ELECTORAL POLITICS, INTEREST GROUPS, AND THE SIZE OF GOVERNMENT

PETER J. COUGHLIN, DENNIS C. MUELLER and PETER MURRELL*

This paper considers how government size responds to a change in the influence of interest groups. First, an election model is developed that has an equilibrium and in which interest groups have unequal influence. The authors then show that an increase in a group's influence per se does not cause government size to increase but does cause its size to increase when the government (1) cannot change tax shares or (2) provides a good benefiting one (untaxed) group, whose sole interest is in maximizing its consumption of the good. The paper concludes with a discussion of some of the normative implications.

I. INTRODUCTION

Interest groups have been considered main actors in the political drama for some time now by political scientists (as in Bentley [1908], Schattschneider [1935], and Truman [1951]), and more recently by economists also (such as Olson [1965] and Stigler [1975]). Despite the widespread consensus on their importance there have been few formal attempts to analyze their impact on public policies. Thus, Borooah and Van der Ploeg [1983, 90] were moved to declare that "...theory based on the importance of different interest groups is badly needed in economics."

An important exception is Becker's [1983; 1985] analysis of interest-group competition. Becker draws insights about the nature of the trade-offs that occur when interest groups compete for political influence. But Becker consciously avoids incorporating into his model political institutions such as voting, voting rules, and candidates [1983, 391-4; 1985, 330]. Rather, he assumes that the political process operates as a veil through which the underlying strengths and demands of various interest groups are revealed.

This paper takes a different tack, but one complementary to Becker's approach: the analysis is based on an explicit model of the political process that includes interest groups. In the model, interest group in-

* Department of Economics, University of Maryland at College Park. Peter Coughlin gratefully acknowledges financial support provided by National Science Foundation Grant No. SES-8409352. Dennis Mueller gratefully acknowledges financial support provided by a grant from the Thyssen Foundation of West Germany. This paper has been improved by helpful comments and suggestions provided by Tom Borchering, Mel Hinich, Yew-Kwang Ng, Rodney Smith, the participants in the session "Elections II" at the 1988 Public Choice Society Meeting and seminars at Claremont Graduate School and Stanford University, and two anonymous referees.

Economic Inquiry
Vol. XXVIII, October 1990, 662-705 ©Western Economic Association International

Copyright (c) 2002 ProQuest Information and Learning Company
Copyright (c) Western Economic Association
fluence is a potential supply of votes for which parties (or candidates) compete. Each party seeks a winning coalition of voters and, thus, engages in "implicit logrolling" (Tullock [1959, 572]). An equilibrium exists in the competition for the support of interest groups, and the nature of the equilibrium electoral platforms is identified below. Embodying in the characterization of the equilibria is a set of parameters that can be said to measure the strengths of the various interest groups—and which enable one to identify how candidates change their policy positions in response to a change in the strength of an interest group. It is therefore possible to analyze simultaneously the existence of an electoral equilibrium, its nature (including its normative properties), and changes in equilibrium due to shifts in the balance of forces between competing groups. In contrast to Becker [1983; 1985], these analytical features are not assumed outright. Rather, they are based on an underlying model of elections. The theoretical underpinnings for this model are discussed in section II.

Interest groups are often alleged to be responsible, in part, for recent increases in the size of government in Western democracies. Therefore, in Sections III and IV, a model of the political process is used to study the impact of an increase in the influence of a particular interest group on government size. In the simple models of the public sector considered here, increasing group influence, per se, does not cause the size of the government to increase—but an increase in the government's size does result when (i) there are constraints on the government's ability to change tax shares in response to a shift in group power, or (ii) there is a publicly provided good which benefits one (untaxed) group, and that group's sole concern with government policy is to maximize

1. Previous works have individually examined elements of our interest group model. None, however, have provided results that could be directly applied to carry out these analyses. Becker [1983] models the effects of changing interest group influence, but he avoids discussion of the political system. Enelow and Hinich [1984a, 82-90] establish the existence of an electoral equilibrium and characterize the nature of the winning electoral platform for an alternative interest group model in which voters have quadratic utility functions and each interest group's bias term is normally distributed. They do not, however, address the effects of changing interest-group influence. Borooah and Van der Ploeg [1983] (in developing an empirical model) also model interest groups in a manner similar to the approach used in the present paper but do not address the questions of whether electoral equilibria exist, where such equilibria would be located, and what the effects of changing interest group influence are.

2. In the seminal paper in this literature, Tullock constructs an example of a community voting on road repairs where the presence of interest groups among the farmers causes government spending to be "considerably higher and at greater expense than is rational" [1959, 575]. He then asserts that, for this result to be generalized, "All that is necessary is that the benefits [of governmental activity] be significantly more concentrated than the costs" [1959, 577]. For discussion of subsequent work on the relation between interest groups and the size of government, see Mueller [1989, ch. 17].
the amount of the good that it gets. The paper concludes by considering some of the normative implications of the results.

II. INTEREST GROUPS IN THE ELECTORAL PROCESS

There are two sets of actors in the model, parties and interest groups, and two sets of relationships which must be analyzed: the relationships between interest groups and the parties, and the relationship between the parties themselves. With respect to the first question, this paper investigates the determinants of each interest group's influence on the platforms proposed by the parties. With respect to the second, it explores whether the competition for votes between the parties produces an equilibrium, and if so, what its properties are.

The question of whether candidate competition produces an equilibrium is not superfluous. As is well known, standard microeconomic assumptions about voters' preferences do not ensure that an equilibrium exists when policy spaces are multidimensional (Mueller [1989, ch. 10]). However, several recent studies have identified sufficient conditions for the existence of equilibria in a multidimensional election model under a "probabilistic voting" assumption (Mueller [1989, ch. 11]). These models assume a degree of uncertainty on the part of candidates as to how individuals will vote, or on the part of voters regarding candidate platforms. These assumptions seem plausible and in accordance with observed voter behavior (Fiorina [1981, 155]). Thus in this paper candidates are assumed to be uncertain about individual voter or interest group behavior, which affects both the outcomes of the competition between candidates and the influence of interest groups upon candidate platforms. A more explicit statement of the assumptions underlying the model and the implications regarding an equilibrium follow.

Envisage a "Downsian" political process in which the government faces a single rival, the opposition. These two decision-makers are denoted g and o, respectively. Voters form expectations of their future utility under each party's victory based on the past performances of

3. The probabilistic voting assumption is not a panacea that automatically ensures electoral equilibria, however. See, for instance, Hinich [1977], Enelow and Hinich [1984a, sec. 5.4], and Feldman and Lee [1988].

