Size of Ellsberg Urn

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behavior fundamentally changes when the uncertainty is explicitly specified and vaguely described (Elsberg, 1961)
Literature

Modeling Ambiguity

- multi-prior approach: e.g. Gilboa and Schmeidler (1989); Schmeidler (1989).
- two-stage approach: e.g. Segal (1987); Klibanoff, Marinacci and Mukerji (2005); Ergin and Gul (2009).
Literature

Testing Ambiguity

- Camerer and Weber (1992)-a survey
- Halevy, 2007
- Epstein and Halevy (2017)
- Chew et al. (2017)
Ambiguity Aversion
preference for known risk over unknown uncertainty
Ambiguity Aversion

two jars with yellow and blue beads
10 beads in each jar

Pick a color and a jar

Draw a bead from chosen jar

Won if color matches
Ambiguity Aversion

A robust finding
Our Question

In a typical ambiguity experiment, a subject chooses between bets on an ambiguous jar and a risky jar.

*How about comparing two ambiguous jars?*
why care?
Motivation

- Several real life decision problems involve evaluations of uncertainties generated by different underlying processes, i.e. two ambiguous jars are often compared.

- Identifying what matters in the underlying process generating ambiguity → Theories

I'm much more comfortable putting my money into football bets than into the stock market.

someecards
Day Laborer Example

Suppose:

You need a day laborer for a low skill job. Any worker with good intentions should be suitable.
(two outcomes: good or bad)

There are day laborers outside any home improvement retailer- where workers congregate.

One location with tens of workers & one with fewer workers.

Which location would you choose from?
Day Laborer Example

Location 1

vs.

Location 2
Wine Example

- no strategic considerations
- no observable difference
- no pairing
- no implication

Customer
- Ignorant
- first commit the color
- then choose a bottle randomly
- Outcome can be “good” or “bad”
Preference for the size

In two ambiguous processes:

- When the most optimistic and pessimistic scenarios are the same, is the level of ambiguity the same?

- Any preferences for the number of states in the state space generating ambiguity when the payoff relevant state spaces are the same?
Experiment

- design a context free experiment
- preferences between two ambiguous jars
- learn about underlying mechanism
120 UMich students participated in 40 min experiments conducted at Exp. Lab. of SI (thanks to Dr. Yan Chen)
average earnings about $24 (including $7 participation fee)
Experimental Design

- Black and white beads
- Risky (Rn) or Ambiguous (An)
- \( n \): # beads in the jar (2, 10, 1000)
- Risky (Rn) -- half-half
- Ambiguous (An) -- unknown composition
Each subject

- picks a color for each jar
- compares always two jars
- total of 14 binary comparisons
- only paid for one decision
Problem

- interpretation of choice

Jar M

- 10 beads
- pays $30

Jar N

- 1000 beads
- pays $30
Problem

- interpretation of choice

- strict preference vs indifference
**Solution:** Two Questions

**Version A**
- Jar M: 10 beads, pays $30.25
- Jar N: 1000 beads, pays $30

**Version B**
- Jar M: 10 beads, pays $30
- Jar N: 1000 beads, pays $30.25
**Solution:** Two Questions

<table>
<thead>
<tr>
<th>Version A</th>
<th>Version B</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>A10</td>
<td>A10</td>
<td>strictly prefer A10</td>
</tr>
<tr>
<td>A1000</td>
<td>A1000</td>
<td>strictly prefer A1000</td>
</tr>
<tr>
<td>A10</td>
<td>A1000</td>
<td>indifferent</td>
</tr>
</tbody>
</table>
# Experimental Design

<table>
<thead>
<tr>
<th>Type</th>
<th>A2</th>
<th>vs</th>
<th>A10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ambiguity</td>
<td>A10</td>
<td>vs</td>
<td>A1000</td>
</tr>
<tr>
<td>Risk</td>
<td>R2</td>
<td>vs</td>
<td>R10</td>
</tr>
<tr>
<td></td>
<td>R10</td>
<td>vs</td>
<td>R1000</td>
</tr>
<tr>
<td>Mixed</td>
<td>A2</td>
<td>vs</td>
<td>R2</td>
</tr>
<tr>
<td></td>
<td>A10</td>
<td>vs</td>
<td>R10</td>
</tr>
<tr>
<td></td>
<td>A1000</td>
<td>vs</td>
<td>R1000</td>
</tr>
</tbody>
</table>
Results: **Ambiguity** (N=116)

![Bar chart showing preference for size in ambiguity conditions.](chart.png)

