Weighted Linear Discrete Choice

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Motivation

- capturing the probabilistic nature of choice
  - applied work demands randomness

- workhorse model is the logit model (a.k.a. MNL)
  - introduced by Bradley-Terry-Luce, popularized by McFadden
  - tractable: "closed-form solution"
  - probability of choosing x from choice set S is

$$\frac{w(x)}{\sum_{y \in S} w(y)}$$

- undesirable implications (Benkard and Bajari, 2001)

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Aim of Today's Talk

- offer a simple model of probabilistic choice
- two parameters
- useful in applications: "closed-form solution"
- microfoundation
- bonus: a simple characterization

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Things that might be covered today

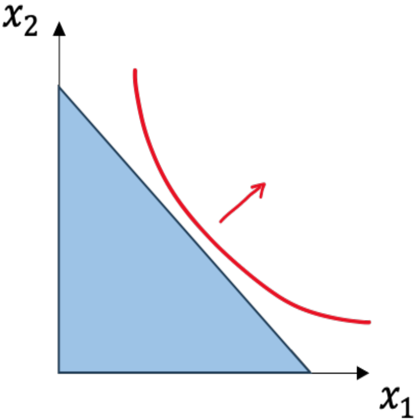
- application: firm competition
  - closed-form solutions for markups and number of firms
- simulations
- show identification when attributes are observable and the choice set is fixed

Some Notation

- X: a finite set of outcomes
- $S \subset X$: available options in the market
- $\rho(x|S)$ is the choice probability of x from S (Market Demand)
  - positive demand: $\rho(x|S) \geq 0$
  - availability : $\rho(x|S) = 0$ whenever $x \notin S$
  - unit demand: $\sum_{x \in S} \rho(x|S) = 1$

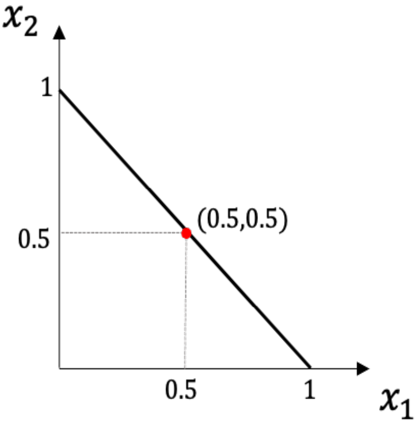
Model

Choice as Optimization



Consumption Bundles

Choice as Optimization



Lotteries with two outcomes

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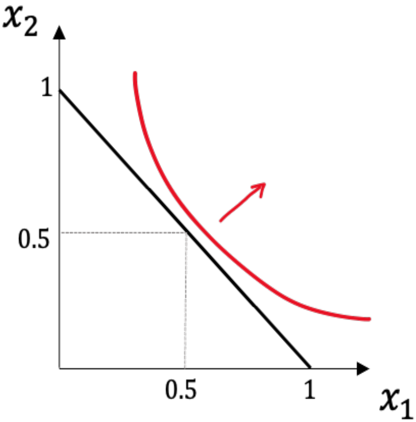
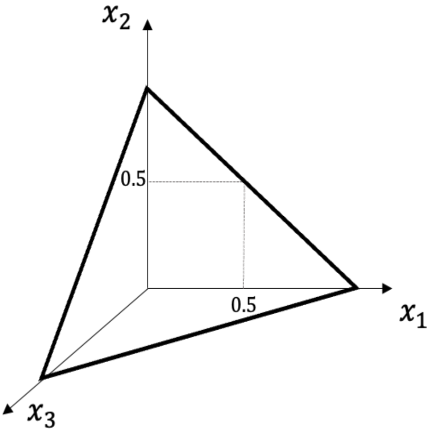
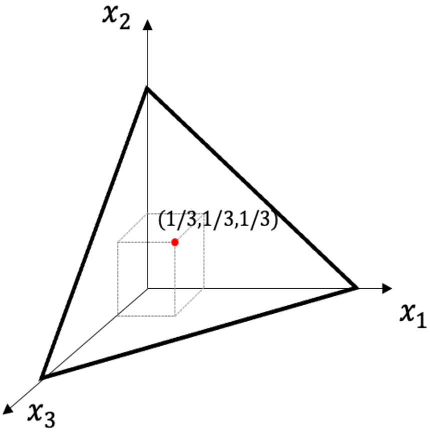


