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THE ANALYSIS OF REVENUE SHARING IN A NEW APPROACH TO COLLECTIVE FISCAL DECISIONS *

DAVID F. BRADFORD AND WALLACE E. OATES

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In spite of the widespread sharing of revenues by central governments with provincial, state, and local public units in a number of countries, there exists little formal analysis of the allocative and distributive effects of these programs. Proponents of revenue sharing in the United States¹ refer to a "fiscal mismatch" between the federal and state-local governments; the ability to generate revenues (particularly in terms of an elastic tax base), they contend, is disproportionately under the control of the federal government, while expenditure "needs" have become increasingly urgent at state-local levels. The solution to this problem is "the regular distribution of a specified portion of the federal income tax to the states primarily on the basis of population and with few strings attached."²

While one can recognize the gravity of the fiscal problems besetting many state and local governments, the existing literature on revenue sharing cannot help but prove somewhat puzzling to students of public finance. We are familiar in theoretical terms with the effects of unconditional, lump sum grants to individual economic units, but the question of the effects of such grants to a collectivity remains unanswered. Are unconditional intergovernmental grants equivalent to a set of lump sum grants directly to the individual members of the collectivity? If so, revenue sharing is simply an implicit reduction in federal personal taxes. Or does revenue sharing have effects on public outputs and disposable incomes that cannot be duplicated through a cut in federal taxes?

* We are indebted to the Ford Foundation for support of this work, and in particular, are grateful to Richard Cornwall, Joseph Pechman, Carl Shoup, and the members of the Committee on Urban Public Economics for many helpful comments on earlier drafts of this paper.

1. See, for example, Walter Heller and Joseph Pechman, "Questions and Answers on Revenue Sharing," Brookings Reprint 135 (Washington, D.C., 1967).

2. *Ibid.*, p. 2.

Previous analyses of intergovernmental grants have been unable to provide answers to these questions, because they typically treat governmental units as if they were individual consumers equipped with indifference curves with the familiar properties.³ Since, however, intergovernmental grants are not grants to individuals but rather grants to groups of people, this analytic approach is clearly inappropriate. What is needed is a theoretical framework that incorporates explicitly the political process, by which we mean the process of collective choice through which the recipients of the grant determine their response to the increment to the public treasury.

In this paper, we present a new approach to the study of the economic behavior of collectivities, one that places a central emphasis on the structure of political decision-making institutions. In Section I, we set forth the model with which we propose to describe revenue sharing and its effects on the individuals receiving the funds. Using this model, we show in Sections II and III that, in two simple and familiar models of budget determination — the Lindahl model and the system of simple majority rule with fixed tax shares — revenue sharing is precisely equivalent, in terms of *both* its allocative and distributive effects, to a reduction of central government taxes of a specified incidence. In Section IV, we begin to explore the generality of this result in terms of a new method of formalizing the properties of political decision systems. We describe in this section a set of sufficient conditions on the political process to assure the equivalence of revenue sharing to a reduction in central government personal taxes. Although these conditions are fulfilled by the simple, illustrative Lindahl and majority rule models, they are not dependent on any particular connection between individual preferences and collective choices. Rather they are properties that could in principle be observed directly from the behavior of collectivities, much as individual indifference maps are in principle observable by revealed-preference techniques.

We find, moreover, in Section V that there are plausible circumstances under which the sufficient conditions and the equivalence between revenue sharing and lump sum grants to individual citizens do not hold. For example, special forms of majority rule in the constitution of the receiving government may result in outcomes being obtainable through grants to the collectivity that could not be generated by the political process following individual grants to its

3. See, for example, an excellent paper by J. Wilde and the references cited therein: "The Expenditure Effects of Grant-in-Aid Programs," *National Tax Journal*, Vol. 21 (Sept. 1968), pp. 340-48.

citizens. We argue in this section that the sources of these differences between grants directly to citizens and grants to their government may have important implications for the desirability of a program of revenue sharing. Section VI examines briefly the implications for our conclusions of tax competition among jurisdictions and of the introduction of fiscal effort clauses into the revenue-sharing formula, and Section VII contains some concluding remarks. Finally, in the Appendix we present a formal proof of the equivalence theorem described in Section IV.

I. THE ANALYTIC FRAMEWORK

To isolate the issues of interest to us, we confine our attention to a model world of "fiscal clubs." We assume initially that each club has a single collective activity, which may be thought of as the provision of a public good for joint consumption by the members of the club. There is, in addition, a single private good that the members consume. A "state of the club" is completely described by a vector $[y_1, y_2, \dots, y_n, g]$, whose first n components indicate the claims of the n members of the club to the private good and whose last component, g , indicates the level of provision of the public good. The set of feasible states of the club is constrained by the requirement that no one have a negative allotment of private goods ("disposable income") and by a relation that specifies the maximum level of the collective activity attainable, given the total quantity of the private good available for distribution. We assume initially that the public good and the private good can be traded off on a constant cost basis, and choose the units so that this conversion is one-for-one.⁴

For any club, the feasible state that actually occurs is a matter of politics, formal or informal. Most feasible states will not be political equilibria. In all likelihood, for example, the state in which everyone has the same amount of the private good but where no public good is provided will be altered by political activity into some other state in which a positive level of the collective activity is undertaken. Similarly, a state in which one individual has a

4. The conversion might be thought of as occurring through production internal to the club, or, alternatively, by exchange in a large market. We should point out that much of the simplification adopted here is purely for expository purposes. In particular, the logic of the argument is compatible with a world of many private and public goods and of more complicated production-exchange relationships. These extensions are developed in the Appendix.

huge command over the private good and all other individuals possess virtually nothing will generally be modified, perhaps through the political process of revolution, into a state with a somewhat more egalitarian distribution of disposable income. The essential point is that, given the endowments of the members, their relative bargaining skills, information costs, etc., and the set of alternative feasible states, the vector of disposable incomes and the level of the collective activity are *simultaneously determined* by the political process.

We can characterize revenue sharing in our model world as a disturbance of equilibrium. An agency external to the club alters the state of the club by making it a gift of a specified sum; we might imagine the immediate effect to be an increment to the level of the club's collective activity. The new state is, of course, a member of a larger set of feasible alternatives than the original one, but there is no reason to expect it to be a political equilibrium within that enlarged set. The political process will in general bring about further changes, ultimately producing a new equilibrium state.

