Understanding the reference effect

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This paper explores how a change in a default—specifically, an exogenously given reference point—affects individual preferences. While reference dependence is extensively studied, very little is known regarding the impact of reference points on individual choice behavior when the reference points themselves are not chosen (Reference Effect). We identify critical properties that differentiate between classes of reference-dependent models and test them. We find that the reference effect exists for asymmetrically dominated reference points, but we do not see any evidence of a reference effect for symmetrically dominated reference points. Some of the existing models are mostly consistent with our data but lack predictive power. None of the models offers the particular predictions that our experiment suggests. Finally, we also tease apart the differences between the reference effect and the asymmetric dominance effect (decoy effect), a well-known phenomenon observed in the literature on context-dependent choice.

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1. Introduction

Many experimental and field studies have shown that preferences are influenced by reference points—a pattern called reference dependence.1 For example, default options have a strong impact on employees' contributions and asset allocations in their 401(k) retirement plans (Madrian and Shea, 2001; Choi et al., 2002; Beshears et al., 2006). Under automatic enrollment, a large majority of employees accept both the low default savings rates and the conservative investment funds (Status Quo Bias). In this paper, we focus on exogenously given reference points such as defaults and study situations in which reference points alter one's choices even when agents do not stick to the reference point itself, which we call the Reference Effect. We provide a laboratory experiment that investigates the reference effect and allows us to contrast the loss aversion models of Tversky and Kahneman (1991) with the status quo constraint models of Masatlioglu and Ok (2005, 2007, forthcoming).

The reference effect was first described by Tversky and Kahneman (1991). Consider the following set of consumption bundles: \{x, y, r, s\}. Each option is a bundle of two goods. Suppose while r is dominated by x but not y, s is dominated by y but not x. Tversky and Kahneman find that x is more likely to be preferred when the reference point is r than when the

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reference point is $s$. However, very little is known regarding how the reference effect operates. In this paper, we study to what extent the position of a reference point matters for individual decision-making when an individual is making a choice between two options and a dominated status quo (reference point). We investigate two different situations: i) only one of the options dominates the reference point (an asymmetrically dominated reference point), and ii) both options dominate the reference point (a symmetrically dominated reference point).

Tversky and Kahneman (1991) and Masatlioglu and Ok (2005, 2007, forthcoming) have proposed different models to capture reference-dependent behaviors. Even though these models were initially constructed to explain the same choice anomalies—endowment effect and status quo bias, they differ with respect to their basic structure and the methodology with which they are derived. To increase our understanding of not only these models but also reference dependence in general, the first part of the paper is devoted to the construction of a framework in which it is possible to study these models in a unified manner. We discuss several behavioral properties suggested by Tversky and Kahneman (1991) and Masatlioglu and Ok (2005) and study whether these properties are satisfied by each model. This allows us to understand these models better and to design an experiment that can distinguish among them. We show that in addition to the status quo bias and the endowment effect, these models also imply that reference points alter choices even though agents do not choose the reference point itself. Surprisingly, we show that even in a simple choice environment their predictions regarding the reference effect differ.

In the second part of the paper, we design a laboratory experiment to study the reference effect. In our experiment, we elicit the preferences of each participant over two consumption bundles with and without reference points. Reference points are always dominated by at least one of the bundles. Hence, we do not expect reference points to be chosen. Instead, we investigate whether these reference points affect individual choices. Our experiment provides a rich environment to test which behavioral properties proposed by previous theoretical models are important to understand reference-dependent behavior and which properties are redundant. Our design also allows us to empirically distinguish between the models of Tversky and Kahneman (1991) and Masatlioglu and Ok (2005, 2007, forthcoming). We document whether a subject’s choice varies as the reference point changes, and we ask whether the observed variation is consistent with any of the models.

Our main finding is that individuals change their choices significantly when the location of the reference point changes. However, this effect is only significant for the asymmetrically dominated reference points. We do not find any evidence of the reference effect for symmetrically dominated reference points. In addition, we demonstrate that the general loss aversion model of Tversky and Kahneman (1991) correctly predicts the reference effect for asymmetrically dominated reference points. However, it fails to predict the absence of a reference effect in the symmetrically dominated region. We illustrate that the model of Masatlioglu and Ok (forthcoming) predicts no reference effect for symmetrically dominated reference points but does not make a particular prediction for asymmetrically dominated reference points.

The paper is organized as follows. In Section 2, we describe the reference-dependent choice models in more detail, and discuss some of the predictions of the models that we make use of in our subsequent experiments. In Section 3, we explain the experimental design and procedures that we followed. We present our experimental results in Sections 4 and 5. In Section 6, we tease apart the differences between the reference effect and the asymmetric dominance effect (decoy effect), a well-known phenomenon observed in the literature on context-dependent choice. The literature review and discussion is presented in Section 7. Section 8 concludes the paper.

2. Theoretical analysis

We review the loss aversion models of Tversky and Kahneman (1991) and the status quo constraint models of Masatlioglu and Ok (2005, 2007, forthcoming). Since not only their basic structures but also their environments are different, we first provide a simple unified framework. In particular, we consider an environment with two-dimensional consumption bundles and make necessary adaptations for each theory so we can study them within the same framework.

2.1. Reference-dependent models

2.1.1. Loss aversion models

Tversky and Kahneman (1991) provide a behavioral model that extends Prospect Theory to the case of riskless consumption bundles. We denote a generic element of two-attribute consumption bundles by $x = (x_1, x_2)$. Tversky and Kahneman (1991) assume that for each reference point, $r = (r_1, r_2)$, the decision maker has a particular preference relation, $\succsim_r$, represented by a utility function, $U_r : \mathbb{R}_+^2 \to \mathbb{R}$. $U_r$ represents the utility of $x$ when it is evaluated relative to reference point $r$.

Tversky and Kahneman note that the reference point usually corresponds to the decision maker’s current position. In addition to reference dependence, Tversky and Kahneman (1991) assume that preferences have the following properties: (i) losses loom larger than gains (loss aversion), and (ii) marginal values of both gains and losses decrease with their distance from the reference point (diminishing sensitivity). In what follows, we refer to this as the loss aversion (LA) model.

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2 There are other reference-dependent models proposed by Munro and Sugden (2003), Sugden (2003) and Sagi (2006). These models either are not applicable to consumption bundles or do not make any particular predictions when adapted to consumption bundles.

3 In our experiment, the decision maker’s current position will be determined by the default option. However, this model is flexible enough to accommodate reference points that are influenced by aspirations, expectations, norms, and social comparisons.
To demonstrate loss aversion and diminishing sensitivity in terms of functional properties, we consider the following, widely used formulation:

$$U_r(x) = g_1(u_1(x_1) - u_1(r_1)) + g_2(u_2(x_2) - u_2(r_2)).$$  \(1\)

where \(u_i\) is the strictly increasing utility function for the \(i\)th attribute and \(g_i\) is a strictly increasing and continuous function on \(\mathbb{R}\) that acts as the value function for the \(i\)th attribute, \(i = 1, 2\). The functional form exhibited in \(1\) allows one to endow various properties of \(g_i\) with nice interpretations. The assumptions \(g_i(0) = 0\) and \(g_i(a) < -g_i(-a)\), for each \(a > 0\), capture the phenomenon of loss aversion.\(^4\) As in Prospect Theory, this model posits that each value function is \(S\)-shaped and is steeper in the case of losses than it is in the case of gains. Concavity of \(g_i(a)\) for \(a > 0\) implies diminishing sensitivity for gains, and convexity of \(g_i(a)\) for \(a < 0\) implies diminishing sensitivity for losses.

Tversky and Kahneman (1991) also propose a more restrictive version of \(1\) in which both \(g_i|_{\mathbb{R}_+}\) and \(g_i|_{\mathbb{R}_-}\) are assumed to be linear functions. We call this model the additive constant loss aversion (CLA) model. In this restrictive model, the linearity of \(g_i\) implies that the sensitivity to a given gain or loss on dimension \(i\) does not depend on whether the reference bundle is distant or near in that dimension.\(^5\)

Since many real life decisions do not include a status quo, a comprehensive decision-making model should also be able to make predictions when there is no reference point. Although Tversky and Kahneman (1991) do not discuss this possibility, in practice the LA and CLA models are commonly adapted to cover such situations by setting \(r = (0, 0)\) and \(u_i(0) = 0\), \(i = 1, 2\), a convention that we shall adopt here as well.\(^6\)

Fig. 1 illustrates how a reference point affects preferences of a decision maker in the LA and CLA models.\(^7\) Note that the indifference curves for both models are kinked at the consumption bundles, along both the vertical and horizontal lines passing through reference point \(r\).

Köszegi and Rabin (2006) extend the loss aversion model by assuming that individuals evaluate alternatives not only by comparing them with a reference point but also by considering the outcomes themselves.\(^8\) Formally, Köszegi and Rabin propose the following form of reference-dependent utility function:

$$U_r(x) = \sum_i u_i(x_i) + \sum_i g_i(u_i(x_i) - u_i(r_i)),$$

where the first component represents the consumption utility and the second component is the gain–loss utility. Even though this functional form is different than the LA model, the effect of moving the reference point on the relative ranking of two alternatives is the same for both of these models. Hence for convenience we only focus on the LA model in this paper.

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\(^4\) This definition was suggested by Wakker and Tversky (1993).

\(^5\) Note that each \(u_i\) could still be a concave function to capture the standard assumption of diminishing marginal utility. Hence, diminishing sensitivity and diminishing marginal utility assumptions are logically independent.

\(^6\) This assumption is made so that the models we compare apply to the same type of decision problems. However, none of our theoretical and experimental results relies on this assumption.

\(^7\) Fig. 1(a) is based on Eq. \(1\) where \(u_i(b) = b^\alpha\), \(g_i(a) = a^\beta\) when \(a \geq 0\), \(g_i(a) = -\lambda(-a)^\beta\) when \(a < 0\) for \(i = 1, 2\) and \(\lambda > 1 > \alpha > 0, \beta > 0\). In Fig. 1(b), \(g_1\) and \(g_2\) are taken to be linear.

\(^8\) Another innovation of this paper is to introduce a theory of endogenous reference points. Since in our paper we consider exogenously given reference points, this feature of the model is irrelevant.
2.1.2. Status quo constraint models

Reference dependence has been commonly treated as a violation of rationality in the literature on consumer choice. Recently, however, Masatlioglu and Ok (2005, 2007, forthcoming) have brought a new perspective on modeling reference dependence. In these models, as opposed to the earlier approach, status quo does not affect the underlying utility function of the agent but instead imposes a psychological constraint on what the decision maker can choose.