Lotteries



Lotteries with three outcomes

Choice as Optimization



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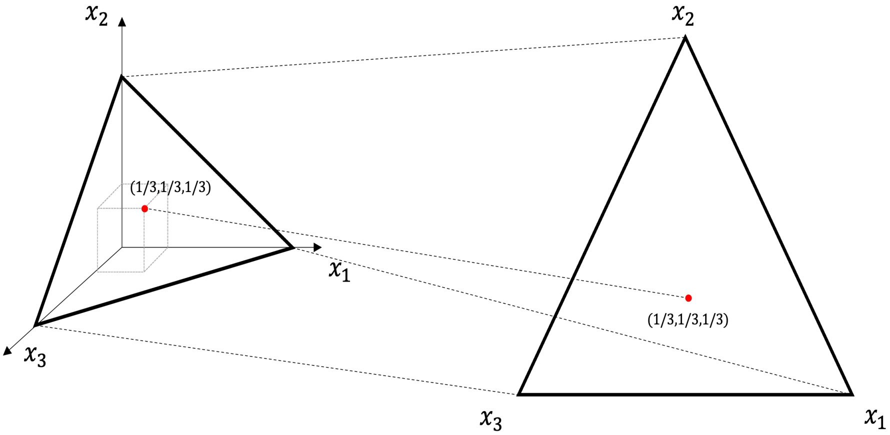
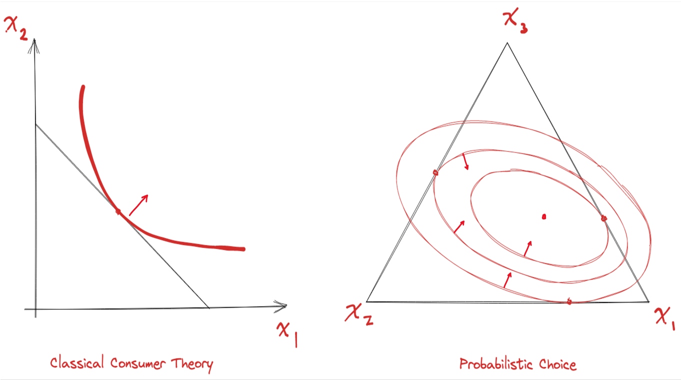


Marshak-Machina Triangle

Choice as Optimization

Machina (1985)



Probabilistic Choice as Optimization

- Which objective function?
- ours is a simple one

$$\max_{\rho(.|S)} \sum_{x\in S} \Big\{ \underbrace{u(x)\rho(x|S)}_{\text{expected utility}} - \underbrace{\frac{1}{2m(x)}\rho(x|S)^2}_{\text{cost}} \Big\}$$

subject to $\sum_{x\in S} \rho(x|S) = 1$

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Probabilistic Choice as Optimization

- Lagrange

$$\max_{\rho(.|S)} \sum_{x \in S} \Big\{ \underbrace{u(x)\rho(x|S)}_{\text{expected utility}} - \underbrace{\frac{1}{2m(x)}\rho(x|S)^2}_{\text{cost}} \Big\} + \underbrace{\Lambda(S)\Big[1 - \sum_{x \in S} \rho(x|S)\Big]}_{\text{constraint}}$$

- FOC

$$u(x) - \frac{\rho(x|S)}{m(x)} - \Lambda(S) = 0$$

Probabilistic Choice as Optimization

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- FOC

$$\rho(x|S) = m(x)u(x) - \Lambda(S)m(x)$$

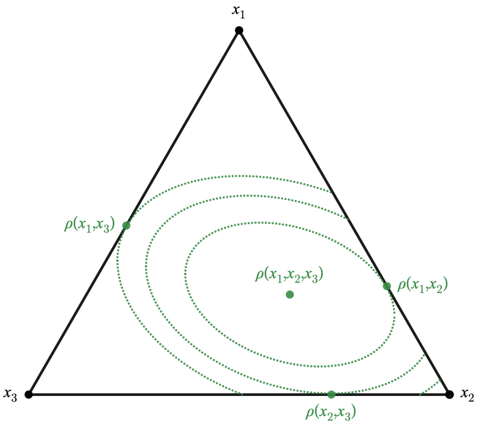
Probabilistic Choice as Optimization

- $\Lambda(S) = \frac{\sum m(x)u(x)-1}{m(S)}$: the Lagrange multiplier on S

$$\begin{aligned} \rho(x|S) &= m(x)[u(x) - \Lambda(S)] \\ &= \frac{m(x)}{m(S)} + m(x)[u(x) - \bar{u}_m(S)] \end{aligned}$$

- (u, m) is a WL representation for some ρ iff $u(x) > \Lambda(X)$ for all x.