In this paper, we pose the question whether the new equilibrium state resulting from the gift to the club could be reached following a quite different initial disturbance, namely, a set of grants to the individual members of the club that totals to the same amount as the revenue-sharing grant to the collectivity. This will immediately be recognized as the question of whether revenue sharing is identical in its effects to a specific program of central government personal tax reductions or rebates. We should emphasize that the thesis we are examining is *not* that revenue sharing is equivalent in its effects to, say, an x percent across-the-board cut in federal taxes, or to any other program of federal tax reduction as normally conceived. Rather, we are exploring the conditions under which the ultimate state attained with revenue sharing is identical in every detail (i.e., in terms of the disposable income of each individual and the level of output of the public good) to that resulting from a grant of, say, \$10 to Mr. 1, \$7 to Mr. 2, etc., where these grants may bear no relation to any politically feasible set of individual grants actually attainable through tax reduction by the central government.

Incidentally, as we shall show later, it is not difficult to describe a political process for which this equivalence does not hold. In the next two sections, however, we show that this equivalence does obtain under reasonable conditions in two simple models of collective budget determination: the Lindahl procedure and majority rule with fixed tax shares. These simple cases serve to clarify the nature of

the proposition and to motivate our speculations about its plausibility in more complex political systems.

II. REVENUE SHARING IN A LINDAHL MODEL

The Lindahl mechanism of collective budget determination can be thought of as the analogue in a world with collective goods to a Walrasian *tâtonnement* in the competitive equilibrium theory of private goods.⁵ In the latter, individuals in the system treat successive price vectors called out by the auctioneer as final, indicating the amounts of the various goods and services that they wish to demand and supply at those prices. For collective goods, the "auctioneer-politician" calls out successive sharing formulas. More specifically, he calls out for each individual i a tax share h_i , which indicates the fraction of the cost per unit of the public good that the i^{th} person will have to pay through the sacrifice of units of the private good. Tentatively taking this "price" per unit of the public good as fixed, each individual indicates the number of units he would like to see provided. The auctioneer then varies tax shares until the number of units of the public good demanded by all the members of the club is the same. This is the equilibrium quantity, which is financed by taxing each member according to his indicated tax share.

The position of a particular member of the club under a Lindahl equilibrium is depicted in Figure I. With a "before-tax" income of OA and a tax share of h (which equals the slope of AB), the line

5. Useful discussions of the Lindahl process are available in L. Johansen, "Some Notes on the Lindahl Theory of Determination of Public Expenditure," *International Economic Review*, Vol. 4 (Sept. 1963), pp. 346-58, and *Public Economics* (Chicago: Rand McNally, 1965); J. Head, "Lindahl's Theory of the Budget," *Finanzarchiv*, Vol. 23 (Oct. 1964), pp. 421-54; and H. Aaron and M. McGuire, "Efficiency and Equity in the Optimal Supply of a Public Good," *Review of Economics and Statistics*, Vol. 51 (Feb. 1969), pp. 31-39. Foley derives a theorem for Lindahl equilibria in a world with public goods that parallels precisely the fundamental theorem of welfare economics in a world with only private goods (D. Foley, "Resource Allocation and the Public Sector," *Yale Economic Essays*, Vol. 7 (Spring 1967), pp. 45-73). For a general analysis, including a description of the historical development of the Lindahl model, see R. Musgrave, *The Theory of Public Finance* (New York: McGraw-Hill, 1959), Ch. 4, and P. Samuelson, "Pure Theory of Public Expenditure and Taxation," in J. Margolis and H. Guitton, eds., *Public Economics* (London: MacMillan, 1969), pp. 98-123. The shortcomings of the Lindahl model as a description of actual budget determination are well known. In particular, as many writers have pointed out, there is an incentive for the individual to understate his true preferences so as to induce others to pay a larger share of the costs of the public good. Such behavior will in general prevent the attainment of the Lindahl solution. However, because of its familiarity and "nice" properties, we have found it very useful for pedagogical purposes to explore the effects of revenue sharing in a Lindahl framework.

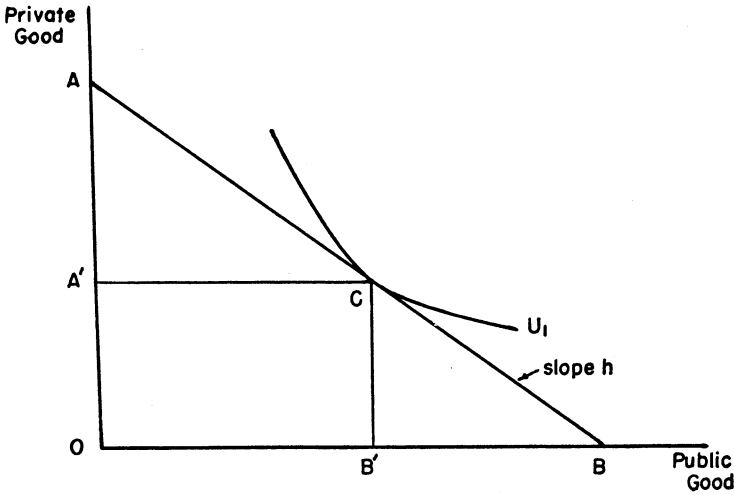


FIGURE I

segment AB indicates the level of consumption of the private good that will be available to the member for each alternative level of provision of the collective good. The individual is assumed to have a preference relation of the usual sort defined over bundles of private and public goods. Given his pretax income and tax share, his chosen level of provision of the public good will be his most preferred bundle along AB , shown in Figure I as point C , with public good provision OB' and after-tax income OA' . Since this position was assumed to be one of Lindahl equilibrium, the corresponding diagrams for all other citizens would display the same desired provision of the public good; however, before- and after-tax incomes and tax shares will generally be different for each citizen.

