Limitations on the Use of Information-Revealing Incentive Schemes in Economic Organizations

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Hierarchical organizations often use formal information channels to facilitate communication between center and subordinate units. Unfortunately subgoal pursuit may lead subordinates to be dishonest when responding to information requests. One way to overcome this problem is to design incentive schemes that encourage honesty. While incentive schemes that successfully encourage accurate information revelation exist for certain environments, this will not always be the case. In particular, if managerial effort is considered or if the center's objective is defined net of bonuses, there may not exist any incentive scheme leading simultaneously to honest reporting and to maximization of the center's objective function. J. Comp. Econ. Sept. 1981, 8(3), pp. 251–271. University of Delaware, Newark, Delaware 19711 and University of Maryland, College Park, Maryland 20742.

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Hayek (1945, pp. 519, 520) has argued that the fundamental economic problem is one "... of the utilization of knowledge not given to anyone in its totality." In hierarchical organizations this problem is addressed by establishing formal information channels between central decision makers and subordinates. However, unless the goals of the organization

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are shared by all, pursuit of subgoals may lead some members of the organization to be less than honest when responding to information requests made by the center.

Faced with the difficulty of obtaining reliable information through established channels, alternative procedures must be adopted. One possibility is the development of separate monitoring agencies to gain information (Downs, 1967, p. 148). However, monitoring requires the use of additional resources and may, in some cases, be prohibitively costly. Alternatively, decision makers could use information on past performance to guide present decisions, but this could be unacceptable in a rapidly changing environment. A third method of obtaining the information relevant to central decision making is for the center to create incentive schemes that encourage honest reporting. Such incentive schemes have been analyzed by Bonin (1976), Weitzman (1976), and Groves (1973) and are the focus of this paper.

As originally formulated, both the Bonin–Weitzman mechanism and the Groves mechanism have been shown to work well in situations where the managers of subgroups simply seek higher incomes and where the goal of the organization (or center) is the maximization of a performance indicator or a sum of performance indicators such as output or profit. Under these circumstances, the choice between the two mechanisms has been shown to depend on the way in which inputs are allocated to subgroups. Loeb and Magat (1978) and Conn (1979) have shown that, if input allocations depend upon the information supplied by subgroup managers, the Groves mechanism will induce managers to send accurate information while the Bonin–Weitzman mechanism will not. On the other hand, if the supplied information is not used for input allocation, the Groves mechanism provides no incentive for accurate information transmittal, whereas the Bonin–Weitzman mechanism encourages accurate reporting.

In this paper, we evaluate the performance of information-revealing incentive mechanisms when alternative assumptions about the goals of managers and the center are adopted. We relax the assumptions that

the center maximizes output and that the subgroup simply maximizes its bonus. We show that when the goals of the participants are more complex, it is possible that neither the Bonin–Weitzman mechanism nor the Groves mechanism will be immune to manipulation by subgroup managers. Furthermore, given certain constraints on the information initially available to the center, we prove that there may be no mechanism that encourages honest reporting. However, if some of these informational constraints are relaxed it may be possible to find a mechanism that can be used by the center to obtain information on the production function. Hence the value and usefulness of the incentive mechanisms may be greatly expanded if they are used in conjunction with other methods of information gathering.

The paper is divided into two major parts. In the first, we examine the performance of information-revealing incentive mechanisms when the managers' objective functions include the disutility of effort as well as a monetary bonus. In the second part, we examine the effectiveness of these incentive schemes when the center's objective is the maximization of profits net of the managers' bonuses rather than gross output (or profits as previously defined). Each part is further broken down into a separate analysis of two cases: (i) where the information submitted by the managers is used to allocate inputs and (ii) where the information is not used to allocate inputs.

In Section 2(a) we examine the introduction of effort into the model where inputs are allocated by the center using information submitted by managers. We show that use of the Groves mechanism causes managers not only to disguise true production capabilities but also to mislead the center about the level of output that they will choose to produce. Thus, misallocation of inputs will be highly probable. More generally, we prove that, unless the center knows the managers' utility functions, there does not exist an incentive mechanism that will encourage managers to reveal their production functions. However, if the managers' utility function is known and effort can be observed, an incentive system can be found that encourages honest revelation.

In Section 2(b) we analyze the effect of including effort in the utility function of the subgroup manager when the information that is passed to the center is not used for allocating inputs. Miller and Thornton (1978), Bonin and Marcus (1979), and Snowberger (1977) have already analyzed the Bonin–Weitzman mechanism when effort affects both production and the utility of subgroup managers. They analyze a model where production levels are subject to uncertainty. In such a model, the difference between the manager's forecast and the actual output level will be smaller than that in a model without effort. Thus, the introduction of effort can be said to
increase the accuracy of forecasts. Also of importance, however, is whether or not subgroup managers will willingly reveal production capabilities. We show that if a maximum output level exists, the use of a bonus function that depends only on information provided by the subgroup manager and on the level of output will not lead to output maximization. Thus, use of a Bonin–Weitzman mechanism encourages accurate reporting of managerial intentions but not of production capabilities. If additional information concerning the utility function and effort expended is made available, it is possible for the center to induce maximum production. However, the value associated with using such a method would depend upon the costs of procuring the additional information.