4. That is, a model in which public policies are determined by competition between political candidates. The assumption that the competition is between the government and an opposition party is made on pp. 54-56 in Downs [1957] (in his discussion of "the basic logic of government decision-making").

5. Here, as is common, "decision-maker" can refer to an individual (such as a Presidential candidate) or an organization (such as a political party).
the government and opposition (denoted by \( m_\text{g} \) and \( m_\text{o} \) respectively), and the currently proposed policy platforms of the government and opposition, \( s_\text{g} \) and \( s_\text{o} \). In any given election, the government has a fixed \( m_\text{g} \in M_\text{g} \) and the opposition has a fixed \( m_\text{o} \in M_\text{o} \). There is no need to restrict the nature of the two measures of past performance that are used. Current policy choices \( s_\text{g} \) and \( s_\text{o} \) are made from the sets \( S_\text{g} \) and \( S_\text{o} \) respectively. \( S_\text{g} \) and \( S_\text{o} \) are assumed to be nonempty, compact subsets of finite dimensional Euclidean spaces. A useful way to think of the difference between \( m \) and \( s \) is to think of a party’s \( m \) as a measure of past outcomes while previously in office and to think of its \( s \) as the way it says it will use its policy instruments.

A natural way to introduce interest groups is to represent “common interests” as a common utility function. Thus, assume that each member of interest group \( i \) has the same utility function, \( U_i(m, s, i) \), over the government’s past performance and current proposals, and the same utility function, \( U_i(m, s, o) \), over those of the opposition. Further, assume that \( U_i \) is continuous in both \( S_\text{g} \) and \( S_\text{o} \). Each individual is assumed to be a member of one and only one interest group, with an unassociated individual treated as a group of one.

If all members of interest group \( i \) were to vote for the government if \( U_i(m, s, g) > U_i(m, s, o) \); for the opposition if the reverse inequality holds; and abstain if \( U_i(m, s, g) = U_i(m, s, o) \), then all of the assumptions of the standard spatial competition model with deterministic voting would hold, with the concomitant implication that an equilibrium to the electoral competition process could not be presumed. Probabilistic elements are introduced by assuming, as in Fair [1978], that each voter has an expected utility bias in favor of \( (b_{ij} > 0) \) or against \( (b_{ij} < 0) \) the government. These biases can be thought of as arising from ideological and other non-policy related factors, the personal characteristics of the candidates, idiosyncratic historical factors and the like. In assessing the likelihood of obtaining the vote of the \( ij \) voter, the government and opposition must consider both the \( U_i \) common to all members of this voter’s interest group, and the individual’s non-policy bias, \( b_{ij} \). The random nature of these biases leads the candidates to view individual voter choices as random variables.

6. See, also, Fair [1978], Fiorina [1981, ch. 4], and Van der Ploeg [1987, 483-4].
7. See also Enelow and Hinich [1982; 1984a, sec. 5.2] and Borooah and Van der Ploeg [1983, sec. 6.1, 6.4].
8. For discussions of nonpolicy factors that may enter into voters’ decisions, see Enelow and Hinich [1982, 115-8; 1984a, 80] and Borooah and Van der Ploeg [1983, sec. 6.2, 6.3].
9. For a survey of other models where individual voter choices are viewed as random variables, see Mueller [1989, ch. 11].
government and the opposition believe that, for each voter $ij$, the bias term, $b_{ij}$, is a random variable with a uniform distribution over a real interval $(l_i, r_i)$.  

To guarantee the existence of an equilibrium, given the assumption of a uniform distribution of biases, the following assumption is also made: For any given $(m_g, m_o) \in M_g \times M_o$ and interest group, $i$, it is true that $l_i < U_i(m_g, s_g) - U_i(m_o, s_o) < r_i$. $(s_g, s_o) \in (S_g \times S_o)$. When it is convenient, $a_i$ is used to denote the density of $b_{ij}$ between $l_i$ and $r_i$, i.e., $a_i = 1/(r_i - l_i)$. Stated informally, this assumption says that, for each interest group, a randomly selected voter could always vote for either party with some non-zero probability. Thus, neither party can completely write off any particular interest group—since, no matter what feasible strategies are selected, there is always some chance of getting votes from each interest group. Here one is assuming that the feasible strategy sets are not "too big," the parties' uncertainty about the voters' choices is not "too small," or both.

The probabilistic nature of voter $ij$'s choice can now be made explicit. Let $P^g_{ij}(m_g, s_g, m_o, s_o)$ be the probability that voter $ij$ votes for the government, conditional on proposals of the government and those of the opposition. This probability is defined as

$$P^g_{ij}(m_g, s_g, m_o, s_o) = \begin{cases} 1 & \text{if } U_i(m_o, s_o) - U_i(m_g, s_g) < b_{ij} \\ 0 & \text{if } U_i(m_o, s_o) - U_i(m_g, s_g) \geq b_{ij} \end{cases} \quad (2.1)$$

where $b_{ij}$ is a random variable that satisfies the assumptions made in the two preceding paragraphs. Equations for $P^o_{ij}(\cdot)$ can be written analogously. As in Downs's [1957] formulation of "the basic logic of voting" [see, in particular, page 39], it is assumed that an individual votes if and only if there is a perceived difference between the candidates (since, in this model, such a difference exists for a given voter $ij$ if and only if $U_i(m_o, s_o) - U_i(m_g, s_g) \neq b_{ij}$).

Standard deterministic voting models assume that a voter votes with probability one for the candidate promising her the higher utility. In terms of modeling of interest groups, this assumption implies that a candidate either receives all of the votes of an interest group or none.

10. It is also assumed that there is a finite number of voters. Alternatively, following Fair [1978, 161-2] and Borooah and Van der Ploeg [1983, sec. 3.2], one could assume that there is a continuum of voters in each interest group, with the fixed bias terms evenly distributed across the voters in each interest group. Under this alternative, government and opposition would be assumed to believe that, for each interest group $i$, $b_{ij}$ is a random variable with a uniform distribution over $(l_i, r_i)$.

11. This assumption or one similar to it is used by Fair [1978], Hinich [1978], and Enelow and Hinich [1982; 1984a].
of them. Viewed spatially, the deterministic voting assumption implies that the probability of getting voter $ij$'s vote is zero for all points which lie outside of the indifference contour passing through the point representing the opposition's platform, and one inside this contour. The probability of getting $ij$'s vote is a discontinuous step function that jumps from zero to one as the government crosses the indifference contour at the utility level promised by the opposition.