Consider now the nature of the new Lindahl equilibrium that will obtain if the club as a collectivity receives a gift of $B'G$ units of the public good (see Figure II). The Lindahl "auction" process, instead of beginning with each member on his vertical axis, now commences with everyone having the same disposable income as in the previous equilibrium but with the collective activity at least tentatively set at a level $B'G$ units above its previous equilibrium level. The process now determines a new equilibrium level of provision of the public good and set of tax shares. For our typical individual in Figure II, the Lindahl process, following the gift to the club, begins at point D . At an announced tax share h' , he confronts a hypothetical choice among the alternative bundles of private and public goods indicated by the straight line $A''B''$ passing through

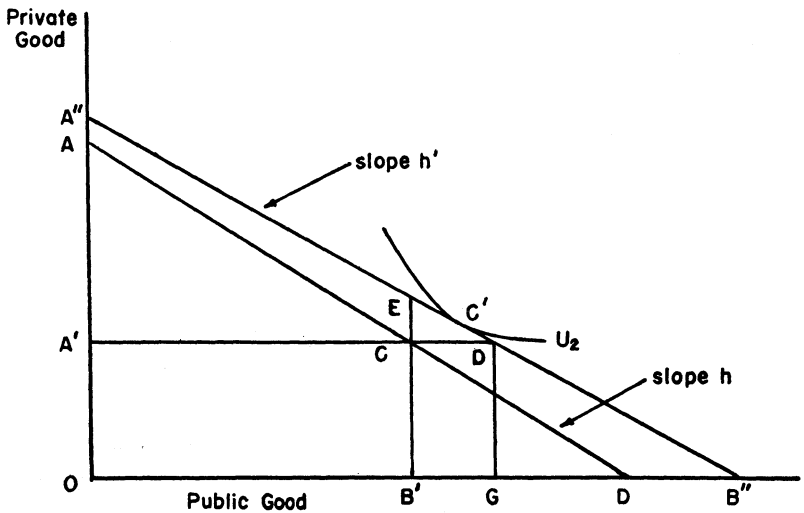


FIGURE II

D with slope h' . Suppose that from among these points the individual prefers C' ; if h' is an equilibrium tax share, a set of shares for the other club members has been found such that they all are in agreement on the level of provision of the public good implied by C' .

In the new political equilibrium in the case depicted in Figure II, the level of the collective activity exceeds that prevailing before the revenue-sharing program, but by an amount less than the full sum of the grant. This implies that members' disposable income is correspondingly increased. Before turning to our central proposition, we might point out the formal fact that the effect of the grant could conceivably be an absolute reduction in the provision of the public good, or an increase by an amount exceeding the grant, depending on the preference maps of the members. This simply says that the public good may be an inferior good, a good with a very high income elasticity of demand, or somewhere in between.

We now turn to the question of whether the gift to the club may be considered as equivalent to a set of gifts to the individual members. On the assumption that each individual has a single most preferred level of the collective activity for his equilibrium tax share, a glance at Figure II enables us to see this equivalence. If, instead of making a grant to the club, the central agency had given CE of the private good to our typical individual, then, confronted with tax share h' , he would have chosen among precisely the same

alternative combinations of public and private goods, with C' his desired outcome. With corresponding amounts (i.e., $B'G$ times the individual's tax share) given to other members and with each again confronted with the postgift tax share previously derived, precisely the same Lindahl equilibrium would emerge. Moreover, such a set of gifts to the members of the club costs the central agency exactly the same sum as the gift to the club. By simple geometry, distance CE equals $(h' \cdot B'G)$; since the tax shares add to one, summing this quantity of the private good over all the members yields a total gift of the private good of $B'G$.

It is important to emphasize that what has been shown here for the case of the Lindahl method of budget determination is that revenue sharing is equivalent to a highly specific program of individual tax reduction by the central government, *not* simply in the sense that the programs generate the same provision of public good by the club, for in addition they generate *precisely* the same after-tax income for each member. That is, they result in precisely the same vector describing the state of the club.

III. REVENUE SHARING AND MAJORITY RULE

Critics have objected that the Lindahl model is an unsatisfactory example of collective decision making because it is unrealistic, especially in ignoring the problem of inducing citizens to reveal their demands for public services when their taxes depend directly upon this variable. Furthermore, it may be thought that our result is somehow dependent upon the special optimality characteristics of the Lindahl solution. Later in the paper, we describe a set of properties of budget determination systems that are sufficient to assure this result, properties that are both plausible and unrelated to any optimality conditions. Before proceeding to the more general analysis, however, it is useful to show that the same conclusions hold for a familiar collective decision rule that does not in general generate optimal outcomes: simple majority rule.

Although majority rule as a political process of budget determination has a certain appeal as being more "realistic" than the Lindahl rule, it requires more restrictions on the admissible outcomes of the process to assure determinacy. The central difficulty is that the "game" of determining the distribution of income does not have a solution under majority rule when all possible assignments of posttax income to citizens are considered allowable. While a majority can assure itself of the entire "pie," those in the excluded

minority will always be able to offer a subset of the winners even greater rewards for joining a new majority.⁶ While the Lindahl rule will generate a "solution" consisting of a government budget plus individual tax shares, majority rule will not, therefore, reach a stable outcome unless some restrictions are placed on the values of these variables. We consider here the case in which individual tax shares are taken as fixed in advance; the level of the club's budget then becomes the sole variable to be determined by the political process under majority rule.⁷ That is, alternative pairs of levels of provision of the public good are put to the voting test until one is found that at least $(n/2 + 1)$ of the members prefer to any other.

In Figure III line AB , with slope h , shows the consumption alternatives in terms of the private and public goods for an indi-