In Section 3 we analyze situations where managers are employees of an organization, and where as a result it can be argued that maximization of output is not a realistic criterion for the center. Here we examine the implications of assuming that the center’s goal is to maximize output or profits net of the rewards paid to subgroup managers. This model is especially appropriate in a situation where net profits are the objective because “net profits,” as defined here, are exactly equivalent to profits as usually defined in the economics literature. Use of the “net” criterion is common in the discussion of incentive systems. For example, it has been used in papers analyzing risk-sharing in the principal–agency relationship (e.g., Ross, 1973; Harris and Raviv, 1979; Shavell, 1979; Holmstrom, 1979a). When the objective is maximization of net profits we find that whatever the process of input allocation, there is no incentive system that will enable the center to obtain accurate information about production possibilities.

To summarize, these results suggest that the usefulness of both the Bonin–Weitzman and the Groves mechanism is limited. The problem, however, is more general. When managers use discretion and adjust their level of effort, it is not possible to find an information-revealing incentive scheme that can be used by the center to achieve the dual objectives of accurate reporting of production possibilities and of output maximization unless extensive information exists on the managers’ utility functions. Furthermore, attempts by the center to maximize net profits will also create an environment where no incentive scheme can be found that will enable the center to obtain accurate production information.

1 Snowberger’s (1977) results for the analysis when effort is included in the model are unfortunately incorrect. He fails to take into account the range where the manager will choose to produce at the target. For a discussion of these issues and contrasting results see Miller and Thornton (1978).
2 Gross output will be measured in monetary units so that it is commensurable with managerial bonuses.

1. THE MODEL

In this section we present the most general version of the model that is used throughout the paper. We are interested in a game being played between a principal who is at the center and is responsible for organizing overall production activity, and N agents or subgroup managers who are responsible for carrying out production in their particular sphere. Initially, the production (or profits) functions are known only by the subgroup managers. We assume that certain changes in the behavior of the managers can produce changes in output levels even though input levels remain constant. Such output changes are usually said to be induced by changes in managerial “effort” (Keren, 1972, p. 470).

In practice, it is appropriate to think of effort as a multidimensional phenomenon. A simple characterization of effort would distinguish three elements. First, output will vary with the number of hours that the manager is present at the workplace, since subordinates are more likely to work diligently if they know the manager is close by. Second, once at the workplace, managers may choose to engage in activities unrelated to production. Thus, output will vary with the number of hours the manager is engaging in production-related activities. Third, the manager can increase the degree of exertion in order to increase the effectiveness of working time. (This is the interpretation that Keren (1972, p. 470) gives to effort.)

The way in which these three elements of effort affect output will not be independent, nor will the manager always be able to vary each independently. Thus, a full description of the nature of effort would require a specification of the interrelationships between these variables and output. However, to simplify the analysis in this paper we will assume that effort is a one-dimensional variable and that a change in any one of the three elements of effort will cause an increase in output and an increase in disutility to the manager. Thus, the production function can be represented by

$$Q_i = f_i(e_i, k_i),$$

where

$$Q_i = \text{gross output (or profits)},$$

$$e_i = \text{level of effort expended by the } i\text{th manager},$$

$$k_i = \text{input level allocated to the } i\text{th sector},$$

$$f_{i\theta} = \frac{\partial f_i}{\partial k_i} > 0 \text{ for } i = 1, \ldots, N.$$
that there is a level beyond which the manager cannot increase production if all other inputs remain constant (Keren (1972) also makes this assumption). Hence, we assume that

\[ f_i = \frac{\partial f_i}{\partial e_i} > 0 \quad \text{for } e_i < \hat{e}_i, \]

\[ = 0 \quad \text{for } e_i \equiv \hat{e}_i, \]

\[ = \frac{\partial^2 f_i}{\partial e_i^2} < 0 \quad \text{for } e_i < \hat{e}_i, \]

\[ = 0 \quad \text{for } e_i > \hat{e}_i, \]  

(2)

where the value \( \hat{e}_i \) may vary with \( k_i \). The usual declining-marginal-productivity assumptions now become (noting that there may be a discontinuity in the second differential at \( \hat{e}_i \))

(3)

The rest of our assumptions are straightforward and are summarized in Eqs. (4)–(6). Inputs are allocated by the center subject to the availability constraint

\[ \sum_{i=1}^{n} k_i = K, \]  

(4)

where \( K \) is the initial amount available. We assume that the utility functions of the managers are separable in income and effort,

\[ u_i(B_i, e_i) = B_i - V(e_i) \quad i = 1, \ldots, N, \]  

(5)

where \( B_i \) = monetary bonus for agent \( i \) and \( V(e_i) > 0 \) for all \( e_i \). The functional form of \( B_i \) depends on the bonus formula chosen by the center. The center aims to maximize the expression

\[ \sum_{i=1}^{n} Q_i - A \sum_{i=1}^{n} B_i, \]  

(6)

\( A \) is a constant and takes on the values 0 or 1 depending on whether the center explicitly recognizes the costs of managerial rewards in determining the level of profit.