In contrast, this paper's assumptions imply that the probability of getting $ij$'s vote is a continuous function that rises smoothly as the government's platform gets closer to a voter's ideal point. A candidate wins some but not all of an interest group's vote, and the higher the utility level it promises a representative member of an interest group relative to the other candidate's platform, the larger the fraction of the interest group's vote that he wins. Not surprisingly, the continuity in the voter's response introduced through the probabilistic voting assumption plays a crucial role in establishing the existence of an equilibrium.

Each political party is assumed to choose its electoral platform to maximize its expected margin of victory, or plurality, at the present time—ignoring the impact of the choice on the future values of the measures of its performance (i.e., on future $m_g$ and $m_o$). Given these objectives: For any given $m_g \in M_g$ and $m_o \in M_o$, a pair of strategies, $(s^*_g, s^*_o)$, is an equilibrium in the two-party electoral competition if and only if $s^*_g$ maximizes

$$\sum_{i=1}^{N} n_i a_i \cdot U_i(m_g, s_g) \text{ over } S_g,$$ and $s^*_o$ maximizes

$$\sum_{i=1}^{N} n_i a_i \cdot U_i(m_o, s_o) \text{ over } S_o.$$  

This conclusion about what results from the competition between the political parties can be stated in a simple, and easily interpretable, manner: each party acts as if it maximizes an objective function which is a specific weighted sum of the utilities of all citizens. Equilibrium in a multidimensional context comes not at the median of voter ideal points, but at a weighted mean. This conclusion therefore provides, within the context of our model, the answer to one of the fundamental questions of political economy: whose preferences affect collective decisions and with what weight?

---

12. This conclusion follows because the payoff function is separable with respect to $s_g$ and $s_o$. For a more detailed analysis of two-candidate games with separable payoff functions, see Enelow and Hinich [1984a; 1984b]. A proof of this conclusion is contained in Coughlin, Mueller and Murrell [1990].
Note that parties do not necessarily choose the same strategies \( (s^*_g \neq s^*_o) \), since the model includes the possibilities that (1) \( S_g \neq S_o \) (2) \( m_g \neq m_o \) and (3) even when \( S_g = S_o \) and \( m_g = m_o = m \), the function
\[
\omega(s \mid m) = \sum_{i=1}^{N} n_i a_i U_i(m,s)
\]  \( (2.2) \)
may have more than one maximum. Also, in those situations in which \( s^*_g = s^*_o \) and \( m_g = m_o \) it is not the case that no one votes. Rather by (2.1), in such situations each citizen for whom \( b_{ij} \neq 0 \) votes. When \( s^*_g = s^*_o \), the distribution of votes between government and opposition is determined entirely by the distributions of the bias terms. Finally, the equilibria in our model also have the following normative property: For any given \( m_g \in M_g \) and \( m_o \in M_o \) each equilibrium, \( (s^*_g, s^*_o) \), is such that \( s^*_g \) is in the Pareto optimal set in \( S_g \), and \( s^*_o \) is in the Pareto optimal set in \( S_o \).

III. THE INFLUENCE OF INTEREST GROUPS

The social choice literature is replete with proofs of the non-existence of voting equilibria or of the inefficiency of the outcomes chosen. In contrast, the results with probabilistic voting models like the one discussed in section II often have a different tenor. Under the assumptions made above, there is an equilibrium, and this equilibrium is Pareto optimal relative to the government and the opposition's feasible strategy sets.

It is assumed that some subsets of voters have identical utility functions. This assumption captures what we believe to be a salient feature of modern political processes. In choosing strategies, both government and opposition weigh the impact of their policies not on the individual voter, but on groups of voters. Competition is for the votes of steelworkers, religious fundamentalists, environmentalists, teachers, and gun owners. Members of each group are given the same weight by government and opposition, but the groups themselves can have different weights. The influence of any one voter on the outcome of the political process depends upon the group to which he or she belongs. Moreover, voters wishing to increase their influence must work

13. These Pareto optima might be second-best if the policy choices of the parties are constrained.

14. Kennelly and Murrell [forthcoming] show empirically that such weights can be expected to vary systematically across the different sectors of society.
through interest groups. Efforts to increase one's benefits from the political process translate into efforts to increase the influence of one's own group.

The influence that a group has on the political parties' choices of platforms is inversely related to the degree to which politicians are uncertain about the preferences of members of the interest group. In the model, this uncertainty occurs because politicians know only the distribution of the biases of the members of an interest group, not the individual biases themselves. The degree of uncertainty is measured by the parameter \( a_i \), which is inversely related to the dispersion of the bias terms. An increase in the homogeneity of the group (a reduction in the variance of the bias terms) increases the influence of the group. Moreover, interest groups can increase their influence by reducing the candidates' perceptions of the variance in the bias terms for their group, and thereby increasing the magnitude of their \( a_i \) weight in the candidates' objective functions.

There are many ways in which an interest group can reduce politicians' uncertainty about the views of its members. A primary purpose of group formation is to make politicians aware of the existence of the group and its common interest. Lobbyists attempt to convince politicians that a certain policy is paramount in the minds of particular voters—that is, that the bias terms are less important than the policy-related utilities \( U_i(l) \). Campaign contributions enable the group to obtain the ear of the candidate to better convey this information.

Campaign contributions are not built directly into the model, however. To do so would require modeling both the flow of campaign contributions from interest groups to candidates, and the allocation of these funds by the candidates.\(^{15}\) Such an exercise would greatly complicate the model without altering the basic properties used in the subsequent analysis of the size of government: that an equilibrium exists, and that interest groups receive different weights in the objective function implicitly maximized through the choice of equilibrium platform.

Given these properties of the model, one can now analyze the effects of changes in interest group influence on government policies. An increase in the strength of interest group \( i \) is represented by an increase in \( a_i \). Let us now examine the effects of changes in interest group strength on the size of government.