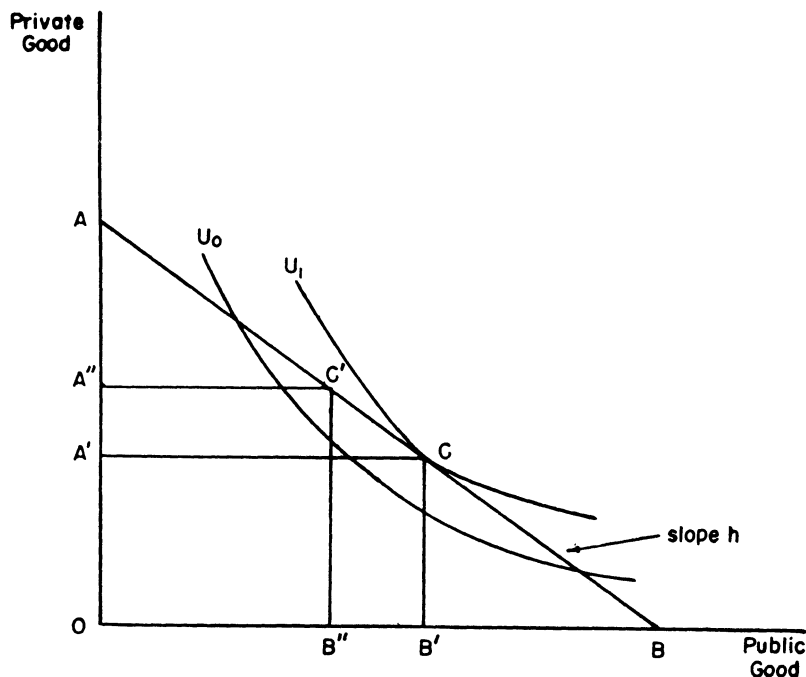


FIGURE III

6. This proposition has been analyzed formally by Benjamin Ward, "Majority Rule and Allocation," *Journal of Conflict Resolution*, Vol. 5 (Dec. 1961), pp. 379-89.

7. Duncan Foley has explored the possibility of stable outcomes under several different types of restrictions on tax shares (*op. cit.*). Foley, however, takes the level of the budget as fixed in his analysis.

vidual who has pretax income OA and is assigned tax share h . Under the assumption that each person's preference structure generates indifference curves of the usual shape, every member's preference relation among club budget levels will be single-peaked. The individual represented in Figure III, for example, prefers budget level OB' to all others. Furthermore, of any two proposed budgets less than OB' he prefers the larger, while of any two greater than OB' he prefers the smaller. In this situation, the well-known theorem of Duncan Black informs us that the equilibrium level of provision of the collective good is the median peak (i.e., the median of the members' most preferred budgets).⁸ Note in addition that, since tax shares are fixed in advance, this majority rule process does not in general lead to a Pareto optimal level of provision of the public good.⁹ This equilibrium budget level might be OB'' , in Figure III, which would place our typical club member at point C' with after-tax income OA'' .

Figure IV, derived in much the same way as was Figure II, illustrates the member's situation after his club has received a gift of $B''G$ units of the public good. Since his tax share is assumed

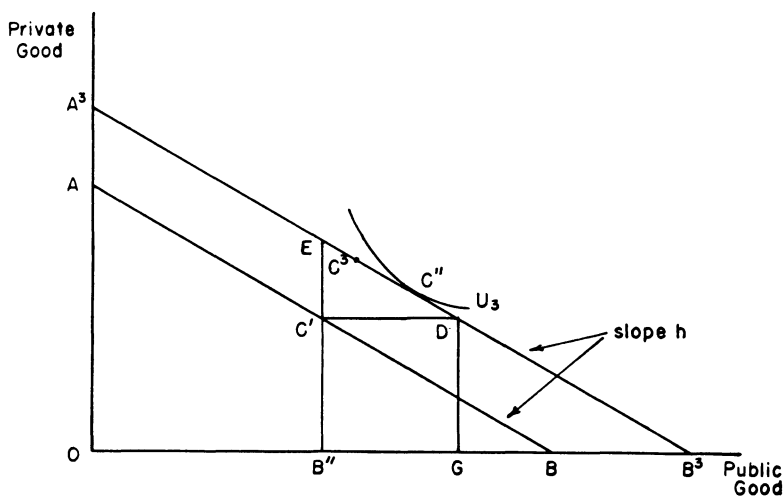


FIGURE IV

8. D. Black, "On the Rationale of Group Decision Making," *Journal of Political Economy*, Vol. 56 (Feb. 1948), pp. 23-34.

9. Since Black's analysis only considers a one-dimensional set of alternatives, our discussion of the case of majority rule, unlike that of the Lindahl mechanism, does not generalize directly into a world with many public goods. Tullock has made some headway toward generalizing Black's results (G. Tullock, "The General Irrelevance of the General Impossibility Theorem," *this Journal*, Vol. 81 (May 1967), pp. 327-52).

fixed in advance, the effect of the gift is to shift uniformly outward the locus of possible alternative bundles of private and public goods. The new set of alternatives lies along the straight line A^3B^3 , with slope h , passing through D . The most preferred outcome of the member depicted is labeled C'' , where the abscissa of C'' is the new peak of his preference structure over club budgets. Majority rule selects the median of such peaks, perhaps the abscissa of C^3 , as the equilibrium level of provision of the public good, and our club member receives the bundle of private and public goods implied by C^3 .

Could the external authority have achieved the same state of the club by a program of gifts directly to the members? Once again, the answer is apparent from an examination of Figure IV. Following a grant of $C'E$ of the private good (and assuming that he keeps the same assigned tax share, h), the member confronts the same alternatives that he does after the gift to the club; his preferences over these alternatives are unchanged, and in particular his preferred or "peak" level of provision of the public good is the same. Consequently, a set of grants to the members of the club where the i^{th} member receives $h_i \cdot B''G$ (where h_i is his tax share) will generate precisely the same state of the club as the grant of $B''G$ of the public good to the collectivity.

Before we turn to a discussion of our efforts to generalize the results of these simple analyses, it may be worthwhile pointing out the approach they suggest to the measurement of the distributive effects of revenue sharing. The implicit gift to the individual involved in a gift of OG to the club, in both the models considered, is $(h \cdot OG)$, where h is the individual's tax share in the *new* equilibrium. Roughly speaking, the gift to the individual is the increment in taxes he would experience if the club expanded its budget, without the gift, by the amount OG . We must say "roughly" because (a) it is difficult to contrive a rigorous concept of a tax increment that would not be generated by the political process in fact, and (b) the tax share really needed is that prevailing in the new equilibrium. However, if we can devise some sensible approximations to the *marginal* tax shares of different classes of individuals, we can make some reasonable estimates of the net distributional effect of the adoption of a revenue-sharing program as contrasted to various patterns of reductions in federal taxes.

IV. REVENUE SHARING IN A MORE GENERAL FRAMEWORK

An interesting result of the preceding analysis is that, under two quite different types of collective decision-making rules, pre-

cisely the same conclusions concerning the nature of the effects of revenue sharing emerge: revenue sharing is in both cases equivalent to gifts directly to the members of the club. And even more specifically, we were able to show under both systems that gifts to the club were equivalent to a distribution of the total sum among the members such that each member's gift is proportional to his tax share. This immediately raises the issue of how general these results are. What characteristic of these collective decision-making rules is it that gives rise to this common outcome?