The interaction described by the model can be thought of as a game being played between the center and the subgroup manager. The center determines the rules for the calculation of the bonus and the allocation of inputs among the subgroups. The managers send messages to the center and determine the level of output. The game consists of a series of steps. Initially the center announces the rules for bonus calculation and input allocation. The managers then send messages to the center. If the information submitted by the managers is to be used for input allocation, input decisions are made at this stage. (In the model where input-allocation decisions are not affected by the information submitted by the managers, one can assume that managers know the level of outputs before the messages are sent to the center.) The last step occurs when output is produced and the managers' bonuses are paid.

The information about production levels contained in the managers' messages will be extremely valuable to the center. In a situation where the level of inputs going to each subgroup has already been decided, the center may need information on outputs in order to coordinate delivery schedules between firms. This situation might hold if in a highly imperfect planning environment input allocation is functionally separate from supply coordination and the supply organization relies on producers rather than planners for information (see Bonin and Marcus, 1979). We examine such situations in Sections 2(b) and 3(a). Another reason for obtaining input information is applicable to a more sophisticated planning environment. In this environment, inputs are allocated using the information acquired from the incentive scheme. We examine such environments in Sections 2(a) and 3(b).

2. EFFORT AND INCENTIVES

In this section, in order to focus attention solely on the introduction of effort, we assume \( A = 0 \) in (6) and therefore the center maximizes output. The assumptions on the way in which effort affects output, given in Eqs. (1), (2), and (3), ensure that if the center had complete information it could find a maximum output. Thus assumptions (1)–(3) allow us to focus on informational and incentive problems rather than on problems that might be present even when the center has full information.

(a) Input Allocation Dependent on Information from Subgroups

When the center uses subgroup information to allocate inputs, and effort does not affect output, the Groves mechanism, rather than the Bonin–Weitzman mechanism, is appropriate (Loeb and Magat, 1978). For a class of environments in which input allocation depends on subgroup messages and effort is of no importance, Green and Laffont (1977) have demonstrated that the Groves mechanism is the only procedure that will give an appropriate incentive for honest revelation. (See Holmstrom (1979b) and Laffont and Maskin (1980) for discussion of the Green and Laffont result when other classes of environments are considered.) Thus, when effort is added to the model, it is natural to begin the search for incentive systems with an examination of the Groves mechanism. We show, however, that when effort is introduced into the model even the Groves mechanism fails. In this section first we focus on the performance of the Groves
mechanism. Then, we prove a general result that shows that there will be no bonus scheme that allows the center to accomplish both objectives simultaneously.

When effort is introduced and the Groves mechanism is used, we show that, not only does the manager have an incentive to conceal true production capabilities, but also the mechanism may induce the manager to mislead the center about intended production levels. To demonstrate the performance of the Groves mechanism, let us focus on a simple example with only two subgroups: \( N = 2 \). In constructing an example we assume simple forms for the functions being used. Assume

\[
 f_1(e, k) = e_1^\gamma k_1^{1-\alpha} \quad e_1 \leq \bar{e}_1 \\
 f_2(e, k) = e_2^\gamma k_2^{1-\alpha} \quad e_2 \geq \bar{e}_2,
\]

where in order to simplify the exposition we assume that \( \bar{e}_1 = k_1 \). Let us assume that the utility function is

\[
 u_i(B, e_i) = B_i - \gamma e_i, \quad \gamma > 0 \text{ for both } i.
\]

By direct analogy with the Groves model, the bonus function for the first manager could be given by

\[
 B_i = f_1(e_i, k_i) + f_2(e_i, k_2), \quad (9)
\]

(An superscript indicates the message transmitted to the center about a variable or function.) Formula (9) is different from Groves's original formula, of course, because it includes a message about the variable effect that the center cannot observe directly. However, (9) can be simplified to give a form exactly analogous to Groves's formula. To show this simplification, first note that after all information has been transmitted and inputs have been allocated, variations in the bonus of the \( i \)th subgroup will be due solely to variations in the output of the \( i \)th subgroup and, hence, caused by adjustments in effort by the \( i \)th subgroup. Therefore, the choice of effort level will be solely a function of input allocated. Solving (7), (8), and (9) for the amount of effort that will maximize utility and then substituting this expression into (7), one can obtain a function \( \Pi_i(k) \) that relates output produced to inputs allocated:

\[
 \Pi_i(k_i) = T_i k_i \quad \text{if } T_i \geq 1 \\
 = k_i \quad \text{if } T_i \leq 1. \quad (10)
\]

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Thus, in order to allocate inputs, the center needs to know only the \( \Pi_i(k) \). The managers will send forecasts of the \( \Pi_i(k) \) which we can denote by \( \Pi^*_i(k) \). Equation (9) can therefore be rewritten as

\[
 B_i = \Pi_i(k_i) + \Pi^*_i(k_2), \quad (11)
\]

where \( k_1 \) and \( k_2 \) are the actual values of input allocations. Formula (11) is exactly analogous to the Groves mechanism.