- **Ambiguity 2 vs 10**
  - Strictly Prefer Larger: 75%
  - Strictly Prefer Smaller: 25%
  - Choosing High Prize: 25%

- **Ambiguity 10 vs 1000**
  - Strictly Prefer Larger: 75%
  - Strictly Prefer Smaller: 25%
  - Choosing High Prize: 25%
Results: **Ambiguity**  (N=116)

<table>
<thead>
<tr>
<th>Preference for</th>
<th>A2 vs A10</th>
<th>A10 vs A1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Larger Jar</td>
<td>62.93%</td>
<td>59.48%</td>
</tr>
<tr>
<td>Smaller Jar</td>
<td>8.62%</td>
<td>10.34%</td>
</tr>
<tr>
<td>Higher Prize</td>
<td>28.45</td>
<td>30.17%</td>
</tr>
</tbody>
</table>
Ratio Bias

the tendency for people to judge an event as more likely when presented as a large-numbered ratio:

For example, 10-in-100 is preferred to 1-in-10

Results: Risk (N=116)
**Ratio Bias: Ambiguity vs Risk**

Preferences for the jar size under ambiguity and risk, N=116

- **Ambiguity 2 vs 10**
  - Strictly Prefer Larger: 62.93%
  - Strictly Prefer Smaller: 8.62%
  - Choosing High Prize: 28.45%

- **Ambiguity 10 vs 1000**
  - Strictly Prefer Larger: 59.48%
  - Strictly Prefer Smaller: 10.34%
  - Choosing High Prize: 30.17%

- **Risk 2 vs 10**
  - Strictly Prefer Larger: 24.34%
  - Strictly Prefer Smaller: 16.38%
  - Choosing High Prize: 59.48%

- **Risk 10 vs 1000**
  - Strictly Prefer Larger: 31.03%
  - Strictly Prefer Smaller: 12.93%
  - Choosing High Prize: 56.03%
Control Ratio Bias (N=56)
Ambiguity Attitude (N=116)

- Ambiguity aversion diminishes (p<0.05)
- Ambiguity seeking does not change (p>0.05)
- Ambiguity neutrality increases (p<0.05)
Control Ambiguity Attitude

# of ambiguity averse = 63
# of ambiguity neutral = 11
Theories of Ambiguity

- Typically, ambiguity models take the state space as given and the process generating the state space is ignored.

- In our experiments two bets - each one on different size jars - (say, A2 and A10) have the same state spaces \{Black, White\}.

- What are the restrictions that our findings impose on the existing theories?
A Two-Stage Problem

Evaluation of a bet on drawing Black from an ambiguous jar of size $n$ as a two-stage procedure
Smooth Ambiguity Model (KMM, 2005)

$$\sum_{i=0}^{n} \varphi[EU(i\text{Black}/(n-i)\text{White})]p_i$$

where $\varphi$ determines the ambiguity attitude and
where $(p_i)_{i=0}^{n}$ is the subjective probability assigned to each arm.
**Smooth Ambiguity Model**

**Remark:** A decision maker, who uses the smooth ambiguity model with a concave $\varphi$, will prefer the second order stochastically dominating lottery, i.e. the larger jar.

- 68.25% of the ambiguity averse subjects (N=63), preferred larger jar under ambiguity.

- 90.91% of the ambiguity neutral subjects (N=11) preferred the higher prize.
Maxmin Expected Utility Model (Gilboa and Schmeidler, 1989)

- multiple beliefs are formed and evaluation based on the worst scenario that she believes.

- Note that there is no restriction on how multiple belief set depends on the size of the jar.

- To explain our data, for $N > n$, it must be more “plausible” not to have any paying color in the jar in size-$n$ than in size-$N$.

$$\min_{p \in \pi_n} p \cdot u(30) < \min_{p \in \pi_N} p \cdot u(30)$$

where $\pi_N$ and $\pi_n$ are the multi prior belief set.
Source Models

Source preference hypothesis (Fox, Tversky, 1995) modeled by Chew and Sagi (2008) as limited probabilistic sophistication and distinguished preference from different sources of uncertainty.

It is flexible to explain any behavior in our setup. The subjects need to be perceiving each jar as a different source.
Wrap-up

**Size Matters**
preference for larger ambiguous jar

**Ratio Bias**
has a bite, but there is more to it

**Ambiguity Attitude**
connection between preference for size and ambiguity attitude

**Guidance for new theories**
The size of the ambiguous state space matters and no existing model is sensitive to this aspect.
Thanks!

Any questions/comments?