In the Appendix of this paper, we attempt to develop a more general analysis of this issue. The analytic framework is generalized in two ways. First, the definition of a state of the club is expanded to allow for any number of distinct collective activities. A state of the club is thus defined by a specification of the disposable incomes of the n members, of the levels of each of m collective activities, and of the cash balance of the club (which, of course, is only a particular collective activity). Second, no "constant cost" assumptions are made; instead it is simply assumed that there is at any time a given set of possible states of the club called the "feasible set." Although the intuitive motivation for this feasible set is the production possibility set of standard theory, we require only the restrictions that the set is closed and bounded, that no one be assigned a negative disposable income, and that any state attainable by redistribution of the total disposable income of a feasible state is itself feasible.

The notion of a political process is also put in a more generalized form. Specifically, we conceive of a political process as a mapping that indicates for any initial feasible state the set of possible ultimate political equilibria. The problem of the general analysis is then to describe the sorts of mappings for which our conclusion continues to hold that revenue sharing is precisely equivalent to a specific program of grants to individuals. That is, we seek to describe sufficient conditions on a political process to assure this result.

In the Appendix, we prove formally the sufficiency of one set of four conditions. They are:

- (1) the mapping representing the political process is single-valued;
- (2) the mapping is continuous;
- (3) for a given feasible set, no two *distinct* equilibrium states exist for which it is true that in one everyone has a disposable income greater than or equal to his disposable income in the other; and

(4) if an individual has zero disposable income initially, he should not attain a greater disposable income via the political process than he would if in the initial state the collective cash balance were higher (with the other initial collective activities held the same).

Condition (1) is a kind of "determinacy condition." It says that, if you understood completely all the complexities of a club's political system, you could predict accurately the final state that would be reached from any initial state. This rules out randomness in the political process, although we consider it likely that analogous conditions to the four given ones would suffice to assure the validity of a similar theorem in a model that systematically incorporates uncertainty.

Condition (2) rules out threshold effects. It says that small changes in initial conditions will not generate abruptly large changes in political equilibrium, and might therefore be called the "no-revolution condition."

It may have been noticed that nothing has been said about the responsiveness of the political system to the preferences of the members. Nowhere have we mentioned maximizing behavior by the individual members, much less by the collectivity. And in fact, all of our conditions could be fulfilled by political processes completely unconnected with the members' preferences. However, we do require a property that we believe political processes highly responsive to individual preferences are likely to have, and that is condition (3). We therefore, with some hesitation, call it a "responsiveness condition." It says, in a sense, that the pattern of collective activities chosen by the club is not the result of whim. Some idea of its force is suggested by the things it rules out. It implies, for example, that there are no *politically feasible* ways to provide the collective service vector more cheaply for everyone than the way actually realized in any particular equilibrium (although there may be cheaper ways feasible under a different distribution of the tax burden). Moreover, it rules out "indifference": if the set of collective activities in an equilibrium state is disturbed in some feasible way — say, the level of police protection increased and the level of fire protection decreased — while leaving the disposable incomes of the members unchanged, then the resulting state is not a political equilibrium. Similarly, if the amount spent on collective activities in an equilibrium is changed with the change being financed by an increase (or decrease, as appropriate) in *every-one's* disposable income, the resulting state is not a political equilibrium. Note that it does *not* say, in either of these cases, that the

political process will tend to restore the original state; it simply says that the state will be further changed from its initial disturbance.

Condition (4) is the "it doesn't pay to be poor" condition. It says that, if you start off with zero income, you will not do better than you would *if initially* the club were providing the same levels of collective services *and* more money were in the collective cash balance. Since this condition does seem essential for our result, it obviously is not trivial, and it may have hidden implications that we have not yet discovered. However, on its face it does not seem unreasonable.¹

These four conditions are sufficient to assure the equivalence of revenue sharing and a particular pattern of gifts (central government tax cuts) to members. We find the conditions interesting not only because we think it plausible that they are approximately true in many cases, but also because there are identifiable situations where they are false, and where our equivalence conclusion does not hold. (Note, however, that we have not proved that failure of our conditions implies failure of the equivalence conclusion; we prove sufficiency, not necessity.)

The four conditions are not implausible ones. Conditions (1) and (2), making allowances for uncertainty, information costs, and time, seem tame enough. Condition (3), however, is both key to the results and somewhat more questionable. A little experimenting will convince the reader that it holds (as do (1) and (2), and (4) by default) in the simple political systems examined in the previous sections providing that individual members have strictly convex preferences.² Its applicability to the real world, however, depends upon the view that the results ruled out by condition (3), as discussed above, are properly excludable, say, for Princeton Borough, or the State of New Jersey. While we find condition (3) acceptable in many cases, there are clear examples where it is not

1. As Professor William Vickrey has pointed out to us, condition (4) can be made apparently even more innocuous by reducing the bound of "zero disposable income" to any finite negative number, since all that we require for the theorem is a lower bound to the set. We remain concerned, however, that, even in this form, it may have some implications we do not yet appreciate.

2. Note that the phrase, "for a given feasible set," must be taken seriously in this and other experiments. This means that the fact that an increase in all budgets might lead to no increase in the public good, only increases in disposable income, in either the Lindahl or majority-rule models does not violate condition (3), since these changes involve alterations to the feasible set. Condition (3) is perhaps weaker than may be first supposed.

fulfilled. These and other exceptions to the equivalence conclusion are discussed in the next section.

We should also point out that we have found it difficult to generalize the result, true for the simple models, that the gift to a member, which is identical in effect to the gift to the club, is equal to his tax share times the total amount of the gift to the club. This is because the concept of tax share in more general models is not well defined. To find the procedures for computing the equivalent gifts to individuals in more general models, which is precisely the problem of estimating equivalent gifts empirically under the assumption that our theorem is true of the real world, is a problem awaiting further work.