The center receives the messages, \( \Pi^*_i(k) \), from the managers and then allocates input in order to maximize gross output. \( \Pi_i(k) \) is assumed that the subgroup managers are honest. From (10) it is obvious that the subgroup that transmits the message with the highest \( T_i^* \) will receive all the input. Using this fact, a payoff table for subgroup 1 can be constructed using (8), (10), and (11).

Given these payoffs it is possible to find an optimal strategy for subgroup 1, a strategy that is independent of the behavior of subgroup two. If \( T_i \geq 1 \), then subgroup 1 wants subgroup 2 to receive all input if \( (1 - \gamma_i) \leq T_i^* \), and wants to receive all input itself if the reverse is true (since \( T_i^* \leq (1 - \gamma_i) \) implies \( T_i \leq 1 \)). Thus, when \( T_i \geq 1 \), an optimal message for subgroup 1 guarantees that if \( (1 - \gamma_i) \leq T_i^* \), then \( T_i^* \geq T_i^* \).

Therefore, in this case the optimal message is \( T_i^* = (1 - \gamma_i) \). Similarly, if \( T_i \leq 1 \) subgroup 1 would like all input assigned to subgroup 2 if \( T_i (1 - \alpha_i) \leq T_i^* \) and would like to receive all input if the reverse is true. Thus, the optimal message when \( T_i \leq 1 \) is \( T_i^* = T_i (1 - \alpha_i) \). Therefore, in this example, not only is it in the interests of the managers not to reveal actual production capabilities, but it is also in the interests of the managers to disguise their production intentions. Regardless of the strategy pursued by the second manager, the strategy that the first manager pursues is the best in the interests of the

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\*\* The strategy of sending biased information is therefore a dominant strategy. This suggests that subgroup 2 would also pursue a similar strategy leading to a dominant Nash equilibrium. This equilibrium can, for example, be configured for parameters leading to misallocation of the input and to underproduction. The interested reader could verify this point by examining the example: \( \gamma_1 = 3/4, \alpha_1 = 2/4, \gamma_2 = 3/4, \alpha_2 = 1/2 \).
first manager to exaggerate the extent to which effort leads to disutility and therefore to indicate that the production level will be lower than the production level given in Eq. (10).

The above example shows not only that inputs will be misallocated in some circumstances but also that the center will not be able to predict output levels. When output levels are unpredictable, coordination of delivery schedules between producing units will become impossible. Of course, by rewarding consistency between forecast and actual output, it would be possible to construct an incentive system that encourages honest revelation of production intentions. The problem, however, is to encourage honest revelation of productive potential without reducing the incentive to produce to the full extent of production capabilities. Below we show that there is no incentive scheme that can simultaneously accomplish the dual objectives of accurate revelation of production possibilities and maximization of the center’s objectives. Thus, the problems encountered in the foregoing example are not peculiar to the Groves mechanism but rather are problems intrinsic in the design of incentive mechanisms when effort affects both output and subgroup utility.

To show that no incentive scheme can simultaneously accomplish these dual objectives we use the general model introduced in Section 1 together with the specific assumptions outlined in the opening paragraph of Section 2. Then, the center’s objective is to maximize \( \sum_{i} \delta_i Q_i \). In order to accomplish its objective, the center will require information, either explicitly or implicitly, from \( i \), because (as was shown in the foregoing example) \( V_i \) will affect the manager’s production decisions. How much the manager intends to produce will, in turn, affect the allocation of inputs between agents. Thus, in addition to sending a message about the production function, the subgroup manager will communicate a message, \( M_i \), telling the center the nature of the effort disutility function, \( V_i \). The message sent by the manager concerning the nature of \( V_i \) may be sent implicitly (as is the case in the example given previously, where \( \Pi_i \) is a function of \( V_i \)). Given the foregoing definitions and assumptions, we proceed to prove the following:

**Theorem 1.** There exists no bonus system that simultaneously enables the center to maximize output and induces honesty as the optimal strategy for subgroup managers when the bonus payments must be solely functions of messages from the subgroups and the center’s observations of output.

*Proof.* The proof proceeds by contradiction. Assume that such a bonus function exists. Write the bonus as \( B_i(\tilde{M}(e_i), f_i(e_i, k_i)) \), where all other arguments of the bonus function not relevant to the proof are omitted. In particular, one would expect that the bonus would depend, either explicitly or implicitly, upon the messages of the other subgroups and other information the subgroup might provide. \( M_i(e_i) \) will be calculated from the messages of the subgroup managers, \( f_i(e_i, k_i) \) is output, and can be directly observed. The first step in the proof is to show that the bonus must necessarily depend upon the message sent by the managers.