The model of Masatlioglu and Ok (2005)—hereafter referred to as the Status Quo Bias (SQB) model—is derived through behavioral axioms, in the tradition of the classical revealed preference theory. In particular, the primary descriptive axiom of Masatlioglu and Ok (2005) is “status quo bias,” which says that an alternative is more likely to be chosen when it is the default option of a decision maker. While the SQB model applies in any arbitrary finite choice space $X$, one can easily extend this model to uncountable spaces (such as $\mathbb{R}^2$) by imposing an appropriate continuity assumption. For comparison purposes, here the set of all alternatives is assumed to be $\mathbb{R}^2$.

There are two main components of this model: (i) the reference point imposes a mental constraint, and hence, some alternatives are discarded by the decision maker; (ii) the final decision is made according to a reference-independent utility function from surviving alternatives.

One of the interpretations of the model is that the decision maker has multiple selves, say two selves, 1 and 2. While self 1 values the first attribute more than the second one, self 2 puts more weight on the second attribute relative to the first one. Each self has a utility function (say $u_1$ and $u_2$). The decision maker sticks to her status quo unless both selves are willing to move away from it. Put differently, if an alternative, say $x$, provides higher utilities for both self 1 and self 2, then both selves agree that $x$ is better than the status quo. Consequently, we define the set $Q(r)$ as the set of alternatives that are preferred by both selves when $r$ is the reference point. Formally,

$$Q(r) = \{ x \in \mathbb{R}^2 \mid u_1(x) \geq u_1(r) \text{ and } u_2(x) \geq u_2(r) \}. \quad (2)$$

If there are multiple feasible alternatives belonging to this set, the decision maker maximizes an aggregation of these utility functions of the two selves, $U = w_1u_1 + w_2u_2$, where $w_i$ is the weight assigned to the utility of self $i$. This is the reference-independent utility function. If there is no reference point, the decision maker solves her problem by maximizing her utility function over all feasible alternatives.

Fig. 2 demonstrates the SQB model. The shaded region containing the reference point represents $Q(r)$. Since the material preference is not affected by reference points, in this model the thick indifference curves (with reference point) coincide with the thin indifference curves (without reference point). Thick indifference curves, however, are only defined within the mental constraint set, $Q$, since any point outside of this set is dominated by the reference point.

Essentially, in this model, the decision maker maximizes her utility function subject to a psychological constraint imposed by the status quo. In particular, agents solve the following maximization problem given budget set $B$:

$$\begin{align*}
\max & \quad w_1u_1(x) + w_2u_2(x) \\
\text{s.t. } & x \in B \cap Q(r),
\end{align*} \quad (3)$$

where $Q(r) = \{ x \in \mathbb{R}^2 \mid u_1(x) \geq u_1(r) \text{ and } u_2(x) \geq u_2(r) \}$.

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9 See Tapki (2007), Kawai (2008), Houy (2007) for extensions and variations of this model.
10 Assume the slope of indifference curves of $u_1$ is strictly greater than the slope of indifference curves of $u_2$.
11 Since $u_i$ is increasing, all alternatives dominating the reference point in every dimension belong to this set.
12 Here, the linear weighting function is not crucial as long as $U$ is an increasing function of $(u_1, u_2)$.
The formulation above elucidates that the SQB model is a “constrained utility maximization model,” where the constraint set, \( Q \), is induced by one’s reference point. The mental constraint set consists of alternatives for which the decision maker is willing to abandon her status quo \( r \).

The mental constraint set satisfies the following properties: (i) strict improvement: if someone prefers an alternative to the status quo, such an alternative must yield strictly higher utility to the agent (if \( x \in Q^1(r) \setminus \{ r \} \), then \( U(r) < U(x) \)), (ii) attainability of status quo: the status quo is always feasible, i.e., \( r \in Q(r) \), and (iii) monotonicity: if \( r' \) is revealed to be better than the status quo \( r \), then any element that is preferred to \( r' \) when \( r' \) is the status quo must be revealed to be better than \( r \) when \( r \) is the status quo.\(^{14}\)

Unlike loss aversion models, the SQB model allows us to construct a ranking of alternatives based on unambiguous comparisons that can be used to carry out a meaningful welfare analysis. Since the SQB model accommodates behavioral anomalies, we cannot summarize status quo bias choices with a consistent preference ordering as in classical choice theory. Nevertheless, the existence of a transitive (not necessarily complete) ranking is guaranteed by the SQB model for welfare comparison purposes.\(^{14}\)

Masatlioglu and Ok (2007, forthcoming) have recently provided two independent extensions of the SQB model. We refer to the model in Masatlioglu and Ok (forthcoming) as the general SQB (GSQB) model and the model in Masatlioglu and Ok (2007) as the procedural reference-dependent choice (PRD) model. We will discuss them in turn.\(^{15}\)

The GSQB model operates similarly to the SQB model, except that the mental constraint set might not have the multiplicity interpretation. Formally, the decision maker solves the following maximization problem given budget set \( B \):

\[
\max \ U(x) \\
\text{s.t. } x \in B \cap Q(r). \quad (4)
\]

The GSQB model satisfies (i) weak improvement: if \( x \in Q(r) \setminus \{ r \} \), then \( U(r) \leq U(x) \) and (ii) attainability of status quo. In addition, since the choice space is endowed with an order structure, as in \( \mathbb{R}^2 \), it is natural to assume that an alternative that dominates another alternative in every dimension should have higher utility value. Moreover, if an alternative dominates the reference point in every dimension, it is unambiguously better than the reference point; hence, this alternative should be in the mental constraint set. That is, \( \{ x \in \mathbb{R}^2 \mid x_1 \geq r_1 \text{ and } x_2 \geq r_2 \} \subseteq Q(r) \).

The important difference between the GSQB model and the SQB model is that the mental constraint set does not satisfy the monotonicity property \( (r' \in Q(r) \text{ implies } Q(r') \subset Q(r) \text{). After discarding this condition, the GSQB model enjoys more explanatory power than the SQB model.}\)

The PRD model also resembles the SQB model, yet it functions through two nested mental constraint sets \( Q^1(r) \) and \( Q^2(r) \) for a given reference point, \( r \), such that \( Q^1(r) \subset Q^2(r) \). In the elimination stage, either \( Q^1 \) or \( Q^2 \) will be utilized and \( Q^1 \) has a priority. If \( B \cap Q^1(r) \neq \{ r \} \), then she proceeds to the optimization stage, as in the SQB model. If there is no common element in \( B \) and \( Q^1(r) \) other than \( r \), then the decision maker relaxes her mental constraint set to a larger set \( Q^2(r) \) and employs \( Q^2(r) \) in the elimination stage. Therefore, at the optimization stage, if \( B \cap Q^1(r) \neq \{ r \} \), the decision maker settles her problem by selecting alternatives in \( B \cap Q^1(r) \); otherwise, she selects from \( B \cap Q^2(r) \). More formally, the optimization problem of the decision maker in the PRD model can be written as follows:

\[
\max \ U(x) \\
\text{s.t. } \begin{cases} x \in B \cap Q^1(r) & \text{if } B \cap Q^1(r) \neq \{ r \} \\ x \in B \cap Q^2(r) & \text{otherwise.} \end{cases} \quad (5)
\]

One can think of \( Q^1(r) \) as the set of alternatives that dominate the reference point, \( r \), in an obvious way: no cognitive effort is needed to figure out the domination. In contrast, \( Q^2(r) \) consists of those alternatives where the dominance over \( r \) is less straightforward: at least some cognitive work on the part of the individual is needed to discern this dominance. Both \( Q^1 \) and \( Q^2 \) satisfy the three properties of the SQB model, and \( Q^1(r) \subset \{ x \in \mathbb{R}^2 \mid x_1 \geq r_1 \text{ and } x_2 \geq r_2 \} \subseteq Q^2(r) \).\(^{16}\)

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\(^{13}\) This property prevents potential cycles: if \( x \) is chosen in some budget set when \( y \) is the status quo, \( U(y) < U(x) \), and \( z \) is chosen in some budget set when \( x \) is the status quo, \( U(z) < U(x) \), then it is impossible that \( y \) is chosen in any budget set when \( z \) is the status quo. Indeed, this argument also works for weak improvements.

\(^{14}\) To see this, define a “welfare” ranking in the following way: \( x \) is “strictly better” than \( y \) (\( x > y \)) if and only if \( x \) is uniquely chosen from \( \{ x, y \} \) when \( y \) is the status quo. The SQB model implies that there is no alternative \( z \) such that \( y \) is chosen from \( \{ x, y, z \} \) when \( z \) is the status quo. Hence, independent of status quo, \( x > y \) entails that whenever \( x \) and \( y \) are available, either \( x \) is chosen but not \( y \) or neither are chosen. Property (iii) guarantees that \( > \) is transitive. Therefore, \( > \) can be used for welfare comparisons.

\(^{15}\) Dean (2008) challenges both loss aversion and status quo constraint models. Dean finds that status quo is more frequently chosen when the choice set gets larger (decision avoidance), which cannot be explained by these models. However, Dean also shows that these models are needed to understand behavior in small choice sets.

\(^{16}\) Although the original model does not impose these requirements, given the interpretation of \( Q^1 \) and \( Q^2 \), we find these assumptions to be reasonable when the domain of choices is the set of two-dimensional consumption bundles.
2.2. Behavioral properties

So far we have reviewed two classes of reference-dependent choice models in a unified manner and have discussed their natures. The previous section makes apparent how models within each group relate to one another. We have yet to show how models in the two different classes are related. To build these connections, we state several properties suggested by Tversky and Kahneman (1991) and Masatlioglu and Ok (2005, 2007) and study whether these properties are satisfied by each model. This will allow us not only to understand these models better but also to design an experiment that can distinguish among them.

Before we state the properties, we introduce some notation. Throughout the paper, \((S, x)\) denotes a choice problem with a status quo, where the set of available options are \(S\) and the status quo is \(x \in S\). If there is no status quo, we use the notation \((S, \circ)\) to represent the corresponding choice problem. The choice behavior is denoted by \(c\), which is a mapping from all choice problems to the set of alternatives.

The first property, Status Quo Bias, is a simple formulation of experimental findings on reference-dependent choice. The property says that if a decision maker chooses \(x\) among \([x, y]\) when there is no reference point, then \(x\) must be chosen when it is the reference point.

**Status Quo Bias Property (SQBP):** If \(x \in c([x, y], \circ)\) then \(x \in c([x, y], x)\).

Given the large literature on status quo bias, this is a very weak limitation to satisfy since it only requires the relative ranking of the status quo point to be at least as good as the case where it is not labeled as the status quo. Among others, Knegtsch (1989) provides strong evidence in favor of the SQBP. The experiment involves two treatments with two options, a mug and a candy bar. In the first treatment, subjects are equally split between selecting these two options. In the second treatment, one of the options is given as an initial endowment and subjects have the opportunity to exchange their endowment for the other good. Here, the vast majority of subjects (around 90 percent) keep their endowments regardless of whether the endowment is a mug or a candy bar.