V. WHEN REVENUE SHARING DIFFERS FROM INCOME REDISTRIBUTION

We distinguish two sorts of cases in which the equivalence conclusion does not hold. In the first class are cases of the failure of condition (3), and, generalizing from the examples we can think of, we call this "failure by institutional structure." In the other class, the problem is more fundamental; the failure of equivalence arises from the invalidity of the modeling of the political process as a determinate mapping, given the set of feasible states. The cases we have constructed of this sort justify calling this "failure by learning or habit."

The first class may be illustrated by an extreme example. Suppose that Princeton Borough is forbidden by higher authority, say the state constitution, from collecting more than some fixed amount in taxes, perhaps based on the town's acreage. And suppose that the "real" equilibrium budget is significantly in excess of this amount, so that the town is continually spending its limit. In this situation, it may well be possible to find a combination of reduction in *all* citizens' incomes and increase in the town budget that would not be changed by the town's political process. This contradicts condition (3), and it is not difficult to see that the equivalence conclusion is likely to fail as well.

Much less extreme examples give similar results; in particular, no constraint by higher authority is needed to produce them. To take such a case, suppose that Princeton's own bylaws (amendable by a two-thirds majority) require a two-thirds majority approval of any property tax rate in excess of 3 percent, with lesser tax assessments requiring only a simple majority approval. Suppose

further that seven-twelfths of the voters would approve a budget in excess of that which can be financed by the 3 percent tax. In this case also, it is not difficult to imagine a reduction in the incomes of all citizens, accompanied by an increase in town spending, such that the resulting situation is stable under the political process, since it could only be altered by a reduction in the property tax rate, preventable by a simple majority. Here again we have failure of condition (3); failure of the equivalence conclusion could certainly also be shown.

Clearly, the last example of violation of condition (3) is not far-fetched, and even the first example may not be too far from describing the situation of some city governments. This suggests that a rationale for revenue sharing could be constructed on the basis of interest in the higher government in circumventing such structural characteristics of the lower-level governments as those described. The implications of this point of departure for sharing formulas (and for other specifics of revenue-sharing plans) appear to be a promising and interesting subject for further research.³

The second class of failures of the equivalence conclusion involves dynamic elements. In his textbook on public finance, Otto Eckstein points out: "An old program is a good program. Once it has existed for a period, a program generates its own clientele, both inside and outside the government, which has a vested interest in its continuance."⁴ This could be interpreted to mean that the mapping that represents the choice from the set of alternative feasible states, given any starting point, itself depends upon the history of the system prior to the starting point. This phenomenon is akin to that of changing tastes in ordinary economic theory and could certainly upset the equivalence conclusion. A special case might be a sort of fiscal illusion whereby citizens don't know, and can't correctly imagine, the consequences of some proposed collective action. If they had experienced it, they would choose it; if not, they would reject it. If the sequence of events following revenue sharing should involve some experience with this action, the choice generated by the political process from a *given* starting point out of a *given* feasible set might well be different from that which would occur in the absence of that experience. As does the first class, the second

3. An obvious question that must be addressed in such an analysis is that of the functions of such structural features. James Buchanan has suggested some possible answers (*The Demand and Supply of Public Goods*, Chicago: Rand McNally, 1968).

4. O. Eckstein, *Public Finance*, 2nd ed. (Englewood Cliffs, N.J.: Prentice-Hall, 1967), p. 33.

class of failures of the equivalence conclusion might support a revenue-sharing program, and closer analysis of it would certainly be desirable.

VI. A SYSTEM WITH MANY CLUBS

The analysis to this point has been conducted wholly in terms of revenue sharing by an external agency with a single, isolated group of individuals. When, however, we expand the system to include many fiscal clubs and allow mobility of individuals among clubs, the possibility arises of a new phenomenon, which may have implications both for resource use and for the distribution of income: competition among clubs for members. It has been argued by some advocates of revenue sharing that decentralized levels of government, engaged in competition to attract not only new residents but also additional business investment, are reluctant to raise tax rates for fear of discouraging potential entrants; such tax competition, so the argument goes, leads systematically to less than efficient levels of provision of the public services consumed in these jurisdictions. A program of revenue sharing, the advocates suggest, will help to resolve this problem by providing additional funds and thereby stimulating levels of expenditures on the public services provided by subcentral levels of government.⁵

It seems to us, however, that the extension of the analysis to a system with many clubs does not alter the substance of our earlier conclusions. There are, for example, real grounds on which to question the proposition that there in fact exists pervasive tax competition that leads to a systematic underprovision of state and local public services.⁶ However, even if this contention is true, even if in our model all fiscal clubs hold their levels of provision of public goods below the desired level for fear of discouraging potential

5. See, for example, W. Heller, *New Dimensions of Political Economy* (Cambridge: Harvard University Press, 1966), p. 126.

6. Since potential residents (members) presumably consider fiscal benefits as well as costs in making choices, a jurisdiction (club) might, for example, have greater success in expanding its membership by extending, rather than contracting, levels of provision of public goods relative to what they would have been in the absence of competition. For an empirical study of 53 suburban communities in New Jersey, which suggests that the quality of local public schools, as well as local tax rates, influences significantly the location decisions of at least some families, see W. Oates, "The Effects of Property Taxes and Local Public Spending on Property Values: An Empirical Study of Tax Capitalization and the Tiebout Hypothesis," *Journal of Political Economy*, Vol. 77 (Nov./Dec. 1969), pp. 957-71. An interesting theoretical analysis of the formation of fiscal clubs is contained in J. Buchanan, "An Economic Theory of Clubs," *Economica*, Vol. 32 (Feb. 1965), pp. 1-14.

members by excessive tax bills, it does not follow that a revenue-sharing program will correct the situation.