---

15. For recent models of the political process that do incorporate campaign contributions, see Denzau and Munger [1986] and Austen-Smith [1987].
IV. THE COMPARATIVE STATICS OF INTEREST GROUP STRENGTH AND GOVERNMENT SIZE

Assume that there is one pure public good and one private good, with units defined so that each price equals one. The amount of the public good consumed by each individual, $G_i$, is selected by the government as one of its policies. To pay for the public good, the government chooses tax shares, which are the same for individuals in the same interest group and are denoted by $t_i$. The government’s budget balance equation is then

$$ G = \sum_{i=1}^{N} t_i \cdot n_i \cdot G. \quad (4.1) $$

When $G > 0$, (4.1) is equivalent to

$$ \sum_{i=1}^{N} t_i \cdot n_i = 1. \quad (4.2) $$

Every individual in group $i$ has the same income, $y_i$, which is used to pay taxes or to buy a private good. Thus,

$$ t_i \cdot G \leq y_i, \ i = 1, \ldots, N. \quad (4.3) $$

It is assumed that, in the next election, voters judge the government by its present policies alone, more specifically, that the set of feasible alternatives for the government is

$$ S_g = \{ (G, t_1, \ldots, t_N) \in E^{N+1} : G \geq 0; \sum_{i=1}^{N} t_i \cdot n_i = 1; \} $$

$$ t_i \geq 0 \ \& \ t_i \cdot G \leq y_i, \ i = 1, \ldots, N \}. \quad (4.4) $$

As all individuals in group $i$ are identical, except for their bias terms (which do not affect their private consumption decisions), their consumption of the private good is identical and denoted $X_i$. It is assumed that in evaluating the incumbent government voters have preferences only over their own final consumption, $G$ and $X_i$. These preferences are represented by a utility function, $u_i(G, X_i)$. To make the analysis tractable, the utility function is assumed to be strongly additive.\(^{17}\)

\(^{16}\) These can be thought of as actually being single goods or, alternatively, as being composite commodities.

\(^{17}\) Strong additivity implies that all goods are normal and substitutes. The latter holds by necessity with two goods.
\[ v_i(G, X_{ij}) = u_i(G) + v_i(X_i). \] (4.5)

\( u_i(\cdot) \) and \( v_i(\cdot) \) are both assumed to be strictly increasing, twice continuously differentiable, and strictly concave. The budget constraint for voter \( ij \) is then exactly satisfied:

\[ X_i = y_i - t_i \cdot G. \] (4.6)

Thus the indirect utility function on \( S_g \) for any voter \( ij \) can be written as:

\[ u_i(G) + v_i(y_i - t_i \cdot G). \] (4.7)

Given the previous assumptions:

\[ U_i(m_g; G, t_1, \ldots, t_N) = u_i(G) + v_i(y_i - t_i \cdot G). \] (4.8)

It is easy to see that the set \( S_g \) defined by (4.4) is non-empty and compact and that \( U_i(m_g; G, t_1, \ldots, t_N) \) is a continuous function of \((G, t_1, \ldots, t_N)\) for each \( m_g \in M_g \). Thus the assumptions made in this section are consistent with those on \( S_g \) and \( U_i(m_g; s_g) \) in section II. As a consequence, the characterization of the electoral equilibria that was stated at the end of that section immediately implies: In equilibrium, the government chooses an \( s_g^* = (G^*, t_1^*, \ldots, t_N^*) \) that solves

\[
\max_{(G, t_1 \ldots, t_N)} \sum_{i=1}^{N} a_i \cdot n_i \cdot [u_i(G) + v_i(y_i - t_i \cdot G)]
\]

subject to \( t_i \cdot G \leq y_i, \ i=1, \ldots, N; \)

\[
\sum_{i=1}^{N} t_i \cdot n_i = 1; \quad t_i \geq 0, \ i=1, \ldots, N; \quad G \geq 0. \] (4.9)

This conclusion gives a particularly simple characterization of government policy, which makes analyses of equilibrium tractable. For example, it is straightforward to show that the government’s policies satisfy the usual marginal conditions for Pareto optimality. When an interior solution exists to the maximization problem (4.9), that solution must satisfy

\[
\sum_{i=1}^{N} \frac{[n_i \cdot (\partial v_i / \partial G)] / (\partial v_i / \partial X_i)}{1} = 1 \] (4.10)
at the maximizing value, \((G^*, t_1^*, \ldots, t_n^*)\), of the government policies. Equation (4.10) is Samuelson's [1954] familiar condition for Pareto optimality when the marginal rate of product transformation equals the goods-price ratio.

This characterization of the government's decision can now be used to study the impact of changes in the \(a_i\) on the size of government. The following example shows that, when government policies are unconstrained, no determinate sign can be given to the change in \(G\) resulting from an increase in the strength of one interest group. Since this is a negative result, an example suffices to show the indeterminacy.

**Example:** Assume there are two interest groups \((i = 1, 2)\) and that they have one voter each (i.e., \(n_1 = 1\) and \(n_2 = 1\)). Assume that \(y_1 = 7\) and \(y_2 = 10\). Finally, assume for both \(i = 1\) and \(i = 2\), that \(u_i(G) = -4 \cdot (17 - G)^2\) and \(v_i(X_{ij}) = -(y_i - X_{ij})^2\).

In this example, (4.9) becomes

\[
\max_{(G, t_1, t_2)} \left[ -a_1 \cdot 4 \cdot (17 - G)^2 - a_1 \cdot (t_1 \cdot G)^2 - a_2 \cdot 4 \cdot (17 - G)^2 - a_2 \cdot (t_2 \cdot G)^2 \right]
\]

subject to \(t_1 + t_2 = 1, t_1 \cdot G \leq 7, t_2 \cdot G \leq 10, t_1 \geq 0, t_2 \geq 0, G \geq 0.\)

(4.11)

By substituting the equality constraint into the objective function, this optimization problem can be rewritten as

\[
\max_{(G, t_1)} \left[ -a_1 \cdot 4 \cdot (17 - G)^2 - a_1 \cdot (t_1 \cdot G)^2 - a_2 \cdot 4 \cdot (17 - G)^2 - a_2 \cdot \left[ (1 - t_1) \cdot G \right]^2 \right]
\]

subject to \(0 \leq t_1 \leq 1, 0 \leq G \leq 17.\)

(4.12)

It can be shown that the objective function at (4.12) is strictly concave over the entire feasible region.\(^{18}\) Therefore, one can characterize the choice of government policy using the usual calculus methods. If the government chooses a positive \(G\), its policies are:

\[
t_1 = a_2 / (a_1 + a_2), \quad t_2 = (1 - t_1) = a_1 / (a_1 + a_2)
\]

(4.13)

18. The demonstration of this point is omitted for the sake of brevity.
\[ G = \frac{[4 \cdot (a_1 + a_2)^2]}{[4 \cdot (a_1 + a_2)^2 + a_1 \cdot a_2]} \cdot 17. \] (4.14)

Therefore \(^{19}\)

\[ \text{sgn}(\partial G/\partial a_1) = \text{sgn}(a_1 - a_2). \] (4.15)

Since the assumptions rule out neither \(a_1 > a_2\) nor \(a_1 < a_2\) (nor indeed equality), the example shows that an increase in the strength of the first interest group (i.e., \(a_1\)) is consistent with either a decrease or an increase in government size. Hence the example has established:

**PROPOSITION 1:** When government can choose policy values freely within the feasible set, an increase in the strength of an interest group can lead to either an increase or a decrease in the size of government.