In order to define the rest of the properties, we need a multi-dimensional commodity space. For simplicity, we assume two dimensions. First, we fix two consumption bundles denoted by \(x\) and \(y\), which are positioned so that neither dominates the other, i.e., each has a superior dimension. The following properties concern how choices are affected when the location of the reference point shifts along one dimension. Reference points are always dominated by either \(x\) or \(y\) or both.

The following properties utilize four pairs of reference points: \(a - a', s - s', l - l', \) and \(l' - l''\). The \(a - a'\) pair is dominated by \(y\) but not by \(x\), hence an asymmetrically dominated pair. The \(s - s'\) pair is dominated by both \(y\) and \(x\), hence a symmetrically dominated pair. Finally, the \(l - l'\) and \(l' - l''\) pairs are dominated by \(x\) but not by \(y\) (Fig. 3). For simplicity, we will state all the properties in terms of horizontal shifts. Readers should keep in mind that each property also applies for the vertical shifts, which can be obtained by interchanging the subscripts 1 and 2 throughout.

The second property we introduce is “Weak Loss Aversion Property”.

**Weak Loss Aversion Property (WLAP):** Suppose \(x_1 \geq l'_1 > l_1 = y_1\) and \(y_2 > l_2 = l'_2\). Then \(x \in c([x, y], l, l)\) implies \(x \in c([x, y], l', l')\).

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17 When the choice is unique, we abuse notation and write \(x = c(S, r)\) instead of \([x] = c(S, r)\).

18 A similar property is defined by Tversky and Kahneman (1991) under Loss Aversion. We prefer to call this Weak Loss Aversion since it applies to a boundary case where the shift introduces a loss for \(y\).
This property captures the basic intuition that “losses looming larger than gains.” Consider a shift in the reference point from \( I \) to \( I' \) (Fig. 3). Here, \( x \)'s gain with respect to the reference point is decreased by exactly the same amount as \( y \)'s loss is increased. Since losses are weighted more heavily than gains, the relative attractiveness of \( y \) is diminished. Hence, the effect of the reference shift must be positive for \( x \). In sum, if \( x \) is chosen initially, it must be chosen when \( I' \) is the reference point. This property does not rule out the possibility that \( y \) is initially chosen and then \( x \) is chosen after the shift. It rules out the case where \( x \) is initially chosen but \( y \) is uniquely chosen after the shift: \( x \in c((x, y, l), l) \) and \( y = c((x, y, I'), I') \).

Notice that \( y \) incurs no losses when the reference point is \( I \). Hence, the \( I \rightarrow I' \) shift introduces a loss for \( y \). The Strong Loss Aversion Property considers \( I \rightarrow I' \) shifts where choosing \( y \) at the initial reference point implies some losses, and the reference shift induces additional losses. The requirement that \( l_1 = y_1 \) is replaced by \( l_2' > y_1 \), but the same intuition applies.

**Strong Loss Aversion Property (SLAP):** Suppose \( x_1 \geq l_1' > l_1 > y_1 \) and \( y_2 > x_2 > l_2' = l_2 \). Then \( x \in c((x, y, I'), I') \) implies \( x \in c((x, y, I'), I') \).

The fourth property, Irrelevance of Symmetrically Dominated Reference Points, considers cases where reference points \( s \) and \( s' \) are dominated by both \( x \) and \( y \). The property says that individual choice should not be affected by a shift in the reference point from \( s \) to \( s' \). Indeed, any shift in this region—including a shift from the origin—does not have any impact on choices. This property captures the idea that a reference shift has an effect on choices only when it makes one of the alternatives more desirable or salient than the other alternative.

**Irrelevance of Symmetrically Dominated Reference Points (ISD):** If \( x_1 > y_1 \geq s_1' > s_1 \) and \( y_2 > x_2 > s_2 = s_2' \), then \( c((x, y, s), s) = c((x, y, s'), s') \).

The next property, Irrelevance of Asymmetrically Dominated Reference Points, is similar to ISD but applies to asymmetrically dominated reference points. That is, both reference points, \( a \) and \( a' \), are dominated by \( y \), but not \( x \) (Fig. 3). The property implies that the choices are not affected by moving the reference point from \( a \) to \( a' \). Note that while the location of asymmetrically dominated reference points does not matter, this property allows choices to be affected by the existence of asymmetrically dominated reference points.

**Irrelevance of Asymmetrically Dominated Reference Points (IAD):** If \( x_1 > y_1 \geq a_1' > a_1 \) and \( y_2 > a_2 = a_2' > x_2 \), then \( c((x, y, a), a) = c((x, y, a'), a') \).

The combination of ISD and IAD is dubbed “constant sensitivity” by Tversky and Kahneman (1991). These two properties imply that there is no choice reversal for the \( a \rightarrow a' \) and \( s \rightarrow s' \) shifts. Hence, they have high predictive power. The next two properties replace constant sensitivity with diminishing sensitivity and allow a certain choice reversal.

**Diminishing Sensitivity for Symmetrically Dominated Reference Points (DSS):** If \( x_1 > y_1 \geq s_1' > s_1 \) and \( y_2 > x_2 > s_2 = s_2' \), then \( x \in c((x, y, s), s) \) implies \( x \in c((x, y, s'), s') \).

**Diminishing Sensitivity for Asymmetrically Dominated Reference Points (DSA):** If \( x_1 > y_1 \geq a_1' > a_1 \) and \( y_2 > a_2 = a_2' > x_2 \), then \( x \in c((x, y, a), a) \) implies \( x \in c((x, y, a'), a') \).

These two properties are conceptually similar to the well-known phenomenon of diminishing marginal utility. Tversky and Kahneman (1991) state that “The marginal value decreases with the distance from the reference point.” For instance, the marginal value of $1000 in addition to a salary of $60,000 is smaller when the current salary is $40,000 than when it is $55,000. Nevertheless, they are logically different from diminishing marginal utility. The combination of DSS and DSA is coined “diminishing sensitivity for gains” by Tversky and Kahneman (1991). These two properties rule out cases where \( x \) is initially chosen at reference point \( r \) and then \( y \) is chosen at reference point \( r' \) where \( (r, r') \in \{(a, a'), (s, s')\} \). Choice reversal in the opposite direction is allowed: \( y \) is chosen from \( (x, y, r) \) at \( r \) and \( x \) is chosen from \( (x, y, r') \) at \( r' \).

Propositions 1 and 2 provide our main theoretical results. Proofs of the propositions are provided in Appendix A.

**Proposition 1.** Let the domain be a two-dimensional commodity space. Then

(i) The CLA model satisfies SQBP, WLAP, SLAP, ISD, and IAD,
(ii) The LA model satisfies WLAP, DSS and DSA.

This proposition makes it clear that the CLA model enjoys much higher prediction power than the LA model. The LA model is not consistent with the predictions of SQBP, SLAP, ISD and IAD.

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Both Tversky and Kahneman (1991) and Masatlioglu and Uler (forthcoming) provide a similar property under the name “sign dependence” and “status quo irrelevance”, respectively.
Table 1
Properties of reference-dependent models under consideration.

<table>
<thead>
<tr>
<th>Tversky–Kahneman</th>
<th>Masatlioglu–Ok</th>
</tr>
</thead>
<tbody>
<tr>
<td>SQBP</td>
<td>SQBP</td>
</tr>
<tr>
<td>WLAP</td>
<td>WLAP</td>
</tr>
<tr>
<td>SLAP</td>
<td>SLAP</td>
</tr>
<tr>
<td>ISD</td>
<td>ISD</td>
</tr>
<tr>
<td>IAD</td>
<td>IAD</td>
</tr>
<tr>
<td>DSS</td>
<td>DSS</td>
</tr>
<tr>
<td>DSA</td>
<td>DSA</td>
</tr>
</tbody>
</table>

Proposition 2. Let the domain be a two-dimensional commodity space. Then

(i) The SQB model satisfies SQBP, WLAP, SLAP, and ISD,
(ii) The GSQB model satisfies SQBP, WLAP, and ISD,
(iii) The PRD model satisfies SQBP.

Within status quo constraint models, the SQB model has much higher prediction power. The PRD model is only consistent with SQBP (see Table 1).

Section 3 develops an experimental design that allows us to contrast the loss aversion models of Tversky and Kahneman with the status quo constraint models of Masatlioglu and Ok. Not every property is useful for this purpose. First of all, SQBP is not only well-established but also satisfied by all models except the LA model. Therefore, we have not included this property in our experiment. While WLAP and SLAP may differentiate between the special and generalized versions of these models, these are not very helpful to distinguish between these two groups of models. Finding evidence against these two properties would show that more general models are needed. However, we will not be able to judge between the general versions of loss aversion and status quo constraint models.

On the contrary, a joint test of ISS, IAD, DSS and DSA would help us contrast between models. By focusing on $a \rightarrow a'$ and $s \rightarrow s'$ shifts, Section 4 tests which properties are satisfied. Section 5 shows that these properties only partially identify the predictions of different models. When these properties are violated, more needs to be done to figure out the predictions of each model for each shift. Section 5 is devoted to theoretical predictions of the models and their explanatory powers.

3. Experimental design

We conducted individual decision-making experiments at the Center for Experimental Social Science (C.E.S.S.) at New York University. There were a total of 99 subjects. The experiment was conducted using the z-Tree software and lasted for approximately half an hour. Subjects earned $14 on average (including the $7 show-up fee) and some Belgian chocolates. Each subject answered 16 questions with only one of them randomly selected at the end of the experiment to determine the payoff. Among these 16 questions, on average 10 of them are relevant for this paper. The other choice problems should not have affected behavior in a particular way.

We settled on a simple choice experiment where we elicited preferences of subjects over two or three alternatives. In our experiments, we avoid the use of value elicitation mechanisms. Ariely et al. (2003) argue that compared to high volatility of absolute valuation, relative preferences are quite stable. They provide evidence that even when subjects may not know how much they value different items, they still know the relative ordering. Subjects’ decisions in a choice experiment only require knowledge of their relative ranking over alternatives.

In our experiment, each session consists of two parts. In the first part, subjects answer several questions in which they choose between two consumption bundles with no reference point. In the second part, they answer the same questions but with a reference point. Reference points are always dominated by at least one of the alternatives.

Fig. 4 demonstrates the experimental environment. Each bundle consists of a combination of money and a number of Belgian chocolate boxes. In the graph, two chocolate–money bundles, $x$ and $y$, are located so that neither dominates the

---

20 An exception to this is finding evidence against WLAP since it is satisfied by all models except PRD. However, since the PRD model does not provide any intuitive reason as to why this property may be violated, we do not expect to see that this property will fail. Hence, we do not consider this property either.