Probabilistic Choice as Optimization



APU

Model

- $u(x) \in \mathbb{R}$, $m(x) > 0$
  - u: utility
  - m: salience/attractiveness

$$\rho(x|S) = \underbrace{\frac{m(x)}{\sum_{y \in S} m(y)}}_{\text{base probability}} + \underbrace{m(x)[u(x) - \bar{u}_m(S)]}_{\text{comparative probability}}$$

weighted average: $\bar{u}_m(S) \equiv \frac{\sum_{y \in S} u(y)m(y)}{\sum_{y \in S} m(y)}$

Weighted Linear Model (WL)

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Weighted Linear Model (WL)

An Example

- two distinct products: x and y
  - $u(x) = 3$ and $m(x) = 0.5$
  - $u(y) = 2$ and $m(y) = 0.5$
- base probability for x: $\frac{1}{2} = \frac{0.5}{0.5+0.5}$
- comparative advantage of x:
  - weighted average utility is $\frac{5}{2} = \frac{0.5*3+0.5*2}{0.5+0.5}$
  - $0.5(3 - \frac{5}{2}) = \frac{1}{4}$
- market share of x

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An Example

- lower $m(x)$ to 0.25
  - $u(x) = 3$ and $\boxed{m(x) = 0.25}$
  - $u(y) = 2$ and $m(y) = 0.50$
- market shares

	base probability	comparative	Total
x	1/3	1/6	0.50
y	2/3	-1/6	0.50

- trade-off

WL-Model

- u: utility
- m: salience/attractiveness

$$\rho(x|S) = \underbrace{\frac{m(x)}{\sum_{y \in S} m(y)}}_{\text{Base Probability}} + \underbrace{m(x)[u(x) - \bar{u}_m(S)]}_{\text{Comparative Probability Transfer}}$$

Special Case: I

$$\rho(x|S) \quad = \quad \frac{m(x)}{\sum_{y \in S} m(y)} + m(x)[u(x) - \bar{u}_m(S)]$$

$\bar{u}$ constant

$$\rho(x|S) = \frac{m(x)}{\sum_{y \in S} m(y)} + \underbrace{m(x)[\bar{u} - \bar{u}]}_{0}$$

$$\bar{u}_m(S) = \bar{u}$$

Special Case: II

$$\rho(x|S) = \frac{m(x)}{\sum_{y\in S} m(y)} + m(x)[u(x) - \bar{u}_m(S)]$$

$\bar{m}$ constant

$$\rho(x|S) = \frac{1}{|S|} + \bar{m}[u(x) - \bar{u}(S)]$$

simple average $\bar{u}(S)$

Special Case: II

$\bar{m}$ constant

$$\rho(x|S) = \frac{1}{|S|} + \bar{m}[u(x) - \bar{u}(S)]$$

$\bar{u}(S)$: simple average

- linear demand system featured prominently in many models of monopolistic competition
- Shubik and Levitan, 1980; Spence, 1976; Dixit and Stiglitz, 1977

Special Case: II

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- linear demand system featured prominently in many models of monopolistic competition
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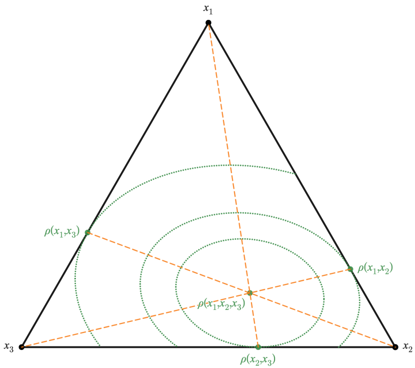
Special Case: II

$\bar{m}$ constant

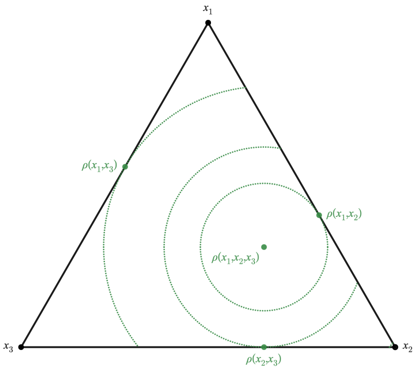
$$\rho(x|S) = \frac{1}{|S|} + \bar{m}[\bar{p}_S - p_x]$$