The intuition behind this result has two elements. First, policies chosen by the government represent a compromise between the various interest groups because the objective function is a weighted sum of utilities. Thus government spending at any time can be “too high” for some group, given its tax share. Second, the political process is a zero-sum game for groups as a whole. An increase in the influence of one group must imply decreasing influence for others. Thus, if there is a transfer of influence toward the group that views government spending as too high, the size of government might decrease. (There is, of course, an opposing effect—the group whose influence has increased has its tax price lowered. Thus this group changes its opinion on the extent to which \(G\) is too high. If this effect dominates, government size increases.)

The analysis above shows that an increase in the size of government cannot be presumed merely by postulating an increase in interest group influence. A model with only one good supplied by the government, however, might be too restrictive to capture fully the activities of interest groups. Such groups often focus on increasing the government’s awareness of programs that particularly favor the group’s members. To include such programs in the model, let us introduce a third type of good. \(Z\) is a mixed private-public good providing \((1 - \theta)\) units of private good benefits to each member of \(i\), and \(\theta\) units of public good benefits to all groups per unit provided, \(0 \leq \theta < 1\). This is a natural way to represent a good favoring an interest group since targeted public

---

19. As usual, the symbol \(\text{sgn}(\cdot)\) is being used for the “signum function” which assigns +1 when its argument is positive, assigns -1 when its argument is negative, and assigns 0 when its argument equals 0.
outlays often have spillover effects that make them semi-public goods. Indeed, it can be argued that these spillover effects might be the reason why this kind of good is introduced into the budget.

Assuming group 1 is the favored group, the government now chooses a program \((G^*, Z^*, t_1^*, \ldots, t_N^*)\) that solves: \(^{20}\)

\[
\max_{(G, Z, t_1, \ldots, t_N)} \left\{ a_1n_1u_1(G + \theta Zn_1) + a_1n_1v_1[y_1 - t_1G + (1 - \theta - n_1t_1)Z] + \sum_{i=2}^{N} [a_nu_i(G + \theta Zn_1) + a_nv_i(y_i - t_iG - n_1t_iZ)] \right\}
\]

subject to the constraints in (4.9) and \(Z \geq 0\). (4.16)

The following result establishes that merely positing the possibility of introducing mixed private-public goods is not sufficient to make definite predictions about the direction of change in the size of government. \(^{21}\)

**PROPOSITION 2:** Suppose that (4.9) has a unique solution and that the maximizing values chosen by the government, \((G^*, t_1^*, \ldots, t_N^*)\) are in the interior of the feasible set. Then, by choosing the same solution and setting \(Z = 0\), one also maximizes (4.16).

Thus, even if an increase in interest-group activity results in demands for the introduction of mixed private-public goods, the government has no incentive to respond to these demands by expanding its size. The government can achieve its objectives by appropriately setting tax shares and the level of spending on the pure public good, while keeping \(Z\) zero. Since the government can achieve its objectives if \(Z\) is equal to zero, the implications of theorem 4 are now relevant. One cannot predict the direction of change in the level of government spending resulting from an increase in interest group activity. The introduction of mixed private-public goods does not lead to any definite predictions on the way in which enhanced interest-group influence affects government size.

An important assumption for propositions 1 and 2 is that all government policy variables can be freely changed from their initial equilibrium positions. It is sometimes argued, however, that taxes are more

---

20. A proof that the present formulation of the government’s problem satisfies the assumptions of section II is omitted for brevity’s sake.

21. Derivations for propositions 2-6 are in the appendix.
difficult to alter than expenditures (Buchanan [1967, ch. 5]). This may be particularly true in a political system in which tax payments are not set for individual citizens, but are related to individual characteristics such as income or expenditure. When tax shares cannot be individually set, the government may find it difficult to target favors to the members of a group that has increased its influence. Given such rigidities, a rise in the influence of the first interest group, $a_1$, causes the government to introduce the good that favors group one into its budget.

PROPOSITION 3: Suppose that the government has chosen a solution to (4.9) such that $Z^* = 0$ and $(G^*, t_1^*, ..., t_N^*)$ is in the interior of the feasible set. Then, if $a_1$ rises and tax shares must remain fixed, the government chooses $Z > 0$.

Once $Z$ is introduced into the budget, it can be shown that total government spending rises. The reason for the increase is simple. If both public and private goods are normal, consumers want to allocate any change in spending power to changes in the consumption of both goods. When $Z$ rises, citizens do not vote for a completely offsetting reduction in $G$ because they also want to reduce their spending on the private good.

PROPOSITION 4: Suppose that the government has chosen a solution to (4.9) such that $Z^* = 0$ and $(G^*, t_1^*, ..., t_N^*)$ is in the interior of the feasible set. Then, if $a_1$ rises and tax shares must remain fixed, total government spending rises.

Thus, when the new good only adds to the quantities of existing goods, some rigidity in the policy process is required to generate an increase in the size of government. However, if a wholly new good is now introduced, then determinate predictions arise without assuming rigidities.

Many interest groups do not lobby for the total welfare of their members. Rather, the activities of such groups focus on a small set of issues that are of deep concern to their members. Often, such groups claim to represent 'single-issue' voters (e.g., tobacco farmers or defense contractors).

The electoral decisions of single-issue voters might be viewed by politicians as largely independent of the political parties' programs on public goods and taxes. This view does not imply that the welfare of such voters is unaffected by spending and taxes, but rather that incentives are too weak to ensure that politicians become informed about preferences on matters other than the single issue. When taxes are primarily a function of income levels and tastes for public goods are sim-
ilar, many individuals might view themselves as roughly identical to a large class of voters on most issues. Given costs of political participation, they do not make their views known on standard issues. They concentrate their efforts on programs in which their interests differ markedly from other voters. The single-issue stance, then, is solely a consequence of the free-rider problem.