21 Knetsch (1989) and Herne (1998) also use choice experiments to test reference-dependent behavior.

22 Another potential problem with complex value elicitation mechanisms is that it may be hard to control for subject misconceptions. Although Plott and Zeiler (2005) successfully control for subject misconceptions, their controls require long procedures which would have been difficult to implement for our purposes.

23 Each box contains 3 chocolates and can be purchased from stores outside. It may be that the price of the chocolate box is approximately known to some of the subjects. Since we have a within-subjects design, this should not bias our experimental findings towards any particular theory of reference dependence.
other. Then we have seven reference points denoted by $a_1$–$a_4$ and $s_1$–$s_3$. Here not having any reference point is treated as if the reference point is the origin, i.e., $\diamond = (0, 0)$. When subjects are provided with a reference point, they are asked whether to keep it or to exchange it for one of the two other alternatives, $x$ or $y$.

The reference points $a_1$–$a_4$ are positioned under the assumption that the agent prefers $x$ over $y$ when there is no reference point. If the opposite holds, then these reference points would be placed symmetrically to the bottom of $x$. More specifically, we first derive each subject’s preferences over $x$ and $y$ (at $\diamond$). If the option $x$ is preferred over $y$, then in another question the third option, $a_i$, which is dominated by $y$, is introduced to the choice problem as the subject’s endowment or the status quo. If the option $y$ is preferred, then the third option is introduced in such a way that it is dominated by $x$ but not by $y$. While eliciting preferences without a reference point was not essential for testing properties and comparing the models, this design leaves more room for choice reversals and hence yields a larger set of relevant data.

The locations of the other reference points, $s_1$–$s_3$, are independent of the relative rankings of $x$ and $y$. While $a_1$–$a_4$ are dominated by $x$ but not by $y$ (asymmetrically dominated), the others are dominated by both $x$ and $y$ (symmetrically dominated). Since all the reference points are dominated by $x$, $y$ or both, we would expect that they will never be chosen.

We are interested in examining the impact of moving the reference points in the direction indicated by the arrows in Fig. 4. Therefore, we repeat the same chocolate–money bundle three times, corresponding to three different reference points, with one being the origin (no reference point). Note that the $x$–$y$ pair differs for different reference shifts. In order to control for possible order effects, we randomize the order of the questions in the experiment. One may be concerned that subjects may artificially try to be consistent in their choices. This would reduce the occurrence of choice reversals and increase the explanatory power of the standard theory. However, this should not have any effect on the comparison between the reference-dependent choice models. Moreover, it is unlikely that the subjects remember their previous choices since these questions are separated from each other and throughout the experiment, once subjects make their decisions, they cannot go back.

Our experiment has 5 sessions. Table 2 demonstrates the treatments in each session. For example, in Session 1, the impact of changing the location of the reference point from $a_1$ to $a_2$ and from $a_3$ to $a_4$ is tested. The within-subjects

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**Table 2**

<table>
<thead>
<tr>
<th>Experimental design.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sessions</td>
</tr>
<tr>
<td>$(a_1, a_2)$</td>
</tr>
<tr>
<td>$(a_3, a_4)$</td>
</tr>
<tr>
<td>$(s_1, s_2)$</td>
</tr>
<tr>
<td>$(s_1, s_3)$</td>
</tr>
<tr>
<td># of subjects</td>
</tr>
</tbody>
</table>

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24 Although Tversky and Kahneman (1991) suggest that money spent to buy goods is not coded as a loss, Bateman et al. (2005) show that loss aversion applies to any loss from the reference point, including money outlays. Also see Kalyanaram and Little (1994), Kalwani et al. (1990), Mayhew and Winer (1992) and Putler (1992).

25 In this case, a reference shift would change the number of chocolate boxes.

26 Questions without any reference point always comes first. Therefore, randomization is done within each part.
design allows us to study the reference effect as well as identify whether, for a given subject, one of the models could accommodate the subject’s choice behavior.

To get a more concrete idea of the design, consider a sample question used in the experiment that changes the location of the reference point from $\diamond$ to $a_1$ and then to $a_2$. In the first stage, the decision maker is offered a choice between the following two bundles: $\$6.10 and 5 chocolate boxes versus $\$7.90 and 2 chocolate boxes. If she prefers the second bundle over the first, then at a later stage of the experiment, the same decision maker will be given an endowment of $\$5.50 and 4 chocolate boxes ($a_1$), which is dominated by $\$6.10 and 5 chocolate boxes. Then, she must choose whether to keep her endowment or switch to one of the two previously mentioned bundles. The same question is repeated one last time for $a_2 = (\$6.10, 4)$. Instructions for the subjects are available in Appendix C. Appendix B provides a screenshot from the experiment.

In the experiment we create only hypothetical endowments by telling subjects that $\$6.10 and 4 chocolate box are theirs to keep—we do not give them anything until the end of the experiment. This should not affect the comparison among the reference-dependent models, but the reference effect would have been stronger if we had actually endowed them with bundles.  

We now summarize the implications of each property for each shift shown in Fig. 4. Since the shifts provided here are simply a subset of Fig. 3, the implications will be based on Section 2.2. Before we explain implications, we introduce some notations and definitions. Given an $r_1 \rightarrow r_2$ shift, $(a, b) r_1, r_2$ denotes choices from $\{x, y, r_1\}$ and $\{x, y, r_2\}$ when the reference point is $r_1$ and $r_2$, respectively. We will omit the subscript if there is no confusion about the reference shift. Note that there are nine possibilities: $\{x, x, (x, y, r_1)\}$, $\{x, y, (x, r_1)\}$, and $\{y, x, (y, r_1)\}$. Since each reference point is dominated by at least one other bundle, we expect that the reference points will never be chosen. Hence, for brevity, we focus on four cases: $\{x, x\}$, $\{x, y\}$, $\{y, x\}$, and $\{y, y\}$. The following implications are valid independent of this simplification.

**Definition 1.** An $r_1 \rightarrow r_2$ shift favors $x$ over $y$ if $x$ must be chosen from $\{x, y, r_2\}$ at $r_2$ whenever an agent chooses $x$ from $\{x, y, r_1\}$ at $r_1$. We simply say the $r_1 \rightarrow r_2$ shift favors $x$ if there is no confusion about $y$.

**Definition 2.** A subject exhibits a choice reversal at $r$ if the subject makes different choices with and without the reference point $r$.

Consider an $s_2 \rightarrow s_1$ shift. Since DSS states that a shift from $s_2$ to $s_1$ favors $y$ (the bundle with more chocolate), the possibility of $(y, x)$ is eliminated, but $(x, x)$, $(y, y)$, $(y, x)$ are still possible. Similarly, an $s_3 \rightarrow s_1$ shift favors $x$ (the bundle with more money). We now list the implications of the properties (see Table 3).

**Implication 1.** Under DSS, the number of subjects who pick the option with more chocolate $(y)$ will be weakly higher at $s_1$ compared to $s_2$ and the number of subjects who pick the option with more money $(x)$ will be higher at $s_1$ compared to $s_3$.

On the other hand, ISD restricts the possible outcomes to $(x, x)$ and $(y, y)$. Thus, it imposes no change in the number of choice reversals in aggregate.

**Implication 2.** Under ISD, there will be no change in the number of subjects choosing option $(x)$ over $(y)$ when the reference points are dominated by both alternatives.

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27. Loewenstein and Adler (1995) show that hypothetical ownership creates less endowment effect. However, this does not mean that hypothetical ownership does not create reference dependence. In Section 6, we argue that a status quo, with hypothetical ownership, has a stronger impact on individual choice behavior compared to a decoy option, where a decoy option is introduced without imposing an ownership.

28. Indeed, there are only three observations out of 394 where a dominated reference point is chosen.

29. Since constraint choice models use a revealed preference approach, we define this over choices and not over preferences.
On the other hand, the number of choice reversals at a2 will not depend on the initial choice when there is no reference point.30 A2 of subjects who exhibit choice reversals at a2 out of 57 subjects reveal opposite choice between the two options whenever the dominated option was selected. We exclude them in the following analysis on the reference effect, since here we study the impact of moving the reference point controls for that. If our results were driven by the indifference or error e is a choice reversal and a value of 0 when there is no change in a subject's choice. The independent variable is also a binary variable. It takes a value of 1 if the reference point is a1, independent of the initial choice between x and y. In the aggregate, the number of subjects who exhibit choice reversals at a2 is less than the number of subjects who exhibit choice reversals at a1. The same applies for a shift from a3 to a4.

**Implication 3.** Under DSA, the number of subjects who exhibit choice reversals at a2 (a4) is weakly less than the number of subjects who exhibit choice reversals at a1 (a3).

On the other hand, IAD restricts the possible outcomes to (x, x) and (y, y), independent of the initial choice. Thus, it imposes no change in the number of choice reversals in aggregate.

**Implication 4.** Under IAD, there is no change in the number of subjects who exhibit choice reversals at a2 (a4) versus at a1 (a3).

Before we move to the next section, we point out that in the case where a subject is indifferent or makes mistakes, Implications 1–4 continue to hold even under such circumstances, as we do not expect to see any particular direction of bias.

### 4. Experimental results

In this section, we analyze how the reference effect operates and if it depends on the location of the reference points. In particular, we compare two regions: asymmetrically dominated and symmetrically dominated reference points. This will help us identify whether the data are consistent with ISS, IAD, DSS and DSA. First, we consider the movement from a1 to a2 (treatment (a1, a2)).31 After we elicit the choice between x and y when there is no reference point, we find out that 22 out of 57 subjects reveal opposite choice between the two options whenever a1 is the new reference point (see Table 4). On the other hand, the number of choice reversals at a2 is 15.

We use logistic regressions to analyze our data. For each choice problem, the dependent variable takes a value of 1 when there is a choice reversal and a value of 0 when there is no change in a subject’s choice. The independent variable is also a binary variable. It takes a value of 1 if the reference point is a1 and 0 if the reference point is a2. Table 4 also presents the regression results. Logistic regression analysis shows that the odds of a choice reversal is 1.76 times greater when the reference point is a1 compared to a2.

One might argue that choice reversal from one option to the other can simply be attributed to subjects being indifferent between these two options, or that subjects may be making a mistake, and not due to the reference effect. Our analysis of moving the location of the reference point controls for that. If our results were driven by the indifference or error.

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30 By pooling our data across cases in which the subjects initially chose the option with more money and those in which they initially chose the option with more chocolate, we are able to increase our power for statistical testing. In our experiment, subjects are more likely to choose the option with more money. Moreover, we observe that reference effect is lower for subjects who pick the option with more money initially. In other words, subjects who initially pick the option with more money demonstrate fewer choice reversals. However, this should not affect the qualitative comparison of the models.