- $\bar{p}_S$ is average price in S
- $\bar{m}$ is a measure of market friction
  - if small, then price differences have little influence on demand
  - if large, market share is determined almost entirely by prices

Probabilistic Choice as Optimization



constant u



constant m

Uniqueness

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Theorem

Let (u, m) be a WL representation of ρ. Then (u', m') is a WL representation of ρ if and only if $u' = au + b$ and $m' = \frac{1}{a}m$ where $a > 0$.

- u unique up to affine transformations
- m unique up to scalar multiplication

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Accommodating Empirical Patterns

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- demand with the introduction of new products
  - "red bus-blue bus" problem
  - introducing a new product
  - larger choice sets
  - zero market demand
- cross-price substitution patterns

#1: Introducing Replicas

- "Red Bus-Blue Bus" problem (Debreu, 1960)

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0.50 0.50

- what happens when a blue bus is introduced?

? ? ?

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- MNL predicts

0.33 0.33 0.33

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- our model predicts

A B B

  - $u(Car) \geq u(Bus) \rightarrow 0.33 \leq A \leq 0.5$
  - $u(Car) \leq u(Bus) \rightarrow 0 \leq A \leq 0.33$

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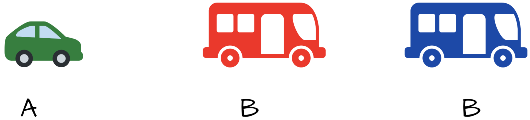

  - $u(Car) \geq u(Bus) \rightarrow 0.33 \leq A \leq 0.5$
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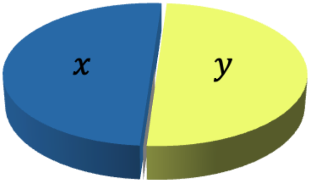
#2: Introducing New Product

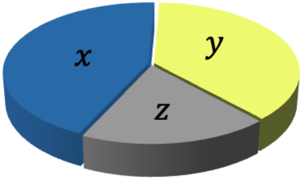
- what happens to the market demand for existing products when a new product is introduced?

Revisit the Example

- two distinct products: x and y
  - $u(x) = 2$ and $m(x) = \frac{1}{4}$
  - $u(y) = 1$ and $m(y) = \frac{1}{2}$



- introduce z: $u(z) = 1$ and $m(z) = \frac{1}{4}$
  - low u low m
  - relatively bad competitor



- introducing a third option increases the relative demand for the higher utility item

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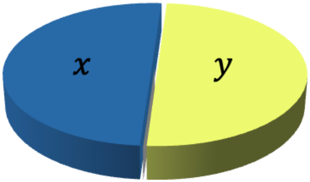

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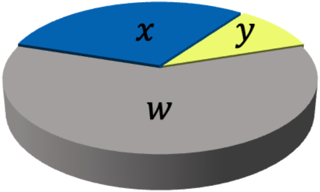


- introduce w: $u(w) = 2$ and $m(w) = \frac{1}{2}$
  - high u high m
  - relatively good competitor

- introducing a third option increases the relative demand for the higher utility item

#2: Introducing New Product

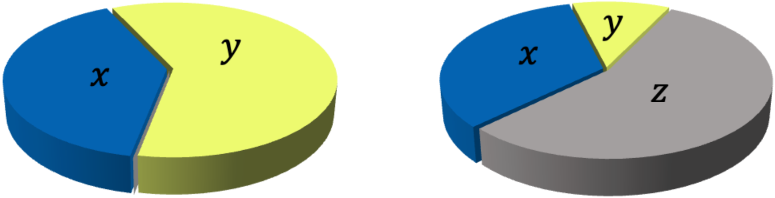
- larger choice sets increase the relative demand for the higher utility item

Proposition

Suppose $u(x) \geq u(y)$. Then $\rho(x|S) \geq \rho(y|S)$ implies $\rho(x|S \cup T) \geq \rho(y|S \cup T)$.