In formulating electoral platforms, politicians might, at an acceptable level of approximation, view some groups as voting purely on the basis of a single issue. This possibility suggests a different way in which the good Z should enter the model. There might exist goods, which provide benefits only to some groups, of which the government is unaware. Many groups organize mainly to point out the potential gains to their members from the provision of such goods, and to bargain with the government for the public provision of these goods. In such a case, Z may be viewed, not as a substitute for existing private and public goods, but rather as a third type of good providing benefits to i that differ from those provided by G or X. Z might be a pure public good for all members of i, but one that also has smaller spillovers (or none) for other groups, for example, a dam in a river, where group i comprises citizens living near the river. For members of i, Z is a public good, but it is not necessarily a perfect substitute for existing public goods. Therefore assume that the policy-related utility for each member of the group in question can be written as \( w(Z) \). As with \( u_i \) and \( v_i \), assume that \( w(\cdot) \) is strictly increasing, twice continuously differentiable, and strictly concave. Although one could allow for spillovers from Z onto other groups, it strengthens the conclusions to abstract from these and assume that Z benefits only one group.

The set of individuals benefitting from Z will be called group \( \mu \). The group has \( n_\mu \) members whose voting decisions focus solely on Z. The density of the bias term, \( b_{\mu \mu} \), between \( l_\mu \) and \( r_\mu \) (for the group \( \mu \)) is denoted by \( a_{\mu} \). Therefore, using our interest group model again, the government ends up acting as if its objective function is

\[
\sum_{i=1}^{N} a_{\mu} n_i[u_i(G) + v_i(y_i - t_iG - t_iZ)] + a_{\mu} n_\mu w(Z). \tag{4.17}
\]

There are two ways in which group \( \mu \) changes government decisions. First, the group’s formation changes the objective function from (4.9) to (4.17). Assume that it is in the government’s interests to set \( Z > 0 \).

---

22. To “prove” that \( Z > 0 \) would involve postulating conditions on the size of \( a_{\mu} n_\mu w'(0) \). Such a proof would add little to our understanding.
The formation of the group then leads to an increase in the size of government.

PROPOSITION 5: Suppose that the government has chosen a solution to (4.9) such that \((G^*, \phi_1^*, \ldots, \phi_N^*)\) is in the interior of the feasible set. Then, if \(Z > 0\) is introduced into the government budget, total government spending rises.

This result is unsurprising given that \(Z\) is introduced into the budget. Nevertheless, the scenario underlying proposition 5 is an important one in examining the effect of increases in interest group strength as a whole. When one thinks of such increases, one often implicitly assumes that the increase is relative to some non-interest group element in the polity. Such a relative increase can be modelled by assuming the formation of a group whose objectives center on an element of government policy that does not benefit other elements of society. Group \(\mu\) is such a group.

The changing fortunes of group \(\mu\) can affect government decisions in a second way as well. If the group already exists and then is able to increase its strength, the following result holds:

PROPOSITION 6: Suppose that the government has chosen a maximum of (4.16) such that \((G^*, \phi_1^*, \ldots, \phi_N^*)\) is in the interior of the feasible set. Then, if \(a_\mu\) increases, the size of government also increases.

The reason for the determinate result found in proposition 6 is straightforward. When the group's strength, \(a_\mu\), rises, the government is moved to help group \(\mu\). Given that this group's preferences about taxes and public goods are not known to the government, help can come only through an increase in \(Z\) and this increase does not cause a matching decrease in \(G\).

To summarize: some interest groups use their influence to shift the burden of taxes to other groups. Others attempt to influence the quantities of public goods provided. But the existence of interest groups of these types does not lead to a systematic relationship between interest-group strength and government size, since interest groups can have a "taste" for either more or less pure public good, and a mere reweighing of these tastes does not generate an unambiguous relationship. It is the existence of goods whose benefits are concentrated on a particular group that can explain why an increase in the number of organized interest groups might lead to an increase in government size. If interest groups are assumed to be of two types, either seeking to influence the levels of taxes and pure public good expenditures or trying to obtain goods like \(Z\), then any increase in the number of organized groups is
likely to bring with it an increase in both types. The increase in the
former has no systematic effect on government size, but the increase
in the latter increases the number of Z-type goods provided and,
thereby, overall government size.23

In concluding, it is important to emphasize the existence of a com-
mon thread that runs through the propositions identifying a determi-
nate sign on the relationship between interest-group strength and gov-
ernment size. All these propositions rely, to some extent, on inflexibil-
ities in the choice of policies. In propositions 3 and 4, tax shares are
fixed. In propositions 5 and 6, government is able to help a group only
through a change in the level of a good consumed by that group. In
the latter two theorems, it is the government's lack of information
about the preferences of the group's members on the more usual policy
issues that leads to its inability to help the group through a change in
tax shares. The results of propositions 3-6, therefore, show that one
must examine institutional constraints on policies to understand more
fully the relationship between government size and interest group
power.

V. SOME NORMATIVE IMPLICATIONS

Conventional wisdom has for a long time seen private goods mar-
kets as functioning rather well and governments functioning rather
badly. One might claim that the central lesson taught by economists is
that competition in private goods markets can produce an equilibrium
which is Pareto optimal. This proposition is restated and proved in
many ways. For example, theorems exist, and are widely cited, showing
that as few as two competitors in a market suffice to produce a Pareto-
optimal equilibrium (Bresnahan [1981]). Even monopolies are forced
to satisfy a 'weak invisible hand theorem' under the threat of entry
(Baumol, Panzer, and Willig [1982, ch. 8]). And when prices do diverge
from marginal costs, they are not thought to diverge by much.
Harberger's [1954] calculation that the social costs of monopoly are
trivial has also become a part of the conventional wisdom (see, e.g.,
Alberts [1984]).

Of course, there is also much discussion of the situations under
which efficient equilibria do not exist. But, such possibilities are not
typically discussed in a manner that undermines the standard lessons
of general equilibrium theory—the case for competitive markets, de-
regulation, free trade and the like. In contrast, the literature that applies

23. Mueller and Murrell [1986] do find that the relative size of the government sector
is positively related to the number of interest groups in a country.
economic analysis to the behavior of government and political markets has a rather different tone.