To provide an incentive to expand the provision of public services to efficient levels, what is needed is a program that reduces, *at the margin*, the effective price of public services to the club. Some type of conditional, matching-grant program, for example, by which the external fiscal authority pays a certain proportion of the cost per unit of the public service will reduce the price of a unit of the good to the club's membership. Under a revenue-sharing program, in contrast, the price of an additional unit remains unchanged, and there is good reason to expect any existing tax competition to continue with the resulting subefficient levels of public output. As James Wilde has put it, "The introduction of an [unconditional] aid program can be expected to result in a tax competition as fierce as ever, though carried on at a lower level of local tax rates than previously."⁷

For this reason, revenue sharing in its pure form (i.e., as a wholly unconditional grant) is not the appropriate program to resolve inefficiencies stemming from tax competition among jurisdictions. Although, as we suggested in the preceding section, revenue sharing may under certain circumstances have more than just distributive effects, it does not provide the sort of systematic incentive for the expanded output of public services that would appear necessary to offset the effects of any existing tax competition. It is interesting in this regard that several proponents of revenue sharing, in their concern that federal assistance should provide a stimulus to the output of public services, have recommended the inclusion of a "fiscal effort" term in the revenue-sharing formula. Typically, the proposal is that the grant to a particular jurisdiction vary directly with the proportion of its income that the jurisdiction raises in tax revenues. Such a provision clearly does introduce a price effect into a revenue-sharing program; for each extra dollar the jurisdiction spends (through its own increased taxation), the size of the grant it receives from the central government is increased by some fraction of a dollar. A dollar of increased public expenditure thus costs the residents of the jurisdiction less than a dollar.

For this reason, the inclusion of a fiscal effort term in the revenue-sharing formula does destroy the formal equivalence of revenue sharing with a set of lump sum grants directly to the individual members of a club; it means a reduced price per unit of the public service to the club and in general higher levels of public outputs.

7. Wilde, *op. cit.*, p. 346.

It is possible, however, under some circumstances for a fiscal effort provision to lead to rather curious results. It can, for example, actually penalize a club or jurisdiction that, because of some institutional constraint (e.g., legal ceilings on property tax rates), is unable to increase its revenues. In this instance, other jurisdictions that respond with an increase in their own levels of taxation can realize a larger portion of the available shared revenues, thereby leaving a relatively smaller grant for the constrained collectivity.⁸

VII. CONCLUDING REMARKS

Using an analytic framework designed to describe models of collective choice, we have found in this paper that, for a reasonably broad class of processes of collective decision making, revenue sharing is precisely equivalent to a specific set of lump sum grants directly to the individual members of the collectivity. We found, however, that there are also a number of not implausible circumstances under which the equivalence theorem does not hold and for which revenue sharing may allow the realization of outcomes not attainable under any set of grants to individuals. An evaluation of the likelihood of these exceptions to the theorem is, we think, a matter of importance for the evaluation of revenue-sharing programs and a particularly promising area for further research.

Finally, we simply point out that, in assessing the empirical plausibility of the equivalence condition, it must be recognized that any actual revenue sharing will take place in a growing system, one in which for a number of reasons state and local budgets are probably going to rise fast enough to call for increases in tax rates (as well as revenues).⁹ Equivalence (or approximate equivalence) does not require in this environment that states and localities cut their tax rates as a consequence of revenue sharing; it may imply only that they will not raise them as rapidly as otherwise.

8. The analytic framework developed in this paper also appears applicable to the study of intergovernmental grants with price effects; in particular, matching grants to a collectivity are, under appropriate conditions, presumably equivalent to some specific set of expenditure subsidies to individuals. This is a matter we are currently exploring.

9. For example, William Baumol has argued that an inevitable bias in technical progress will make state-local government activities ever more expensive relative to manufactured goods ("Macroeconomics of Unbalanced Growth: Anatomy of Urban Crisis," *American Economic Review*, Vol. 57 (June 1967), pp. 415-26). Some evidence on this is available in D. Bradford, R. Malt, and W. Oates, "The Rising Cost of Local Public Services: Some Evidence and Reflections," *National Tax Journal*, Vol. 22 (June 1969), pp. 185-202.

APPENDIX: A GENERAL THEOREM

Let S be the set of feasible states of the club. A typical element s of S is an $(n + m + 1)$ -dimensional vector whose first n components, s_i ($i = 1, \dots, n$), specify the private good claims, y_i (the "disposable incomes"), of the n members, whose next m components, s_{n+j} ($j = 1, \dots, m$), indicate the levels, g_j , of operation of m collective activities, and whose $(n + m + 1)$ *st* component indicates the level, G , of the collective cash balance. The set S is determined by the resources available to the club and by the production and exchange relationships that determine how the private good can be transformed into the different collective goods. It is assumed that S is closed and bounded, that disposable incomes must be nonnegative, and that a state attainable from a feasible state by redistribution of its disposable income is itself feasible.¹ Symbolically, if $s \in S$, then

$$s_i \geq 0, \quad i = 1, \dots, n$$

and

$$s'_j = s_j \quad j = n + 1, \dots, n + m + 1,$$

$$s'_i \geq 0 \quad i = 1, \dots, n$$

$$\sum_{i=1}^n s'_i \leq \sum_{i=1}^n s_i$$

$$\rightarrow s' \in S.$$

It is further assumed that collective activities are not costless or are, at best, free. Let $Y(g_1, \dots, g_m, G)$ be the set of all distributions of disposable income obtainable in S , given government activities g_1, \dots, g_m and collective cash balance G . Then the last assumption can be formulated as the requirement that, for $\epsilon_1, \dots, \epsilon_{m+1} \geq 0$, $Y(g_1 - \epsilon_1, \dots, g_m - \epsilon_m, G - \epsilon_{m+1})$, if defined, contains $Y(g_1, \dots, g_m, G)$. Figure V shows a typical feasible set in the two-person, one-public service case, assuming fixed collective cash balance, G .

We represent a political process for the club by a mapping, p_s , from S into the set of subsets of S , such that $p_s(s)$ is a set containing the possible ultimate equilibrium states in S that might be reached by the political process, given s as a "starting point." The political process, p_s , is thus a set-valued function of state vectors. If s is itself an equilibrium vector, $p_s(s) = \{s\}$; s is mapped into itself by political process.²

Of special interest to us are the restrictions that one must place on this rather general model of collective budget determination in order to be able to demonstrate a proposition about the equivalence

1. It would probably be possible to relax the last part of this restriction to say that the set of possible states having any given vector of collective services and collective cash balance is convex.

2. One might describe as a "constitution" the mapping from the set of all possible sets of feasible states to the set of possible political processes, $\{S\} \rightarrow \{p_s\}$.