An optimal bonus will encourage the manager to produce at \( \tilde{e}_i \). Thus,

\[
B_i(\tilde{M}(\tilde{e}_i), f_i(\tilde{e}_i, \tilde{k}_i)) - V_i(\tilde{e}_i)
\]

must be the maximum value of the utility function. \( \tilde{M}_i \) is the optimal message for the manager and \( \tilde{k}_i \) is the output-maximizing input allocation. Given that the foregoing assumptions imply a unique optimum, the manager’s utility at \( \tilde{e}_i \) must satisfy the following inequality:

\[
B_i(\tilde{M}_i(\tilde{e}_i), f_i(\tilde{e}_i, \tilde{k}_i)) - V_i(\tilde{e}_i) > B_i(\tilde{M}_i(e_i), f_i(e_i, \tilde{k}_i)) - V_i(e_i)
\]

for all \( e_i < \tilde{e}_i \).

If the center ignores the messages and these messages have no effect on the bonus, the center cannot be sure that (12) will hold. In order to ensure that the bonus will be large enough for (12) to hold, the center must use information given by the manager. For the center must be prepared to make larger bonus payments if subgroup managers indicate through their messages that their disutility is greater. However, if the bonus is determined in this way, there will no longer be an optimal message for the subgroup manager.

To demonstrate this, note that since \( \tilde{M}_i \) is the message intended to reveal \( V_i \), if the bonus system is optimal the center can ensure that (12) holds by substituting \( \tilde{M}_i \) for \( V_i \) in (12) obtaining

\[
B_i(\tilde{M}_i(\tilde{e}_i), f_i(e_i, \tilde{k}_i)) - M_i(e_i) > B_i(\tilde{M}_i(e_i), f_i(e_i, \tilde{k}_i)) - M_i(e_i)
\]

for all \( e_i < \tilde{e}_i \).

We have already assumed that an optimal bonus function exists. This implies that there must be an optimal message for the manager. Given that (13) must hold at \( e_i = 0 \), it is known that

\[
B_i(\tilde{M}(\tilde{e}_i), f_i(\tilde{e}_i, \tilde{k}_i)) - \tilde{M}(\tilde{e}_i) > B_i(\tilde{M}(0), f_i(0, \tilde{k}_i)) - \tilde{M}(0).
\]

Thus, write

\[
B_i(\tilde{M}_i(\tilde{e}_i), f_i(\tilde{e}_i, \tilde{k}_i)) = \tilde{M}_i(\tilde{e}_i) + \theta \tilde{e}_i + B_i(\tilde{M}_i(0), f_i(0, \tilde{k}_i)) - \tilde{M}_i(0),
\]

where \( \theta > 0 \). Now suppose that the manager had decided to send the
message $M_i^\phi$ instead of $\hat{M}_i$, where
\[
M_i^\phi(e_i) = \hat{M}_i(e_i) + \phi e_i, \quad \text{for } \phi > \theta.
\]
Then from the analysis leading to the formulation of inequality (13), we know that the center would have had to set the transmission so that
\[
B_i(M_i^\phi(\hat{e}_i, f(k, \hat{k})) > M_i^\phi(\hat{e}_i) + B_i(M_i(0, f(0, \hat{k})) - M_i(0),
= \hat{M}_i(\hat{e}_i) + \phi \hat{e}_i + B_i(M_i(0, f(0, \hat{k})) - M_i(0),
> \hat{M}_i(\hat{e}_i) + \theta \hat{e}_i + B_i(M_i(0, f(0, \hat{k})) - M_i(0),
= B_i(M_i(\hat{e}_i, f(k, \hat{k}))).
\]
$\hat{M}_i$ was the optimal message, yet the manager can earn a larger bonus by sending the message $M_i^\phi$. Thus, there exists no optimal response for the manager contradicting the assumption that the bonus function will induce managers to transmit the information that allows the center to maximize output. Theorem 1 is established.

The proof centers on the observation that the manager can always increase the size of the bonus by exaggerating the dissility of effort. The incentive to exaggerate precludes the possibility that the bonus function will encourage honest revelations about production functions. Stated simply, if the center uses only a bonus function to gather information, the manager will be able to obtain a bonus of any given size by sending an arbitrary production function message and by suitably exaggerating the dissility of effort. (It should be noted at this point that the proof does not establish that the single objectives, output maximization or accurate revelation, are individually unachievable in specific situations, but rather that they are simultaneously unachievable in general.)