In economic analyses of politics, the discussion of Arrow's impossi-

bility theorem plays a role that is analogous to the presentation of the

first welfare theorem in the analysis of markets. It provides the point

of departure and typifies the tone of the debate. (Although, of course,

Arrow [1963, 59] was careful to point out that the ominous implications

of his theorem applied to both market and democratic processes.) The

absence of equilibrium is frequently viewed as undermining the norm-

ative properties of democratic institutions (Riker [1982]). Majority

rule induces an oversized government (Tullock [1959]) and lack of com-

petition among government bureaus leads to bureaucracies of double

the optimal size (Niskanen [1971]). The rational, self-interested behav-

ior of voters leaves them in ignorance (Downs [1957, chs. 11-14]).

The conclusions contained in this paper paint a picture of the outcomes

from political competition that resembles, rather than contrasts with,

that from market competition. Equilibria exist and they are Pareto

optimal; representative democracy does not necessarily lead to a large

government, let alone a government which is too large, even in the

presence of interest group pressures. This is not, however, to suggest

that existence of equilibrium and Pareto efficiency are sufficient condi-

tions to guarantee that policy choices are satisfactory. As Arrow [1985,

107-8] has stated, "a manifestly unjust allocation, with vast wealth for

a few and poverty for many, will nevertheless be Pareto optimal if there

is no way of improving the lot of the many without injuring the few

in some measure." Nevertheless, existence of equilibrium and Pareto

efficiency are usually viewed as necessary conditions for a satisfactory

electoral process.

The election model developed here has the implication that political

competition forces the government to maximize an objective function

that is a weighted sum of the utilities of each member of each interest

group. An increase in the weight attached to a particular group that

leads to an increase in government size may be deemed by some other

group to have produced a government that is too large. But both levels

of government output satisfy Pareto optimality. Therefore, if our

assumptions characterize the political process accurately, one need no

longer focus on the Pareto criterion in discussing the properties of al-

ternative outcomes. Rather, normative discussions of the size and com-

position of government output can be translated into postulates concern-

ing the appropriate weights to be attached to the utilities of different

interest groups.

This conclusion has a further implication. Much of the public choice

literature in responding to the challenge of the Arrow [1963] impossi-
bility result has focussed on the choice of voting rule, with several alternatives to majority rule having been proposed. Without denying the importance of these results, this paper suggests another area for consideration. It is important to examine the means by which individual preferences enter the political process together with the rule for aggregating preferences. In this model, an implicit weighting of individual preferences occurs through the process of political competition in a society in which individuals belong to different groups and where these groups are able to press the interests of their members with different degrees of intensity. Therefore a normative evaluation of the political outcomes must consider the fact that this process implicitly grants different degrees of influence to the votes that different individuals cast.

APPENDIX

Proposition 2. Each member of group 1 receives \((1 - \theta)Z\) units of private good. \(Zn_1\) of the public good is produced. Hence the objective function becomes

\[
a_1n_1u_1(G + \theta Zn_1) + a_1n_1v_1[y_1 - t_1G + (1 - \theta - n_1t_1)Z]
+ \sum_{i=2}^{N} [a_in_iu_i(G + \theta Zn_1) + a_in_iv_i[y_i - t_iG - n_1t_1)]Z]. \tag{A.1}
\]

Maximizing with respect to \(G, Z,\) and \(t_i,\) one obtains

\[
a_1n_1u_1' - a_1n_1v_1't_1 + \sum_{i=2}^{N} (a_in_iu_i' - t_i'a_in_iv_i') = 0, \tag{A.2}
\]

\[
a_1n_1u_1'\theta_n_1 + a_1n_1v_1'(1 - \theta - n_1t_1) + \sum_{i=2}^{N} (a_in_iu_i'\theta_n_1 - a_in_iv_i't_1n_1) = 0, \tag{A.3}
\]

and

\[
a_1n_1v_1'[(n_i/n_1)G + n_1Z] - a_in_i\theta v_i'(G + n_1Z) = 0, \quad i = 2, ..., N \tag{A.4}
\]

where (A.4) is obtained by substituting \(n_1t_1 = 1 - \sum_{i=2}^{N} n_i t_i.\)

Using (A.4) to substitute for \(v_i'\) in both (A.3) and (A.2), one obtains two identical equations. Therefore a solution to (A.2)–(A.4) can be obtained by setting \(Z = 0\) and solving (A.2) and (A.3). This solution is the one that would be obtained from the problem specified in (4.9).
Proposition 3. Initially, (A.2), (A.3), and (A.4) are satisfied. In a move from this equilibrium, two changes occur in these equations. First, (A.4) is not relevant because tax rates remain fixed. Second, one must use a method that allows for either $Z = 0$ or $Z > 0$—the Kuhn-Tucker theorem. Introducing the "multiplier", $\lambda$, and using

$$n_1 t_1 = 1 - \sum_{i=2}^{N} n_i t_i$$  \hspace{1cm} (A.5)

it follows that the first-order necessary conditions are

$$a_1 n_1 u_1' - a_1 n_1 v_1' t_1 + \sum_{i=2}^{N} (a_i n_i u_i' - t_i a_i n_i v_i') = 0,$$  \hspace{1cm} (A.6)

$$a_1 n_1 u_1' \theta n_1 + a_1 n_1 v_1' (1 - \theta - n_1 t_1) + \sum_{i=2}^{N} (a_i n_i u_i' \theta n_1 - a_i n_i v_i' t_i n_1) + \lambda = 0,$$  \hspace{1cm} (A.7)

and

$$\lambda Z = 0; \quad \lambda \geq 0; \quad Z \geq 0.$$  \hspace{1cm} (A.8)

Totally differentiating (A.6) and (A.7), one obtains

$$da_1 [n_1 u_1' - n_1 v_1' t_1] + dG[a_1 n_1 u_1'' + a_1 n_1 v_1'' t_1^2]$$

$$+ \sum_{i=2}^{N} (a_i n_i u_i'' + t_i^2 a_i n_i v_i'')]$$

$$+ dZ[a_1 n_1^2 \theta u_1'' - a_1 n_1 v_1'' t_1 (1 - \theta - n_1 t_1)$$

$$+ \sum_{i=2}^{N} (a_i n_i \theta u_i'' n_1 + t_i^2 a_i n_i v_i''' n_1)] = 0$$  \hspace{1cm} (A.9)

and

$$da_1 [n_1^2 u_1' \theta + n_1 v_1' (1 - \theta - n_1 t_1)] + dG[a_1 n_1^2 \theta u_1''$$

$$- a_1 n_1 v_1'' (1 - \theta - n_1 t_1)t_1$$

$$+ \sum_{i=2}^{N} (a_i n_i \theta n_1 u_i'' + a_i n_i t_i^2 n_1 v_i'')] + dZ[a_1 n_1^3 \theta^2 u_1'']$$
\[ + a_1 n_1 v_1^{''2}(1 - \theta - n_1 t_1)^2 \]
\[ + \sum_{i=2}^{N} (a_i n_i v_i^{''2} + a_i n_i n_i^{''2} t_i^2)] + d\lambda = 0. \quad (A.10) \]