#2: Introducing New Product

- reversal is also possible



#3: Larger Choice Sets

- What happens as choice sets get VERY large?
  - $\rho(x|S) \to ?$ as $|S| \to \infty$
- Benkard and Bajari (2001) show
  - Multinomial logit, nested logit and random coefficients predict
  - $\rho(x|S) \to 0$ as $|S| \to \infty$
- WL model can allow for non-negligible market shares
  - $\rho(x|S)$ away from 0 as $|S| \to \infty$

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#4: Zero Market Demand

- Many models predict that the market share of any item must be positive
  - Benkard and Bajari (2001) show true whenever the conditional error distributions have unbounded upper support and a continuous upper tail
- WL model can easily allow for 0 probabilities
  - Entrants can drive some, but not other products out
  - Need to reformulate axiomatic foundation

#5: Cross-Substitution

- In MNL model,

$$\frac{\partial \rho(x|S)}{\partial m(y)} = -\frac{m(x)}{m(S)^2}$$

  - the same for all y
  - At odds with empirical evidence
  - Very restrictive

- In WL model,

$$\frac{\partial \rho(x|S)}{\partial m(y)} = -\frac{m(x)}{m(y)m(S)}\rho(y|S)$$

axioms

conclusion

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axioms
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Relation to APU

- WL

$$\max_{\rho(.|S)} \sum_{x\in S} \left\{ u(x)\rho(x|S) - \frac{1}{m(x)}\rho(x|S)^2 \right\}$$

- Additive Perturbed Utility (Fudenberg et al., 2015)

$$\max_{\rho(.|S)} \sum_{x\in S} \left\{ u(x)\rho(x|S) - k(\rho(x|S)) \right\}$$

where k is a strictly convex and smooth function.

  - cost: item-specific, but quadratic cost function
  - no closed-form solution
  - satisfies Strong Stochastic Transitivity, but outside of RUM

APU

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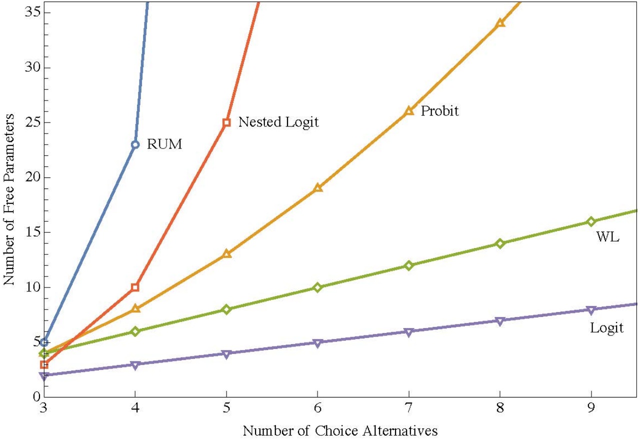
Does WL belong to RUM?

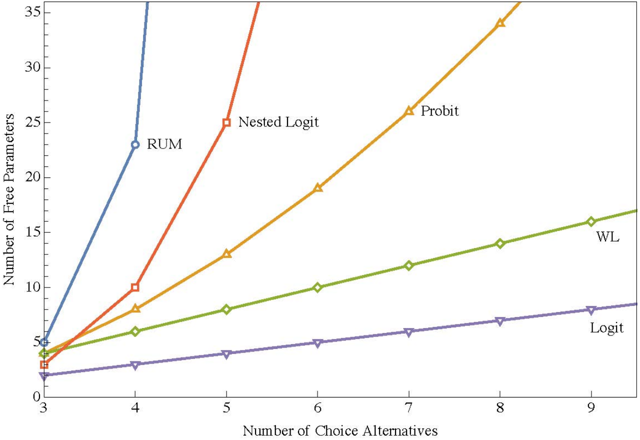
Model Comparisons

Number of Parameters



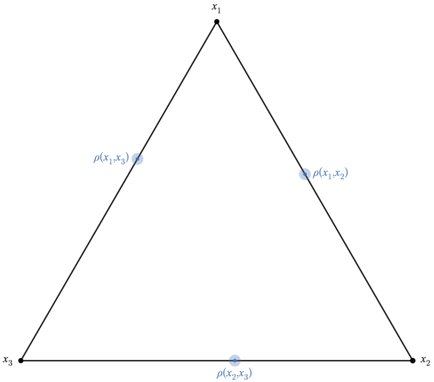
- MNL (RUM) is the most (least) parsimonious model
- WL and MNL's number of parameters increase linearly