31 There are two cases where the dominated option was selected. We exclude them in the following analysis on the reference effect, since here we study the impact of moving the reference point on the x–y choices. We do not exclude them when we study the explanatory power of the models.
arguments, then we should have observed no statistically significant difference in the number of choice reversals at $a_1$ and $a_2$. In contrast, our analysis shows that we can strongly reject the hypothesis that the number of choice reversals are the same: subjects are more likely to choose an option when it dominates the reference point in both components. When the location of the reference point is moved from $a_3$ to $a_4$, we find that 16 and 14 subjects out of 59 switch their decisions at $a_3$ and $a_4$, respectively. Although there are more choice reversals at $a_3$, this difference is not significant ($p = 0.416$).

We also pool the data on asymmetrically dominated reference points to check whether the odds ratio in treatment ($a_1, a_2$) is significantly different than the odds ratio in treatment ($a_3, a_4$). We find that a shift in the asymmetrically dominated reference point ($a_1, a_2$, or $a_3, a_4$) increases the odds of choice reversal by a factor of 1.46 ($p = 0.032$) and that there is no significant difference between ($a_1, a_2$) and ($a_3, a_4$) movements ($p = 0.264$).

This finding is consistent with diminishing sensitivity for asymmetrically dominated reference points (DSA). Hence, IAD is violated.

Next, we look at the region where the reference points are strictly dominated by the two alternatives (symmetrically dominated reference points). First, we investigate moving the reference point from $s_2$ to $s_1$ (treatment ($s_1, s_2$)). At $s_1$ there are 9 subjects (out of 40) who choose the option with more chocolate. The total number of subjects who choose the option with more chocolate is 8 when the reference point is $s_2$. We cannot reject the hypothesis that preferences do not change when the reference points are dominated by both alternatives ($p = 0.658$). Similarly we look at switching the reference point from $s_3$ to $s_1$ (treatment ($s_1, s_3$)). Out of 39 people, 35 people choose the option with more money at $s_1$. Even though a small portion of subjects change their choice when the reference point is $s_3$, the total number of subjects who choose the option with more money does not change with moving the reference point (Table 5).

This finding is consistent with diminishing sensitivity for asymmetrically dominated reference points (DSA). Hence, IAD is violated.

Our results show that moving the reference point affects individual choices even when the reference point itself is not chosen. However, we only observe the reference effect for asymmetrically dominated reference points—in particular when there is a salient dominance by one option but not the other. We do not see a reference effect for symmetrically dominated reference points. Hence, we find that both DSA and ISD are desirable properties. However, there is no single model satisfying both of them at the same time. While the CLA, SQB and the GSQB models satisfy ISD, the LA model satisfies DSA.

At this point further investigation is needed to compare the performances of these models. Propositions 1 and 2 only partially identify what each model predicts under different reference point shifts. For example, the SQB model does not satisfy IAD, but we would like to know what type of prediction this model makes when the reference points are asymmetrically dominated. This will allow us to determine the explanatory power of the models. The next section investigates the predictions and explanatory power of each model.

5. Comparing models

To compare the models under consideration, we now discuss the predictions of each model for different reference shifts.

5.1. Predictions of models

Theoretical predictions of the reference-dependent models for switching the reference points are summarized in Table 6. The proofs can be found in Appendix A. These predictions are based on the assumption that bundle $x$ is chosen over bundle $y$ when there is no reference point. In Table 6, “-” represents the case where models predict no change in choices

<table>
<thead>
<tr>
<th>Table 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetrically dominated reference points.</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$s_1$</td>
</tr>
<tr>
<td>More money ($x$)</td>
</tr>
<tr>
<td>Less money ($y$)</td>
</tr>
<tr>
<td>Logistic regression analysis</td>
</tr>
<tr>
<td>($s_1, s_2$)</td>
</tr>
<tr>
<td>($s_1, s_3$)</td>
</tr>
</tbody>
</table>

Note: Standard errors are clustered at the individual level and reported in parentheses.

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32 Note that the dependent variable is now defined as 1 when a subject chooses the alternative with more money and 0 otherwise.
33 Appendix A does not directly show what the PRD model would predict for the $a_3 → a_4$ shift. For this case, this model favors $y$ as does the SQB model. Here the reference points are only weakly dominated by $y$, and therefore the PRD model predicts the same as the SQB model. Since $Q^1$ includes only
after moving the reference point. If a reference shift favors \( y \) in a particular model, it is indicated by “\( y \)” in the table. That is, the new allocation of the reference point makes \( y \) choices more likely. Finally, “I” represents the cases where the models are indecisive, i.e., they have no particular prediction on choice. For example, consider the models’ predictions when the reference point is moved from \( a_1 \) to \( a_2 \). Assuming \( x \) is chosen when there is no reference point, the CLA model predicts no change in choices and is consistent with only 2 possible choices out of 9. In particular, it is consistent with choosing \( y \) at both \( a_1 \) and \( a_2 \) (denoted by \( (y, y) \)) or with choosing \( x \) at both reference points (denoted by \( (x, x) \)).34 The LA and SQB models are consistent with 3 choices out of 9 since, even though they do not allow for the reference point to be chosen, they allow for choices among \( x \) and \( y \) to change in a particular direction.35 The GSQB and PRD models are consistent with 4 choices out of 9 since they are indecisive in terms of the relative ranking of \( x \) and \( y \).36

Note that the predictions in Table 6 rely on the initial preference. If the initial preference between \( x \) and \( y \) is \( y \), then the reference points would be placed at the bottom of option \( x \) in a symmetric fashion. Hence, for example, the shift from \( a_1 \) to \( a_2 \) would now favor \( y \) for the LA model and \( x \) in the SQB model.

5.2. Explanatory power of the models

We investigate the relative performance of the models assuming subjects are not indifferent and do not make mistakes in their choices. Naturally if one allows for subjects being indifferent between options and errors, then any one of these theories would explain all individual choices perfectly. Therefore, the question is, while being able to make some predictions, is there a model that can also accommodate our data? We first study which models can explain our data. Then, we consider the explanatory power and predictive power of these models at the same time to provide a more natural comparison.

First, we explore to what extent the classical choice theory can accommodate our experimental data. To do this, we examine how many individuals behave in a manner consistent with classical choice theory. Remember that each person is going through different treatments (Table 2). We say an individual is consistent with the classical choice theory if her choices are consistent with this theory at all times. For example, if we see choice reversals at \( a_1 \) compared to her choice with no reference point, then we say this individual displays a choice pattern inconsistent with the classical choice theory.37

We find that 42% of all subjects exhibit some reference dependence. Put differently, roughly half of the time, we observe behavioral patterns inconsistent with the classical choice theory. Plott and Zeiler (2005, 2007) argue that the endowment effect is driven by subjects’ misconceptions and experimental procedures. In our experiment, we provide a simple choice environment in which individual choices appear to be influenced by the reference points, even when the subjects do not stick with the reference point. In other words, reference dependence is not confined to the endowment effect or status quo bias in general.38

Now, we focus our attention on reference-dependent models. Similarly, we say an individual is accommodated by a particular theory if her choices are consistent with this theory at all times. For example, a subject’s choice behavior cannot be explained by the CLA and SQB models if the subject displays a preference reversal at \( a_1 \) but not at \( a_2 \). This choice behavior is, nevertheless, consistent with the LA, GSQB, and PRD models (see Table 6).

Table 6
Predictions of reference-dependent models under consideration.

<table>
<thead>
<tr>
<th></th>
<th>CLA</th>
<th>LA</th>
<th>SQB</th>
<th>GSQB</th>
<th>PRD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 \rightarrow a_2 )</td>
<td>-</td>
<td>( x )</td>
<td>( y )</td>
<td>I</td>
<td>I</td>
</tr>
<tr>
<td>( a_3 \rightarrow a_4 )</td>
<td>-</td>
<td>( x )</td>
<td>( y )</td>
<td>I</td>
<td>( y )</td>
</tr>
<tr>
<td>( s_2 \rightarrow s_1 )</td>
<td>-</td>
<td>( y )</td>
<td>-</td>
<td>-</td>
<td>I</td>
</tr>
<tr>
<td>( s_3 \rightarrow s_1 )</td>
<td>-</td>
<td>( x )</td>
<td>-</td>
<td>-</td>
<td>I</td>
</tr>
</tbody>
</table>

-: “no changes”, \( x \): “favors \( x \)”, \( y \): “favors \( y \)”. I: “indecisive”.
Table 7
Explanatory power of the models (in percentages).

<table>
<thead>
<tr>
<th>Tversky–Kahneman</th>
<th>Masatlioglu–Ok</th>
<th>Classical theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLA</td>
<td>LA</td>
<td>SQB</td>
</tr>
<tr>
<td>66</td>
<td>90</td>
<td>69</td>
</tr>
</tbody>
</table>

Table 8
Explanatory power of the models when reference point is moved (in percentages).

<table>
<thead>
<tr>
<th>N</th>
<th>Tversky–Kahneman</th>
<th>Masatlioglu–Ok</th>
<th>Classical theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLA</td>
<td>LA</td>
<td>SQB</td>
<td>GSQB</td>
</tr>
<tr>
<td>(a₁, a₂)</td>
<td>59</td>
<td>78</td>
<td>93</td>
</tr>
<tr>
<td>(a₃, a₄)</td>
<td>59</td>
<td>90</td>
<td>97</td>
</tr>
<tr>
<td>(a₁, a₃)</td>
<td>40</td>
<td>75</td>
<td>95</td>
</tr>
<tr>
<td>(a₃, a₃)</td>
<td>39</td>
<td>67</td>
<td>92</td>
</tr>
</tbody>
</table>

We see that the PRD and LA models explain approximately 90 percent of the participants (see Table 7).³⁹ Out of 99 individuals, 87 individuals are consistent with both models, and 4 subjects cannot be explained by any model.⁴⁰ While there are 6 participants who can be explained with the PRD model but not the LA model, only 2 participants can be accommodated by the LA model but not the PRD model. The McNemar’s test shows that this difference is not statistically significant (p = 0.157). The LA and PRD models explain significantly more data compared to the CLA, SQB and GSQB models (all p-values are less than 0.002). The SQB model can explain the behavior of 68 subjects. There are 7 participants that can be explained by the GSQB model but not by the SQB model. The GSQB model performs significantly better than the SQB model (p = 0.008). 65 participants can be explained by the CLA model. While the SQB model performs slightly better than the CLA model (p = 0.083), the GSQB model performs significantly better than the CLA model (p = 0.002). Only 57 people are consistent with the predictions of the classical choice theory.⁴¹ Finally, the CLA model accommodates the data significantly better than the classical choice theory (p = 0.005).