These equations can be rewritten, using obvious notation:
\[ da_1 A + dG B + dZ C = 0 \quad da_1 + dG E + dZ F + d\lambda = 0. \quad (A.11) \]

If \( dZ = 0 \), then \( d\lambda = da_1 [(EA/B) - D] \). Since \( B < 0 \) from the second-order conditions for maximization, when \( da_1 > 0 \) and \( dZ = 0 \), \( d\lambda \geq 0 \) if and only if \( DB - AE \geq 0 \). Now, some lengthy algebra leads to:
\[ DB - AE = n_1^2 a_1^2 (1 - \theta)(1 - n_1 t_1')(u_1 v_1^{''2} t_1 + v_1 u_1'') \]
\[ + n_1 v_1'(1 - \theta)(1 - n_1 t_1)(\sum_{i=2}^{N} a_i n_i v_i^{''}) \]
\[ + n_1 (1 - \theta)(v_1' - n_1 u_1')(\sum_{i=2}^{N} a_i n_i v_i^{''}). \quad (A.12) \]

Using the strict concavity of the \( u_i(\cdot) \) and \( v_i(\cdot) \), it follows that when \( dZ = 0 \), \( d\lambda < 0 \). This violates the Kuhn-Tucker conditions. Hence \( dZ > 0 \) when \( da_1 > 0 \).

**Proposition 4.** Since \( dZ > 0 \), changes are given by \((A.9) \) and \((A.10) \) with \( d\lambda = 0 \). The size of the government budget is \( G + n_1 Z \). Using the notation of the derivations for Proposition 3:
\[ \frac{dG + n_1 dZ}{d\lambda} = \frac{(AE - DB + DC - AF)}{(BF - EC)} \]
\[ = [(1 - \theta)^2(1 - n_1 t_1)n_1(a_1 v_1^{''2} + \sum_{i=1}^{N} a_i n_i v_i^{''})]/(EC - BF). \quad (A.13) \]
Since \( EC - BF < 0 \) (from the second-order conditions), proposition 4 follows.
Proposition 5. The first-order conditions for theorems 8 and 9 are

\[ \sum_{i=1}^{N} a_{pi} u_{ir}(u_{ir} - t_{ir}v_{ir}) = 0, \]  
(A.14)

\[ \sum_{i=1}^{N} a_{p1}(-t_{i}v_{i}) + a_{pi}n_{i}w_{i} = 0, \]  
(A.15)

\[ n_{1}a_{1}v_{1}'(G + Z) - \sum_{i=2}^{N} n_{i}a_{i}v_{i}'(G + Z) = 0. \]  
(A.16)

When an increment of \( Z \) is introduced into the budget, the movement of \( G \) and \( t_{i} \) is determined by the total differentials of (A.11) and (A.13):

\[ dZ(\sum_{i=2}^{N} a_{pi}t_{i}^{2}v_{i}'') + dG(\sum_{i=1}^{N} a_{pi}(u_{i}'' + t_{i}^{2}v_{i}'')) + \sum_{i=2}^{N} dt_{i}[-a_{i}n_{1}v_{1}'''(G + Z)t_{i}]
+ a_{pi}t_{i}(G + Z)v_{i}''] = 0 \]  
(A.17)

and

\[ dZ[-t_{1}n_{1}a_{1}v_{1}'''(G + Z) + \sum_{i=2}^{N} n_{i}a_{i}v_{i}'''(G + Z)t_{i}] + dG[-t_{1}n_{1}a_{1}v_{1}'''(G + Z)
+ \sum_{i=2}^{N} n_{i}a_{i}v_{i}'''(G + Z)t_{i}] + \sum_{j=2}^{N} dt_{j}[(n_{i}n_{j})/n_{1}]v_{1}'''(G + Z)^{2}
+ dt_{i}[n_{i}a_{i}v_{i}'''(G + Z)^{2}] = 0, \ i = 2, \ldots , N. \]  
(A.18)

Rewriting in an obvious manner:

\[ dZ \cdot A + dG \cdot (A + B) + \sum_{i=2}^{N} dt_{i} \cdot C_{i} = 0 \]  
(A.19)

and

\[ dZ \cdot D_{i} + dG \cdot D_{i} + \sum_{j=2}^{N} dt_{j} \cdot F_{ij} = 0, \ i = 2, \ldots , N. \]  
(A.20)
Using Cramer's rule, one obtains:

\[ \text{sgn}[(dG + dZ)/dZ] = \text{sgn}(-B) = \text{sgn}(\sum_{i=1}^{N} a_i \eta_i \mu_i''). \] (A.21)

Therefore \((dG + dZ)/dZ > 0\).

**Proposition 6.** The changes in the variables are governed by (A.19)—(A.20), and

\[ da_\mu(n_\mu w') + dG(\sum_{i=1}^{N} a_i \eta_i \mu_i'' t_i) + dZ(a_\mu n_\mu w'' + \sum_{i=1}^{N} a_i \eta_i \mu_i'' t_i) \]

\[ + \sum_{i=2}^{N} dt_i [-a_i \eta_i \mu_i''' t_i (G + Z) t_i + a_i \eta_i t_i (G + Z) t_i''] = 0. \] (A.22)

Using the notation introduced in (A.19), (A.22) can be written as

\[ da_\mu(n_\mu w') + dGA + dZ(a_\mu n_\mu w'' + A) + \sum_{i=2}^{N} dt_i C_i = 0. \] (A.23)

Using Cramer's rule, one obtains:

\[ \text{sgn}[(dG + dZ)/da_\mu] = \text{sgn}[-n_\mu w'(\sum_{i=1}^{N} a_i \eta_i \mu_i'')]. \]

Thus \((dG + dZ)/da_\mu > 0\). (A.24)

**REFERENCES**