The proof that an optimal bonus function does not exist focuses on the center’s ignorance of the subgroup manager’s utility functions. It can be shown that this ignorance is the crucial factor in determining nonexistence. If the center is able to use alternative sources in order to obtain information on the utility function of the manager and is able to observe the level of effort, then it is possible to find an incentive function that will encourage accurate revelation of the production function. For example, the following variation of the Groves incentive mechanism will create incentives for accurate reporting of production possibilities if $V_i$ is known by the center and $e_i$ is observable:
\[
B_i = f_i(\hat{e}_i, \hat{k}_i) + \sum_{j \neq i} f_j(e_j, k_j) + V_i(\hat{e}_i).
\] (14)

In this formula, a caret indicates the values of the variables chosen by the participants and an accordingly superscript indicates the message sent by the subgroup to the center (in contrast to the actual values, which are without a superscript). For this particular model, inputs will be allocated in the normal manner for a Groves scheme (see Magat and Loeb (1978) for the details relevant to this model), and effort will be chosen by the managers in order to maximize utility. Substituting (14) into (5), managers’ utility will in this case be given by
\[
u_i(B_i, e_i) = f_i(\hat{e}_i, \hat{k}_i) + \sum_{j \neq i} f_j(e_j, k_j).
\]

Given any input allocation, the manager will obviously maximize utility by choosing $e_i = \hat{e}_i$. Thus, the manager choosing a message in order to maximize utility will be in a position exactly equivalent to that of a manager trying to maximize the bonus in Groves’s (1973) model. Thus, Groves’s well-known proof, in the version outlined by Magat and Loeb (1978), will be immediately applicable. Use of (14), given that $V_i$ is known to the center and effort can be monitored, leads to honest reporting and maximization of the center’s objective.

The difficulty with implementing this modified bonus is that the information required for implementing it will not be readily available. The utility function of the subunit manager and the level of effort expended, unlike the output level, will not be automatically revealed in the normal course of interaction between the center and the manager. However, in some circumstances, auxiliary sources of information may be available. Monitoring of effort is an obvious possibility. Information from the market for managers could be another. Ross (1979), James et al. (1979), and Stiglitz (1975) all describe situations where self-selection processes lead to information revelation. Although these authors do not discuss revelation of $V_i(\cdot)$ or its equivalent, it is possible that markets will reveal information about this element of the utility function. Such revelation is most likely to be reliable when the employment of the manager extends over a short period. When the manager’s employment is extended over long periods and new information is constantly transmitted, market information will be less reliable. The manager may find it possible to make an initial low bid that will be compensated for in later periods after “first-mover advantages” (Williamson, 1975) have been secured. Thus, acquiring the necessary information to make (7) implementable may be difficult. However, our primarily negative results of this section show that such information acquisition will have a payoff.

(b) Input Allocation Independent of Subgroup Messages

Bonin (1976) and Weitzman (1976) both address the issue of incentive-mechanism design when the subgroup messages are transmitted after
inputs are allocated. While the actual schemes used by the two authors differ, they both can be reduced to a bonus formula of the form

$$B_i = aQ_i - b(Q_i - s_i) \quad \text{if } Q_i \geq s_i$$

$$= aQ_i - c(s_i - Q_i) \quad \text{if } s_i \geq Q_i,$$

(15)

where $a > b > 0$, $a > c > 0$, and $s_i$ is the output forecast transmitted by the subgroup manager. Thomson (1979) has shown that (15) is one of a class of bonus formulas that will encourage "elicitation" of information held by subgroup managers.

Using (1) as the production function (remembering that $k_i$ is fixed) and (5) as the managerial utility function, one can analyze the performance of (15), the Bonin–Weitzman mechanism, when effort is introduced. Since the objective function of the center, Eq. (6), is separable in the output of each unit, one can conduct the analysis by examining the effect of the bonus system on a single manager while ignoring all other subunits. In a certain world, one can look at the manager’s problem of choice of $Q_i$ and $s_i$ as if both choices are made at the same time. Hence, for fixed $Q_i$, the $s_i$ chosen must be optimum. It is easy to see from (15) that if $Q_i > s_i$, then the bonus will be less than if $Q_i = s_i$. Thus, it will always be in the manager’s interests to send an honest report of production intentions.\(^{12}\)

When the objective is solely to produce accurate production forecasts, the Bonin–Weitzman incentive mechanism works well. The center, however, has dual objectives: to obtain accurate information and to maximize its objective function. If the latter objective is to be realized then the following condition must be satisfied:

$$\frac{\partial f_i}{\partial e_i} = f_{w_i} = 0.$$  

(16)

When the subgroup managers maximize their utility functions, they will produce at an output level where the following condition will be satisfied:

$$\frac{\partial V_i}{\partial e_i} = V_i'(e_i) = \frac{\partial B_i}{\partial Q_i} \cdot \frac{\partial Q_i}{\partial e_i} = a \cdot f_{w_i}.$$  

(17)

Since $V_i'(e_i) > 0$ and $\frac{\partial B_i}{\partial Q_i} < 0$, the manager will always choose a level of effort that is less than the output or profit-maximizing level desired by the center. Hence, the Bonin–Weitzman mechanism does not accomplish the dual objectives of the center. By inspection of (16) and (17), it is easy to see that there exists no bonus scheme that depends on output and target alone and that could solve both problems. A general proof that no bonus scheme exists that encourages honest revelation and output maximization would follow the proof of Theorem 1 very closely and is therefore omitted.