Next we investigate the impact of moving the reference point from one location to the other. Table 8 summarizes the explanatory power of the models for each treatment. We report the percentages of subjects that can be explained by each model. It can be seen that the PRD and LA have strong explanatory powers. Although the CLA, SQB and GSQB models can explain less data compared to these models, they can accommodate significantly more data compared to the classical choice theory.

5.3. Measuring the predictive success rates for each model

Although the CLA and the SQB models and the classical choice theory do not have high explanatory power, they make powerful predictions. Both the PRD and LA models have a large percentage of correct predictions. However, they also predict a large area within the set of all possible outcomes. In this section we take into account the “predictive power” of the models. We compare the models by using a measure of predictive success which was first introduced by Selten and Krischker (1983).⁴² They define the measure as the difference between the hit rate and area:

\[ M = R - A \]

where \( M \) = measure of predictive success, \( R \) = hit rate, and \( A \) = area. The hit rate is the percentage of correct predictions and the area is the relative size of the predicted region within the set of all possible outcomes. Since the PRD and LA cannot always make precise predictions, they will have relatively high area. This may imply a very low measure of predictive success for these models compared to the CLA, SQB, GSQB models or classical choice theory.

In our experiment, the area for each theory is different for each session since subjects go through different treatments. Therefore, we compute the predictive success rates of each model for each session. As an example, consider Session 2. Hit rates are simply calculated by checking the percentage of consistent predictions made by each model in a given session (0.45, 0.80, 0.45, 0.55, 0.90, and 0.25 for the CLA, LA, SQB, GSQB, PRD model, and classical theory, respectively). To calculate the area for a given session we first calculate the set of all possible outcomes in a given session. In Session 2 there are two treatments: \((a₁, a₂)\) and \((s₁, s₂)\). Consider the treatment \((a₁, a₂)\). Without loss of generality, we assume the initial choice

³⁹ Note that the two subjects that picked dominated options are also included in this analysis.

⁴⁰ Two of these subjects picked dominated options, and two of them behaved consistent with the PRD in some cases and the LA in some other situations.

⁴¹ A large proportion of subjects that are consistent with classical choice theory always choose an option with more money. One reason for this could be that they focus on maximizing their monetary earnings from the experiment and they do not care about their chocolate earnings. Although this behavior is consistent with the classical choice theory, it does not imply that these subjects are not reference dependent in a different environment which does not include money.

⁴² See Selten (1991) for a detailed explanation of this measure.
is \( \mathbf{x} \). There are 9 possible choice patterns: \( \{\mathbf{a}_1, \mathbf{a}_2\}, \{\mathbf{a}_1, \mathbf{x}\}, \{\mathbf{a}_1, \mathbf{y}\}, \{\mathbf{y}, \mathbf{a}_2\}, \{\mathbf{y}, \mathbf{x}\}, \{\mathbf{x}, \mathbf{a}_2\}, \{\mathbf{x}, \mathbf{y}\}, \{\mathbf{x}, \mathbf{x}\}, \text{ and } \{\mathbf{y}, \mathbf{y}\}. \) While the classical choice theory is only consistent with \( \{\mathbf{x}, \mathbf{x}\} \), the CLA model is also consistent with \( \{\mathbf{y}, \mathbf{y}\} \). Since the LA model favors \( \mathbf{x} \) in this type of shifts, it is also consistent with \( \{\mathbf{y}, \mathbf{x}\} \). On the other hand, the SQB model favors \( \mathbf{y} \), so it is consistent with \( \{\mathbf{x}, \mathbf{x}\}, \{\mathbf{y}, \mathbf{y}\}, \text{ and } \{\mathbf{x}, \mathbf{y}\} \) choices. The GSQB and PRD models allow these four patterns. Out of 9 possible choices, the number of consistent choice patterns is 2, 3, 4, 4 and 1 for the CLA, LA, SQB, GSQB, PRD model, and classical theory, respectively. When we take into account both treatments in Session 2, the area (A) is 0.02, 0.11, 0.04, 0.05, 0.20, and 0.01, respectively. Therefore, the measure of predictive success is 0.43, 0.69, 0.41, 0.50, 0.70, and 0.24, respectively.

Table 9 shows that the predictive success rates of all of the reference-dependent models are superior in every session to that of the classical choice theory. Moreover, the LA and PRD models have a higher predictive success compared to other models of reference dependence, but the gap between the predictive success rates between the LA model and the CLA model and among the PRD model and the SQB and GSQB models are quite small compared to a measure based solely on explanatory power.\(^{43}\)

6. Reference effect versus asymmetric dominance effect

The choice reversals observed in our experiment are similar to behavior observed in the context-dependent choice literature. Indeed, the phenomenon called “Attraction Effect” refers to a situation in which an inferior alternative influences the relative attractiveness of other alternatives in a choice set. The “Asymmetric Dominance Effect” (also called the decoy effect) is a special case of the attraction effect where the binary comparison of two objects, contrary to normative theory, is affected by the introduction of an asymmetrically dominated alternative to an existing choice set.

One could argue that the experimental evidence reported in the reference-dependent literature, including our paper, has nothing to do with ownership, but is rather an observation of the decoy effect. A much more convincing argument would be that, in both situations, the third alternative is used as a reference point and whether it is named as an endowment or not does not make any difference. It is crucial to know whether the influence of the status quo and the decoy option in people’s choice behavior are exactly the same or not. Our experiment has additional choice problems that study the difference between decoy and endowment.\(^{44}\) We study two related research questions. i) Does a status quo option affect the relative rankings of alternatives more than a decoy option? ii) Does endowment matter when it is not presented as the decoy option?

In order to answer our first question, we ask three choice problems for a given \( \mathbf{x-y} \) pair. We first elicit each subject’s preferences over \( \mathbf{x} \) and \( \mathbf{y} \). Hence the first choice problem is \( \{(\mathbf{x}, \mathbf{y}), \emptyset\} \) where \( \emptyset \) denotes that there is no status quo. If the option \( \mathbf{x} \) (\( \mathbf{y} \)) is preferred, then in another question the third option, \( \mathbf{z} \), which is dominated by the other alternative \( \mathbf{y} \) (\( \mathbf{x} \)), is introduced and subjects are asked to choose from \( \{(\mathbf{x}, \mathbf{y}, \mathbf{z}), \emptyset\} \), which is the second choice problem. In the third choice problem, option \( \mathbf{z} \) is given as the endowment while the set of available options is the same set, i.e., \( \{(\mathbf{x}, \mathbf{y}, \mathbf{z}), \emptyset\} \). If being endowed with an option has any bite, we expect to see different outcomes for the second and third decision problems. Particularly, \( \mathbf{y} \) (\( \mathbf{x} \)) is chosen more when \( \mathbf{z} \) is the endowment.

The percentage of switches when a status quo is introduced is 29%, compared to the 18% when a decoy option is introduced.\(^{45}\) Out of 79 subjects, 52 never change their initial choice. 13 subjects change their choice when the third option is introduced as a status quo option but not when it is introduced as a decoy option. Only 4 subjects change their choice when the third option is introduced as a decoy option but not when it is introduced as an endowment. 10 subjects show choice reversal at both times. Logistic regression analysis shows that the odds of a choice reversal is 1.91 times greater when the reference point is introduced as an endowment than when it is introduced as a decoy option (\( p = 0.028 \)).

\(^{43}\) For comparison purposes, it would be interesting to know the predictive success rate of a model that allows for all choices except the choice of dominated options. We find that such a model performs almost as good as the LA and PRD models even though it is much more general. We like to note that Selten’s measure is not without problems since it is very sensitive to the composition of the choice sets and the number of elements. The reason such a general model performs well is because the number of undominated options in our environment is very low. Such a general model would have a very low predictive success rate if we included more undominated options that are not desirable. In any case, we think that the relative comparisons among the models presented in this paper are still very informative.

\(^{44}\) We also control for the order effect in this part of the design by changing the order of the questions across sessions.

\(^{45}\) In this analysis we eliminated 1 observation in which dominated option was selected.
The above analysis shows that while the decoy effect is present in our data, this effect is augmented when the dominated option is introduced as a reference point. However, it is still not clear whether this establishes the importance of ownership. An alternative view could be that when the decoy option is called the endowment, subjects’ attention is diverted to the decoy option, which makes the decoy option more salient. Hence, the decoy effect becomes stronger.

To test this alternative view, we consider a pair of decision problems: \((x, y, z)\) and \((x, y, Z)\), where \(z\) is asymmetrically dominated by \(y\). In the second choice problem, the endowment and the decoy option are two separate bundles. Hence, the decoy option is not highlighted by the presence of the endowment. If being endowed with an option has no influence on choice behavior, i.e., the decoy option is the reference point, then the choice behavior should be the same in these two choice problems.

Out of 97 subjects, 83 did not change their decisions when they were endowed with \(y\). 12 subjects changed their decisions in favor of \(y\), and only 2 subjects changed their decisions in favor of \(x\). When a status quo is introduced, participants are more likely to choose the alternative \(y\) \((p = 0.006, \text{ robust odds ratio } = 1.54)\).

Note that the status quo introduced here is very weak since subjects do not actually own them. However, even in the presence of a weak entitlement we see the effect of a status quo. In a setting where individuals actually own the status quo option, we expect to see even more striking effects on choices.

7. Literature review and discussion

The closest papers to ours are Bateman et al. (1997) and Herne (1998)—they provide comprehensive tests of the model of Tversky and Kahneman (1991). Bateman et al. (1997) elicit individual valuations of private goods using a laboratory experiment with eight different measures. They find that implications of the general loss aversion model on valuations are supported by their data. In particular, Bateman et al. find a divergence between these measures of valuation consistent with what is predicted by the model of Tversky and Kahneman (1991). Herne (1998), which considers a choice environment with asymmetrically dominated reference points, provides direct tests of the loss aversion and diminishing sensitivity hypotheses. The results of Herne’s experiments are consistent with the general reference-dependent model of Tversky and Kahneman (1991). In particular, Herne finds support for loss aversion and diminishing sensitivity phenomena under asymmetrically dominated reference points using a between-subjects design (and, hence, individual factors cannot be controlled for).

To our knowledge, our paper is the first to study different models of reference dependence in a unified manner and disentangle critical behavioral properties that are satisfied by each model. Our paper analyzes the reference effect by distinguishing the possible effects of both asymmetrically and symmetrically dominated reference points. In addition, due to our within-subjects design, we are able to contrast the models of reference dependence according to their predictive success rates (which accounts for both predictive and explanatory powers of the models).