Number of Parameters



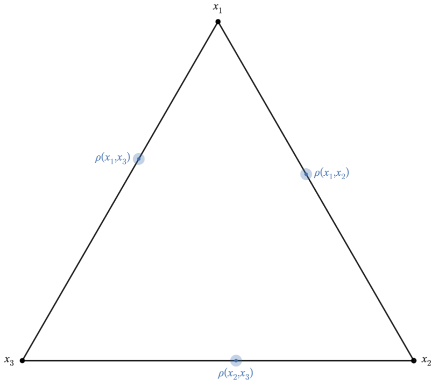
- MNL (RUM) is the most (least) parsimonious model
- WL and MNL's number of parameters increase linearly

Explanatory Power



- binary choices given
- ask possible trinary choices for each model

Explanatory Power



- binary choices given
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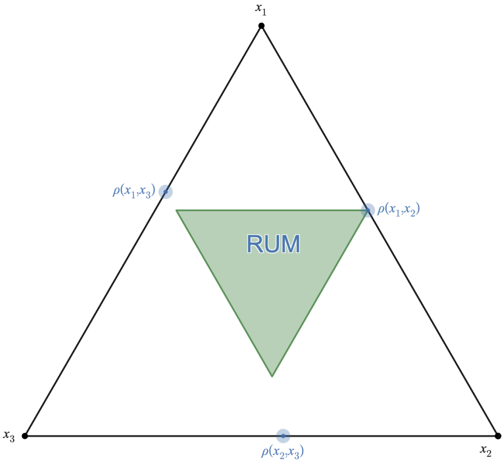
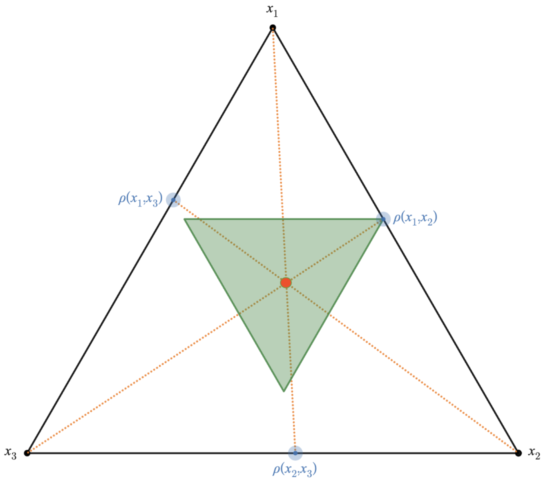
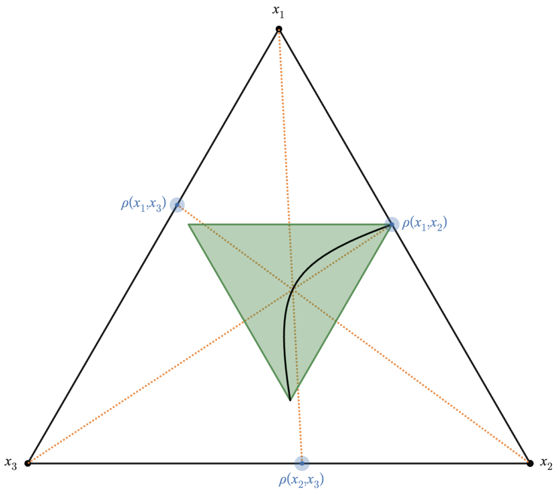
Explanatory Power-RUM