In order to construct a workable incentive system, the center will need to reward greater effort as well as greater output. Such an incentive system would be

$$B_i = aQ_i + V_i'(e_i) - b(Q_i - s_i) \quad \text{if } Q_i \geq s_i$$

$$= aQ_i + V_i'(e_i) - c(s_i - Q_i) \quad \text{if } s_i \geq Q_i.$$  

(18)

The bonus system (18) can only be implemented if the center knows $V_i$ and can observe effort. In such a case, it is easy to show that (18) produces appropriate results. By exactly the same argument as that for the bonus system (15), the manager will make $s_i = Q_i$. Thus, the utility-maximizing condition for the manager is

$$V_i'(e_i) = a f_{w_i} + V_i'(e_i).$$

The solution to this equation is exactly the same as the solution to the center’s maximizing equation (i.e., $f_{w_i} = 0$). Thus, system (15) will accomplish the dual objectives required of an incentive system.

In summary, if the center does not have information on $V_i(e_i)$, it is impossible to construct an optimal bonus function. If information is available on the disutility of effort and effort can be monitored, a simple adaptation of the Bonin–Weitzman mechanism solves the incentive problem. However, acquisition of such information will require alternative, perhaps costly, procedures. Thus, the issues discussed in the last paragraph of Section 2\(^a\) concerning the problems of gaining knowledge of the manager’s utility function are also relevant here.

3. INCENTIVES IN AN ENVIRONMENT WHERE NET BENEFITS ARE THE MAXIMAND

In this section we focus on problems that arise in a situation where the center has preferences on the size of the bonus. The introduction of the cost of managerial rewards into the center’s objective function is an explicit recognition of an important problem that the organization must face. When managers are employees of an organization their bonus payments are funds that the organization has to relinquish in order to induce production. There seems to be no reason why such funds should not be counted as costs to the organization. When bonuses are counted as costs, they must be subtracted from output or profits in order to judge the appropriateness of organizational decisions.\(^{32}\) Subtracting bonuses from $Q_i$ is

\(^{12}\) Miller and Thornton (1979) and Bonin and Marcus (1979) analyze the case where output is uncertain.
most appropriate when \( Q_i \) is interpreted as "gross profits." In such a case, \( Q_i - B_i \) is the organization's profit from the \( i \)th subgroup.\(^{13}\)

Formally, this adjustment in the center's objective function means that in formula (6), \( A = 1 \). In order to focus on the effect of changing the center's criterion to "net" instead of "gross" we assume that effort does not affect output. Thus, \( f(e_i, k_i) = f(k_i) \) for all \( e_i \). It then follows that effort will not appear in the managers' utility function so \( V(\cdot) \) can be ignored in Eq. (2).

Both the Bonin–Weitzman mechanism and the Groves mechanism have been formulated for situations where the center has no preferences on the size of bonuses. Thus, the size of the bonus relative to the size of \( Q_i \) is essentially undetermined in their schemes. The question remains, therefore, whether the two mechanisms can accomplish the new dual objective of maximizing net output or profits and accurate information transmittal. We answer this question in a general manner by showing that no bonus system will accomplish both objectives simultaneously. Because the proof is more straightforward in the case where inputs are allocated without using information obtained from managers, we begin with that case in Section 3(a). A similar proof for the case where the information in the messages sent by managers is used to allocate inputs is presented in Section 3(b).

(a) Input Allocation Independent of Subgroup Messages

Before proceeding with the proof that there is no bonus scheme that can accomplish the center's objectives, we make the following assumption:

**Assumption.** There is a minimum level of the bonus, \( B_i, i = 1, \ldots, N \), below which production in subgroup \( i \) will not take place.

This is a natural assumption since it is merely a representation of the fact that managers will only work for an organization if their remuneration reaches a required level.\(^{15}\) With inputs assigned before messages are transmitted and with output independent of effort, there is a maximum attainable level of production, \( Q^* \). The existence of a maximum attainable level of production and a minimum level for the bonus together will ensure the existence of an optimal level of net output. Therefore, any problem in

attaining an optimum must be due to the informational and incentive characteristics of the problem.

Since the center's objective is separable, one can analyze the construction of an optimal bonus function for one subgroup alone. Denote the message that the manager sends to the center as \( \bar{Q} \), which is a forecast of actual output \( Q \). One can then write any general bonus function as \( B_i(Q_i, \bar{Q}) \). The center will choose \( B_i(\cdot) \) in order to maximize \( Q_i - B_i(\cdot) \) subject to the constraint that \( B_i(\cdot) \equiv B \). An optimal bonus function that satisfies these conditions and also encourages accurate information revelation on the part of the manager will have \( Q_i = \bar{Q} = \bar{Q} \) and \( B_i(Q_i, \bar{Q}) = B_i \). Given the foregoing assumptions, we prove:

**Theorem 2.** There exists no bonus function that will, in general, lead simultaneously to maximization of net output (or profits) and to accurate information revelation.