As we have already mentioned, our paper is also related to the asymmetric dominance effect in the literature on context dependence. Huber et al. (1982) is the first paper to document that the addition of an asymmetrically dominated alternative (decoy) increases the proportion of choices in favor of the target (dominating alternative). Ratneshwar et al. (1987) show that meaningfulness and familiarity with the product category moderates the attraction effect compared to the Huber et al. (1982) study. However, the effect does not disappear.

Simonson (1989), Simonson and Tversky (1992), Tversky and Simonson (1993), Ariely and Wallsten (1995), and Wedell and Pettibone (1996) provide additional evidence and theoretical explanations for the asymmetric dominance effect, such as increasing justifiability and tradeoff contrasts. Kivetz et al. (2004a, 2004b) separate out four different groups of models that explain context effects in general and test them. Three of these models can explain asymmetric dominance effects and share a common feature with the models of Tversky and Kahneman (1991). That is, the decision makers compare each alternative with a single reference point. While in this paper the reference point is simply assumed to be the default option, the models of Kivetz et al. (2004a) assume that the reference point is determined by an exogenously given procedure. Depending on the model, the reference point is either the midpoint or the minimum of the range of objective attribute levels in the choice set. If the choice set changes, so does the reference point, and hence the reference point is context dependent. One important fact is that the reference points in these models do not necessarily belong to the choice set.

In our experiments changing the default option affects the composition of the choice set. It would be interesting to know whether these models are also capable of accommodating our data. To utilize these models, one should implicitly assume that the decision makers are not affected by the default option directly. Nevertheless, the default option indirectly affects the final choices by altering the composition of choice. Given our experimental setup, two of these models, the contextual

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46 Note that we do not elicit the preferences between the \((x, y)\) pair in an effort not to repeat the same bundles too much, and hence the decoy/endowment option is exogenously placed. Therefore, we expect to observe fewer choice reversals, and this does not affect the decoy or endowment option differently. The decoy option is placed near the higher money bundle once and higher chocolate bundle at another choice problem to control for any asymmetries due to preferences across money and chocolate (see instructions to the experiment).

47 In this analysis we eliminated two observations in which dominated option was selected.

48 In addition, Wedell and Pettibone (1996) study context dependence under symmetrically dominated decoy options. They do not observe an effect of adding a symmetrically dominated decoy option on choice. However, they do not study the effect of shifting the decoy option in this region like we do in our study. In any case, it is encouraging to find similar results.

49 These are the contextual concavity model, the normalized contextual concavity model, and the loss aversion model. Please see Kivetz et al. (2004a, 2004b) for the details of these models.
between willingness-to-accept (WTA) and willingness-to-pay (WTP) (Plott and Zeiler, 2005 and List, 2003, 2004).

We have seen that this prediction is not consistent with our data. Hence, it cannot be a canonical model if one wants to investigate the implications of asymmetrically dominated reference points. Similarly, the SQB model is also too restrictive. Unlike the CLA model, the model predicts that asymmetrically dominated reference points influence behavior, but the prediction of this model is not supported in our data for this particular case.

Finally, the GSQ model seems to be the winner of all, since it has the right combination of high predictive and explanatory power. The model predicts that there is no reference effect for symmetrically dominated reference points and allows for reference effect for asymmetrically dominated reference points at the same time. In addition to that, it satisfies the key property of status quo bias. Moreover, the GSQ model is consistent with both the presence and absence of a gap between willingness-to-accept and willingness-to-pay.

Our paper has implications regarding the economic policies that impose default options such as retirement plans. It has been shown that the automatic enrollment tends to anchor employee asset allocations on the automatic enrollment default asset allocation. More strikingly, the default allocation affects the asset allocations of those workers who abandon their default plan. Our findings add to this literature that default options may have a strong impact on choices, even when they are not chosen. More importantly, we demonstrate the impact of different default options on choices. In particular,

This paper provides a bridge between literatures on context dependence and reference dependence. We find that even the presence of a weak entitlement (decoy option is referred to as a default option) has an effect on choice beyond the effect of adding a decoy option to the choice set. It might be argued that this is because a default option reinforces the context effect. While this might be some part of the explanation, we show that there is also an additional and independent effect of entitlement (i.e., entitlement affect choices even when it is not presented as the decoy option). Future research should provide theories that could accommodate both the effect of a default option and context dependence.

8. Conclusion

This paper is an additional step towards understanding reference-dependent choice behaviors. Our contribution to the literature is two-fold. First, we contrast reference-dependent models by providing which behavioral properties that they satisfy. Then, we design an experiment to test these properties and to understand reference effect. In our experiment, we find strong evidence for the existence of the reference effect: reference points alter one’s choices even when agents do not choose the reference point itself. In particular, we find that the reference effect exists for asymmetrically dominated reference points, but we do not observe any evidence of the reference effect for symmetrically dominated reference points. With respect to behavioral properties we find support for ISD and DSA. In addition, we document the predictive success rates of different models. We believe that these results not only help us compare the existing reference-dependent models but also provide guidance for future modeling choices.

We now provide a general assessment of the existing models. All the models under consideration have advantages over others. Some enjoy high predictive power; while others have high explanatory power. In our view, both the LA model and the PRD model are too general to be useful in applications. For example, the LA model does not even satisfy the fundamental and most-illustrated property of SQBP (see Sagi, 2006; Masatlioglu and Ok, forthcoming). Hence, the LA model allows for the prediction that is opposite of what Tversky and Kahneman (1991) initially intended to explain. While we do not test the well-documented SQBP, one can imagine that had we included that in our design, the predictive success rate of the LA model would diminish dramatically. On the other hand, the PRD model satisfies SQBP but fails to make powerful predictions. In sum, even though these models are capable of accommodating most of our data, they lack prediction power.

The CLA model is the most restricted model and requires no reference effect for asymmetrically dominated reference points. We have seen that this prediction is not consistent with our data. Hence, it cannot be a canonical model if one wants to investigate the implications of asymmetrically dominated reference points. Similarly, the SQB model is also too restrictive. Unlike the CLA model, the model predicts that asymmetrically dominated reference points influence behavior, but the prediction of this model is not supported in our data for this particular case.

Finally, the GSQ model seems to be the winner of all, since it has the right combination of high predictive and explanatory power. The model predicts that there is no reference effect for symmetrically dominated reference points and allows for reference effect for asymmetrically dominated reference points at the same time. In addition to that, it satisfies the key property of status quo bias. Moreover, the GSQ model is consistent with both the presence and absence of a gap between willingness-to-accept and willingness-to-pay.

Our paper has implications regarding the economic policies that impose default options such as retirement plans. It has been shown that the automatic enrollment tends to anchor employee asset allocations on the automatic enrollment default asset allocation. More strikingly, the default allocation affects the asset allocations of those workers who abandon their default plan. Our findings add to this literature that default options may have a strong impact on choices, even when they are not chosen. More importantly, we demonstrate the impact of different default options on choices. In particular,

50 Unfortunately, we could not see an obvious way to modify these models to accommodate findings in Section 6.
51 In addition to that, there are other implications of the LA model, which contradict the empirical facts: (i) it permits non-convex indifference curves (Munro and Sugden, 2003), (ii) it is impossible to make welfare comparisons under this model (Mandler, 2004), and (iii) it necessitates a discrepancy between willingness-to-accept (WTA) and willingness-to-pay (WTP) (Plott and Zeiler, 2005 and List, 2003, 2004).
52 While we find the GSQB model to be the most reasonable model among the current models, there is still room for improvement, since the GSQB model does not satisfy the DSA property.
53 For a detailed explanation, see Masatlioglu and Ok (forthcoming).
54 Beshears et al. (2006) show that while 86% of employees who were subject to automatic enrollment have some of their assets allocated to the default fund, this number is only 10% for employees who were not subject to automatic enrollment (they were hired and initiated savings plan participation before automatic enrollment).
asymmetrically dominated reference points would have an impact on decisions; however, we do not expect symmetrically dominated asset allocations to influence employee decisions once the default allocation is deserted. This enables policy makers to derive important policy implications regarding the interaction of a default allocation and employee savings. Consequently, employers may be able to help their employees achieve desired retirement goals.

Appendix A. Theoretical predictions and proofs of Propositions 1 and 2

We now illustrate the predictions of each model for five different reference shifts: \( \diamond \to x \), \( I \to I' \), \( I' \to I'' \), \( s \to s' \), and \( a \to a' \). This will also prove both Propositions 1 and 2.

**The CLA model**: Since each \( g_i \) is linear and satisfies loss aversion, w.l.o.g., we assume \( g_i(a) = a \) when \( a \geq 0 \) and \( g_i(a) = \lambda a \) when \( a < 0 \), for \( i = 1, 2 \) and where \( \lambda > 1 \). We also normalize \( u_1(0) = u_2(0) = 0 \).

\((\diamond \to x)\): \( x \in c(x, y) \), \( \diamond \) implies that \( 0 \leq U_{(0,0)}(x) - U_{(0,0)}(y) = u_1(x_1) - u_1(y_1) + u_2(x_2) - u_2(y_2) \). Hence, \( x \) provides more than \( y \) for at least one of the dimensions, say dimension 1. Then we have \( U_x(y) = \lambda(u_1(y_1) - u_1(x_1)) + u_2(y_2) - u_2(x_2) \). Since \( \lambda > 1 \), we have \( U_x(y) < u_1(y_1) - u_1(x_1) + u_2(y_2) - u_2(x_2) \). By assumption, the right-hand side is less than 0. Therefore, \( U_x(y) < 0 \), \( x \in c(x, y) \). As a result, SQP is satisfied.

\((I \to I')\): The CLA model implies that \( U_I(x) - U_I(y) = u_1(l_1') - u_1(l_1) \) and \( U_I(y) - U_I(y) = \lambda(u_1(l_1') - u_1(l_1)) \). Since \( \lambda > 1 \), we have \( U_I(x) - U_I(y) > U_I(x) - U_I(x) \). Arrange the terms to get \( U_I(x) - U_I(y) > U_I(x) - U_I(y) \). Since \( x \in c(x, y) \), \( I \), we have \( U_I(x) - U_I(y) > 0 \), so \( U_I(x) - U_I(y) \geq 0 \). Therefore, \( x = c(x, y, I') \), which implies WLAP is a necessary condition.

\((I' \to I'')\): Since the illustration is exactly the same as above, we skip it. In sum, SLAP is also a necessary condition.

\((s \to s')\): We have \( U_s(x) - U_s(x) = u_1(s_1') - u_1(s_1) = U_s(y) - U_s(y) \). Therefore, there is no change in the relative ranking of \( x \) and \( y \) as we move the reference point. Hence, the model predicts no change, which means ISD holds. Moreover, DSS is violated.

\((a \to a')\): Since the illustration of the \( s \to s' \) shift also applies here, we skip the proof. In sum, IAD is also a necessary condition.