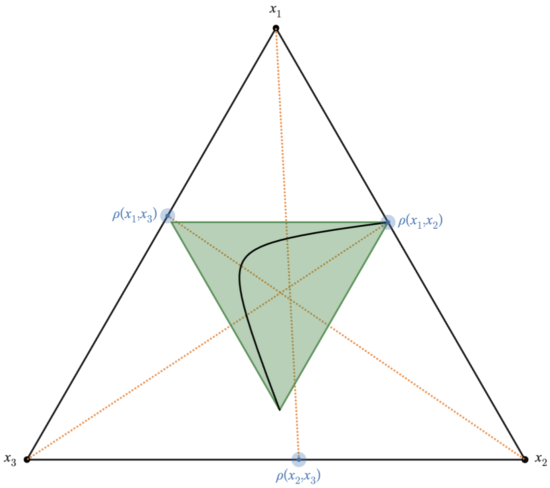
Explanatory Power-Luce



Explanatory Power-WL



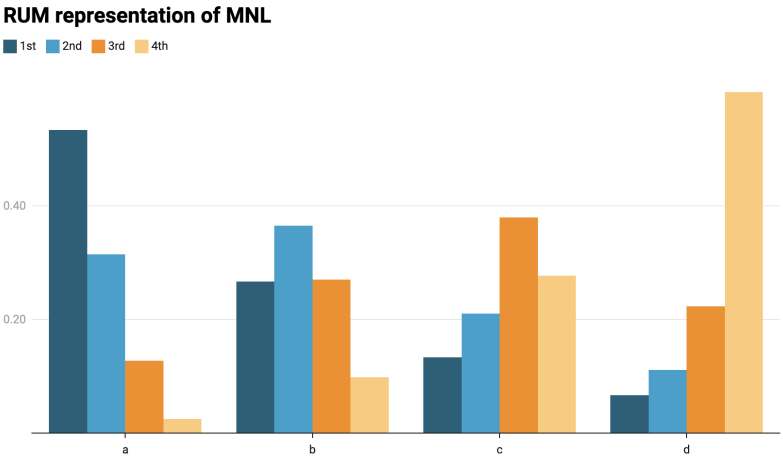
Explanatory Power-WL



Conclusion

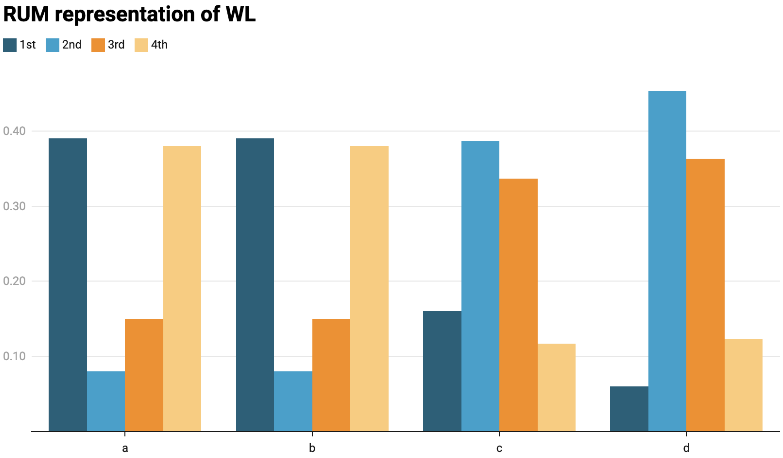
Polarization

No Polarization in MNL



- For products $\{a, b, c, d\}$
- MNL does not allow polarization

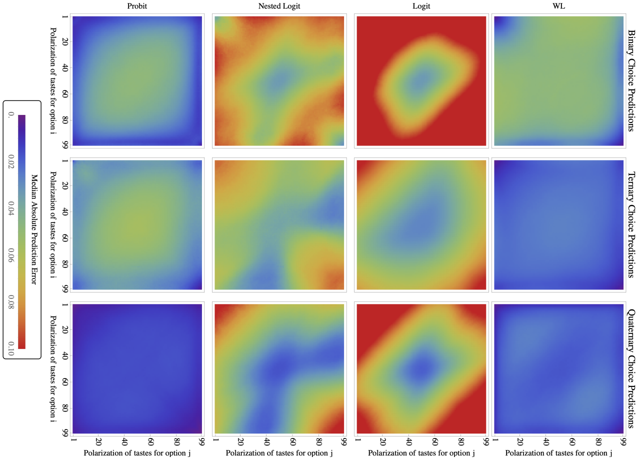
Polarization in WL



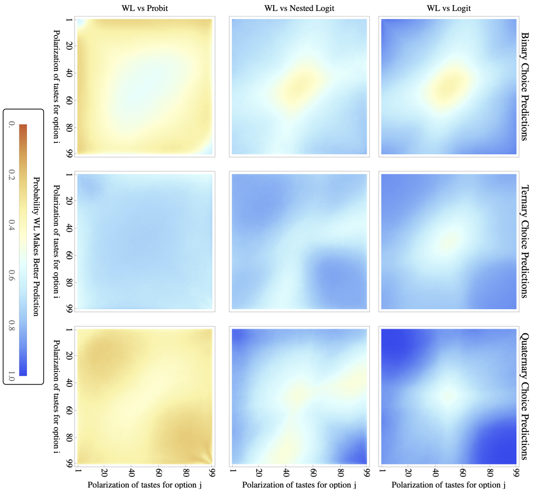
- a and b are polarized options
- WL allows polarization

Simulations

Simulations



Simulations



Conclusion

Axiomatic Foundations

Axiomatic Foundations

- Three axioms
- First axiom is positivity
  - Every alternative is chosen with positive probability

Axiom 1: $\rho(x|S) > 0$ for every $x \in S$ and $S \in \mathcal{D}$.