**Proof.** An optimal function must pay a larger bonus for an outcome where \( Q_i = \bar{Q} = \bar{Q} \) than for an outcome where \( Q_i = \bar{Q} = Q' \), \( Q' \) being any output such that \( Q_i' < Q_i \). If this were not true, then the bonus function would encourage only consistency between forecast and outcome and would provide no incentive for maximum production. Thus, it is required that

\[
\frac{\partial B_i}{\partial Q_i} \bigg|_{Q_i = \bar{Q}} - \frac{\partial B_i}{\partial \bar{Q}} \bigg|_{Q_i = \bar{Q}} > 0.
\]

This inequality implies that \( B_i(Q_i, \bar{Q}) \big|_{Q_i = \bar{Q}} > \bar{B} \) for at most one value of \( \bar{Q} \). As a result, in order to find the maximizing bonus function the center would have to know the value of \( \bar{Q} \) before the bonus function was constructed. However, this is a contradiction, because the bonus function is constructed in order to elicit information about \( \bar{Q} \). Thus, an optimal bonus function does not exist.

(b) Input Allocation Dependent on Information from Subgroups

The proof of nonexistence of an incentive mechanism for environments where the information collected by the center is used for input allocation is similar to the proof in Section 3(a). For exactly the same reason as given in Section 3(a), it is assumed that the bonus must reach some minimum level: \( B_i(\cdot) \equiv B_i \). In its most general form, the bonus function can be written as\(^{16}\)

\[B_i(\cdot) \equiv B_i \]

In writing the bonus function in such a way, it is not assumed that the bonus necessarily depends upon each argument. Rather the bonus is expressed in such a way to allow for full generality. In some cases, for example the Groves mechanism, the \( i \)th bonus may depend on the \( i \)th message only through the message's effect on input allocation.

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\(^{13}\) Bonin and Marcus (1979) have also used a "net" criterion as the objective of the center. For a centrally planned economy, the "net" criterion may be a simple way to model concern over the size of managerial income. Hildebrandt and Tulin (1979) have considered the difficulties that arise when distributional concerns are confounded with incentive problems. However, they do not consider the issue of obtaining information from managers.

\(^{15}\) Ross (1973) makes essentially the same argument. Holmstrom (1979a) and Shavell (1979) both use an equivalent assumption.
4. CONCLUSION

Bonin (1976), Weitzman (1976), and Groves (1977) have established fundamental results on the design of incentive mechanisms to induce optimal effort. The literature considers an environment in which the center observes a principal with private information that allows it to signal the principal's type by making a payment. The principal, in turn, may choose to make payments to agents knowing that these payments will be observed by the center. The problem of incentive design is to determine the payment schedule that induces the principal's optimal effort.

In this section, we discuss the implications of our results for incentive design. We show that our results can be used to design incentive mechanisms that are optimal in a wide range of environments. In particular, we show that our results can be used to design mechanisms that are optimal in environments in which the principal has private information.

First, we consider a simple environment in which the principal has private information. In this environment, we show that our results can be used to design a mechanism that is optimal in the sense that it induces the principal's optimal effort. We also show that our results can be used to design a mechanism that is optimal in the sense that it induces the principal's optimal effort and minimizes the principal's cost.

Second, we consider a more general environment in which the principal has private information and the principal's type is observed by the center. In this environment, we show that our results can be used to design a mechanism that is optimal in the sense that it induces the principal's optimal effort and minimizes the principal's cost.

Finally, we consider a more general environment in which the principal has private information and the principal's type is not observed by the center. In this environment, we show that our results can be used to design a mechanism that is optimal in the sense that it induces the principal's optimal effort and minimizes the principal's cost.
Weitzman and Groves schemes. However, our results extend beyond solely an analysis of these two schemes, because the results examine whether any bonus scheme can accomplish the dual objectives of output maximization and unbiased information revelation.

Pryor (1979) has evaluated the performance of a number of standard bonus schemes under the assumption that effort affects output (with both "gross" and "net" objectives). In his series of examples, he concludes that all the standard, previously used bonus schemes fail in some fundamental manner. The results in this paper thus significantly generalize those of Pryor by showing that, not only do existing schemes fail, but also it is impossible to construct any bonus scheme that is entirely satisfactory from the center's viewpoint. The search for the elusive optimal bonus scheme in command economies (Berliner, 1976) is thus bound to be a failure.

Still, the results in this paper should not be regarded as primarily negative in significance. The main thrust of the results is that one approach to informational problems in hierarchies will often not be viable. Thus, other approaches or a combination of approaches will be needed. For example, as we have shown in Section 2 the monitoring of effort may be a necessary element in the implementation of bonus schemes. In an important sense these results can be regarded as complementary to the work of Williamson (1975), who stresses how the structure of organizations (i.e., internal labor markets, M-form firms, etc.) can be used to encourage subordinates to act in ways consistent with the overall goals of an organization. For it may be necessary to consider both the organizational-structure questions and bonus-design questions together if hierarchical organizations are to achieve greater efficiency.

REFERENCES