**The LA model**: Again normalize \( u_1(0) = u_2(0) = 0 \).

\((\diamond \to x)\): Since the CLA model is a special case, the LA model can accommodate status quo bias. On the other hand, Masatlioglu and Ok (forthcoming) provide an example where the LA model predicts status quo aversion. Hence, this model makes no particular prediction for this type of shifts, so ineducive.

\((I \to I')\): We demonstrate the theoretical predictions for the case of switching the reference point from \( I \) to \( I' \). In the LA model, the utility gains of \( x \) and \( y \) from the \( I - I' \) shift are \( U_I(x) - U_I(x) = g_1(u_1(x_1) - u_1(l_1')) - g_1(u_1(x_1) - u_1(l_1)) \) and \( U_I(y) - U_I(y) = g_1(u_1(y_1) - u_1(l_1')) \). All utility differences are in terms of the first dimension since \( l_2 = l_2' \). Note that \( u_1(x_1) - u_1(l_1) = u_1(x_1) - u_1(l_1') + u_1(l_1') - u_1(l_1) \) since \( y_1 = l_1 \). The concavity of \( g_i \), \( i = 1, 2 \), implies that

\[
\begin{align*}
g_1(u_1(x_1) - u_1(l_1')) - g_1(u_1(x_1) - u_1(l_1)) & \geq g_1(u_1(l_1') - u_1(y_1)) \\
& > g_1(u_1(y_1) - u_1(l_1'))
\end{align*}
\]

since \( g_i(a) = a_0.5 \) when \( a > 0 \) and \( g_i(a) = -2(a_0.5) \) when \( a < 0 \). Rearranging this inequality gives us \( U_I(x) - U_I(y) > U_I(y) - U_I(y) \). This implies that switching from \( I \) to \( I' \) favors \( x \). This implies WLAP is a necessary condition.

\((I' \to I'')\): Since the CLA model is a special case of this model, the LA model also allows that this shift favors \( x \). On the other hand, it is easy to create examples such that \( x_1 = l_1' > l_1' > y_1 \) and \( y_2 > x_2 > l_2' = l_2' \), and while \( x \) is chosen under \( I' \), \( y \) is chosen under \( I'' \). For example, assume that \( g_1(a) = a_0.5 \) when \( a > 0 \) and \( g_1(a) = -2(a_0.5) \) when \( a < 0 \). In addition, assume that the utility differences in dimension 1 are \( u_1(l_1') - u_1(y_1) = 16 \) and \( u_1(x_1) - u_1(l_1') = 9 \), and in dimension 2 are \( u_2(y_2) - u_2(l_2') = 144 \) and \( u_2(x_2) - u_2(l_2') = 1 \). Then we have \( U_I(x) = 96.5 = 5.5 = 4 \) and \( U_I(y) = -2 + 16.5 + 144 = 4 \). Hence, \( x \in c(x, y) \). On the other hand, \( U_I(x) = 0.5 + 4.5 = 1 \) and \( U_I(x) = -2 + 25.5 + 144 = 2 \). Hence, \( x \notin c(x, y, I', I') \). In sum, the model makes no prediction, and SLAP is not a necessary condition.

\((a \to a')\): The utility losses of \( x \) and \( y \) from switching are \( U_A(x) - U_A(x) = g_1(u_1(x_1) - u_1(a_1')) - g_1(u_1(x_1) - u_1(a_1)) \) and \( U_A(y) - U_A(y) = g_1(u_1(y_1) - u_1(a_1')) - g_1(u_1(y_1) - u_1(a_1)) \), respectively. All utility differences are in terms of the first dimension since \( a_2 = a_2' > 0 \). Let \( \beta = u_1(a_1') - u_1(a_1) \). Then we have

\[55\] If \( f \) is an increasing concave function and \( f(0) = 0 \), then we have \( f(x) + f(y) \geq f(x + y) \) for all \( x, y \geq 0 \).
The SQB model:

\[ U_x(x) - U_{\alpha}(x) = g_1(u_1(x_1) - u_1(\alpha_1') + \beta) - g_1(u_1(x_1) - u_1(\alpha_1')) \]

This means that switching from a to \( a' \) increases x's relative ranking with respect to y. In addition, if x is chosen at a \( (U_x(x) - U_{\alpha}(y) > 0) \), then x will be uniquely chosen at \( a' \). Therefore, DSA is a necessary condition for the model.

\( (s \rightarrow s') \): This shift also increases x's relative ranking with respect to y. For the proof, please see the demonstration of a \( \rightarrow a' \) shift. Thus, DSS is also a necessary condition for the model. □

The SQBP model:

The model is built on SQBP: \( x \in c(S, y) \) implies \( x = c(S, x) \) (Masatlioglu and Ok, 2005). Hence, it also satisfies SQBP.

\( (\alpha \rightarrow \alpha) \): The model is built on SQBP (Masatlioglu and Ok, forthcoming).

\( (l \rightarrow l') \): Since x dominates \( l' \) both dimensions, x must be in \( Q(l) \). Similarly, \( y \) is in \( Q(l) \). Given that \( y \) is in the constraint set, if \( x \in c(x, y, l, l) \), then \( U(y) \) must yield a utility as high as \( U(y) \). Hence, \( x \) must be best in \( Q(l) \) since \( x \in c(l) \). Therefore, \( x \in c(x, y, l) \). Switching from \( l \) to \( l' \) favors \( x \), which implies WLAP is a necessary condition.

\( (l' \rightarrow l'^{''}) \): We show that the model does not make any particular prediction for this shift. In other words, the model is indecisive. The argument for the SQB model does not hold since we no longer know whether \( y \) is in \( Q(l') \) or not. Hence, it is possible that \( x \) is chosen while \( y \) is not in \( Q(l') \) and provides higher utility than x. When the reference point shifts to \( l'^{''} \), \( y \) might be included in \( Q(l'^{''}) \), which makes \( y \) best in \( Q(l'^{''}) \). Hence, it is possible that \( y \in c(x, y, l') \). On the other hand, since the SQB model is a special case of this model, the model can generate cases where \( y \) is favored. Therefore, the model does not make any prediction. In other words, SLAP is not a necessary condition for this model.

\( (s \rightarrow s') \): ISD is one of the axioms of the model. Hence, the model predicts no change. Thus, DSS is not a necessary condition for this model.

The GSQBP model:

\( (\alpha \rightarrow \alpha) \): The model is indecisive. Since the SQB model favors \( y \), the model can generate cases where \( y \) is favored. We now show the other possibility. Consider an \( \alpha \rightarrow \alpha' \) shift where \( y \) is chosen while \( x \) is not in \( Q(a) \) and provides higher utility than \( x \). When the reference point shifts to \( \alpha' \), \( y \) might be included in \( Q(a') \). Since \( x \in c(x, y, \alpha') \), \( y \) is selected when \( l \) is the reference point \( (Q(l) \text{ will be utilized}) \). When \( l' \) is the reference point, \( y \) is selected since \( U(y) > U(x) \). Therefore the model favors \( y \) for this case. Switching from \( l \) to \( l' \) has no implication, which implies WLAP is not a necessary condition.

\( (l' \rightarrow l'^{''}) \): Similar to above, and hence SLAP is not a necessary condition.

\( (s \rightarrow s') \): The model is indecisive. Consider a case where \( U(y) > U(x) \), \( x \) is in \( Q(l) \) or \( Q(l') \) and \( y \) is in \( Q^2 \). Therefore the model favors \( y \) for this case. Switching from \( s \) to \( s' \) is a necessary condition for this model. □

The PRD model:

\( (\alpha \rightarrow \alpha) \): The model is built on SQBP (Masatlioglu and Ok, 2007).

\( (l \rightarrow l') \): The model is indecisive. Since the SQB model favors \( x \), this model also favors \( x \) for some cases. We now show that there are other cases where the model favors \( y \). Consider a case where \( U(y) > U(x) \), \( x \) is in \( Q(l) \) and \( y \) is in \( Q(l') \). Since \( x \in c(x, y, l) \), \( x \) is selected when \( l \) is the reference point \( (Q(l) \text{ will be utilized}) \). When \( l' \) is the reference point, \( y \) is selected since \( U(y) > U(x) \). Therefore the model favors \( y \) for this case. Switching from \( l \) to \( l' \) has no implication, which implies WLAP is not a necessary condition.

\( (l' \rightarrow l'^{''}) \): Similar to above, and hence SLAP is not a necessary condition.

\( (s \rightarrow s') \): The model is indecisive. Consider a case where \( U(y) > U(x) \), \( x \) is in \( Q(s) \) or \( Q(s') \) and \( y \) is in \( Q^2(s) \) or \( Q^2(s') \). Then \( x \) is selected when \( s \) is the reference point \( (Q(s) \text{ will be utilized}) \), and \( y \) is selected when \( s' \) is the reference
point. Therefore the model favors \( y \) for this case. Since everything is symmetric, if we switch the roles of \( x \) and \( y \), we get an opposite prediction. Switching from \( s \) to \( s' \) has no implication (ISD and GSS do not hold).

\((a \rightarrow a')\): The model is indecisive except at the boundary (where \( y_2 = a_2 = a'_2 \)).

First, consider the cases without this boundary. Since the SQB model favors \( y \), the model can generate cases where \( y \) is favored. We now illustrate the other possibility. Consider an \( a \rightarrow a' \) shift. Assume that (i) \( x \) provides more utility than \( y \) (\( U(x) > U(y) \)), and (ii) \( x \) is not in \( Q^1(\mathbf{a}) \), but \( y \) is in. Hence \( y \) is chosen when the reference point is \( a \). Further assume (i) when the reference point shifts to \( a' \), \( y \) is excluded in \( Q^1(\mathbf{a}) \), and (ii) both \( x, y \) are in \( Q^2(\mathbf{a'}) \). Then we have \( X \in C((x, y, a'), a') \), so \( x \) is favored in this case. Switching from \( a \) to \( a' \) has no implication (IAD and DSA do not hold).

For the boundary case, these type of shifts favor \( y \) as does the SQB model. Since \( Q^1 \) includes only strictly dominating alternatives, neither \( x \) nor \( y \) belongs to \( Q^1(\mathbf{a}) \) or \( Q^1(\mathbf{a'}) \). Hence, only \( Q^2 \) has a bite in this case. Given that \( Q^2 \) satisfies (i)–(iii), the PRD model behaves same as the SQB model for these type of shifts. \( \square \)

Appendix B. A screenshot (Fig. 5)

![A screenshot from the experiment.](http://dx.doi.org/10.1016/j.geb.2013.07.009)

Appendix C. Supplementary material (Instructions to the experiment)

Supplementary material related to this article can be found online at [http://dx.doi.org/10.1016/j.geb.2013.07.009](http://dx.doi.org/10.1016/j.geb.2013.07.009).

References