Axiomatic Foundations

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Axiom 1: $\rho(x|S) > 0$ for every $x \in S$ and $S \in \mathcal{D}$.

Axiomatic Foundations

- Second axiom is strict regularity
  - When the competition gets fiercer among alternatives (i.e. more alternatives), choice probabilities for any given alternative strictly decrease.

Axiom 2: $\rho(y|S) < \rho(y|S \setminus \{x\})$ for every $x \in S$.

Axiomatic Foundations

- For today's talk, assume $x, y \in S \cap T$ and $\rho(y|S) \neq \rho(y|T)$
- Consider a new auxiliary function

$$r_{S,T}(x,y) = \frac{\rho(x|S) - \rho(x|T)}{\rho(y|S) - \rho(y|T)}$$

  - measures the relative probability change of x and y from S to T
  - the relative probability levels rather than the absolute levels (as in Luce's IIA)

Axiomatic Foundations

- This function is independent of decision problems

$$r_{S_1,T_1}(x,y) = r_{S_2,T_2}(x,y)$$

- a stronger version is needed
- transitivity condition on the function

Axiom 3: For any x, y, z and $S_i, T_i \in \mathcal{D}$,

$$r_{S_1,T_1}(x,z) = r_{S_2,T_2}(x,y) r_{S_3,T_3}(y,z)$$

Characterization

Suppose $\mathcal{D}$ contains all menus with size 2 and 3. Then a stochastic choice function ρ has a WL representation on $\mathcal{D}$ if and only if it satisfies Axioms 1-3.

◂ Conclusion

Intuition of Proof

- We first define the salience of each alternative by using $r_{S,T}$ where S and T are menus with size 2 and 3
  - Fix $y^* \in X$ and define $m(y^*) = 1$
  - Then define $m(x) := r_{\{x,y^*\},\{x,y^*,z\}}(x, y^*)$
- Show that $\frac{m(a)}{m(b)} = r_{S,T}(a, b)$
  - Axioms guarantee that m is well defined
- Using the fact that the "shadow value" of a choice set is the same across all items chosen in the set, can define utility function
  - $u(a) - u(b) = \frac{\rho(a|S)}{m(a)} - \frac{\rho(b|S)}{m(b)}$
  - Again, axioms guarantee this is well defined
- Show that data can be represented by WL model with constructed parameters

◂ Conclusion

Identification with Attributes

- Empirical identification exercises typically fix a choice set
- Items have observable attributes
- Attributes enter into parameters in linear fashion
- Different than axiomatic approach previously
- Show how WL works in this environment

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Identification with Attributes

- A set of observable attributes of cardinality k
- a_i denotes the vector of attributes for product i
- Includes not only things that affect product quality, but also things like price, advertising, etc.
- Assume that there exists a vector β such that $u_i = \beta a_i$ for each i
- Similarly there exists a vector α such that $c_i = \alpha a_i$ for each i.

Proposition

Suppose that $u_i = \beta a_i$ *and* $c_i = \alpha a_i$ *where* a_i *is a* $k \times 1$ *vector. Suppose that we have at least* $2k$ *linearly independent observations of* $(\rho(i)a_i - \rho(j)a_j, a_i - a_j)$ *for* $i, j \in S$. *Then* β *and* α *are identified from choices in* S *up to positive scalar multiplication.*

- Key intuition: from the first order conditions of an optimization problem, we know that $\beta[\rho(i)a_i - \rho(j)a_j] = \alpha[a_i - a_j]$
- Have a set of linear equations

Conclusion

- WL model is a simple model of stochastic choice
- Nests well-known existing models: Luce and linear monopolistic competition
- Deliberate randomization
- Closed-form solution
- Tractable in applications
- Can capture well-known empirical phenomena
- Simple axiomatization
- easy to estimate
- Identifiable in standard empirical applications