ by the construction. Therefore, we only need investigate the case when $y \neq c(T) \neq c(S)$ and $c(S) \notin B$. Note that $c(T) \succ y$ in this case by Lemma 5 (ii).

Now let $S' = S \setminus B$. Since $y \in B$, S' is a proper subset of S.

Case I: $c(S'') \neq c(S)$ for some S'' where $S' \subset S'' \subset S$.

By Lemma 5 (iii), S has a minimum block B' that is a subset of $S \setminus S'' \subset B$. Since $c(S) \notin B'(\subset B)$, every element in B' is worse than $c(S)$ by Lemma 5 (ii). Since y is the worst element in B that is a superset of B', we conclude $c(S) \succ y$.

Case II: $c(S'') = c(S)$ for all S'' where $S' \subset S'' \subset S$.

Since $y \neq c(T) = c(T \setminus (B \setminus y)) \neq c(T \setminus B)$, and $c(S \setminus (B \setminus y)) \in T \setminus (B \setminus y)$, we have $c(S \setminus (B \setminus y))\ P''\ y$. Therefore, $c(S) \succ y$ because of $c(S \setminus (B \setminus y)) = c(S)$. □