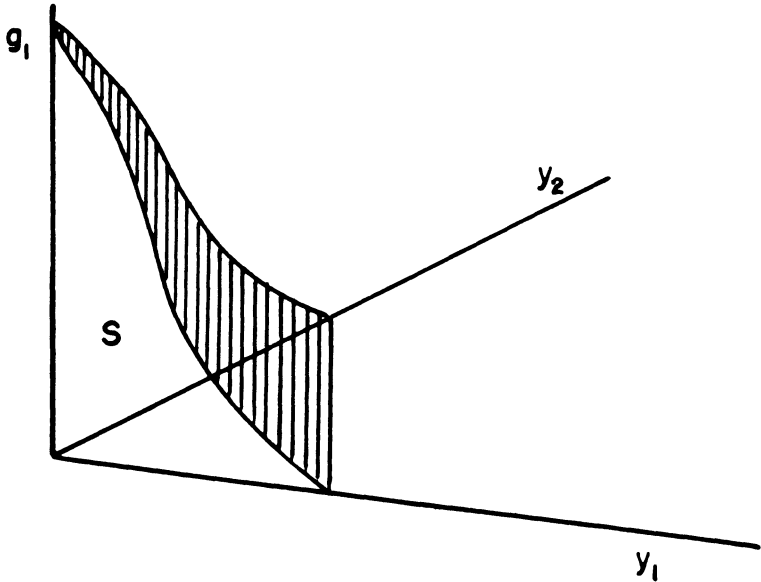


FIGURE V

The feasible set, S , holding collective cash balance at G , is enclosed by the shaded surface and the planes of the axes.

of revenue sharing to a program of income redistributions, as we have done in the simple cases examined in the text. We have found four assumptions about p_s to be sufficient for this:

ASSUMPTION 1. *To any initial state corresponds exactly one ultimate equilibrium state.*

In the text we described this assumption somewhat unrigorously as the single valuedness of the political process. Because of this assumption we can treat p_s as a vector-valued function of state vectors (though it is, strictly, a set-valued function); $p_s(s)$ is an element of S . If s is an equilibrium state, $p_s(s) = s$.

ASSUMPTION 2. *p_s is continuous.*

ASSUMPTION 3. *If s is an equilibrium state under p_s , and $s' \in S$, $s' \neq s$ gives no less disposable income to all citizens ($s'_i > s_i$, $i = 1, \dots, n$), then s' is not an equilibrium under p_s .*

The fourth assumption concerns families of subsets of S . Let $S(g_1, \dots, g_m, G)$ refer to the set of all elements of S having the specified levels of government activities and the specified collective cash balance, and consider the family of such sets defined as G varies from zero to some maximum.

ASSUMPTION 4. *If $s \in S(g_1, \dots, g_m, G)$ and, for some $i \leq n$, $s_i = 0$, and $s' \in S(g_1, \dots, g_m, G')$, $G' > G$, then $p_s(s)_i \leq p_s(s')_i$.*

The meaning of these assumptions was discussed in the text. We can now state:

THEOREM. Given A1–A4, if $G < G'$ and $s' \in S(g_1, \dots, g_m, G')$, then there exists $s \in S(g_1, \dots, g_m, G)$ such that $p_s(s) = p_{s'}(s') \equiv x$.

Before proceeding to a proof of the theorem we should indicate how it implies the equivalence conclusion. It is assumed that, since revenue sharing in its pure form does not affect the technology of producing public services or the terms of trade under which they are purchased, the set of feasible states is the same, no matter how the gift is made to the club—be it to the club's collective cash balance or to the individual citizen. The initial effect of a gift to the club is an increment to the club's cash balance; this new state is further transformed by the political process. The theorem says that among the states in the same feasible set with the same initial government services but with the pre-revenue-sharing cash balance, is a state that maps into exactly the same state as is attained as a result of revenue sharing. That is, the same final outcome can be produced by a gift directly to citizens.

Proof. The idea of the proof is very simple and recalls proofs of existence theorems in competitive equilibrium theory. Suppose we "try out" an arbitrary element of $S(g_1, \dots, g_m, G)$ (to simplify the notation we call this set henceforth simply $S(G)$) to see whether it maps into x under p_s . If, say, individual i ends up with too much disposable income on this trial, the natural next step would be to pick a new starting point in $S(G)$ that gives individual i somewhat less disposable income than the original trial vector. The initial disposable incomes of the other citizens are similarly adjusted. All we need to show is that there definitely is at least one element of $S(G)$ that does not need any adjustment, since A3 says that if two equilibria in S have the same disposable income components they are identical. (This oversimplifies slightly. Since it is against the rules to reduce the disposable income of someone who starts with zero, assumption A4 is needed to make sure this will not be required.)

The proof employs the Brouwer fixed-point theorem, which states that a continuous mapping from a closed and bounded convex set in Euclidean space into itself has a fixed point (i.e., there is a point that is mapped into itself by the function). Having shown $S(G)$ to be closed, bounded, and convex, we define a function that maps any element of $S(G)$ which does *not* lead via the political process to x as an equilibrium into a different element of $S(G)$, so that if this function maps that element into itself, then that element of $S(G)$ does generate the political equilibrium s . This "correction" process will be seen to be continuous, and hence such a fixed point must exist, establishing the theorem.

(1) $S(G)$ is closed, bounded, and convex. $S(G)$ consists of all elements of S with a certain specified collective activities vector and specified cash balance, G . Since it is also assumed that any distribution of a given total disposable income is possible (including distributions summing to less than the given total), clearly $S(G)$ is convex. Since S is bounded, so must be $S(G)$. Furthermore,

$S(G)$ is closed, for suppose a sequence of elements of $S(G)$ converges to a limit. Since S is closed the limit point is in S ; since all elements of $S(G)$ have the same government services components and collective cash balance, so must the limit point. But then the limit point must be in $S(G)$, by definition of $S(G)$. It follows that there is a maximum aggregate disposable income in $S(G)$; call it $y^*(G)$.

(2) We first take care of a pathological case. If $y^*(G) = 0$, by the assumption about the costliness of government activities and cash balances, $y^*(G') = 0$. This means that $S(G')$ and $S(G)$ each contain a single element, s' and s , respectively, in which no one has a positive disposable income. Then, by A4, $p_S(s)_i \leq p_S(s')_i$, $i = 1, \dots, n$. Then, by A3, $p_S(s) = p_S(s')$.

(3) If $y^*(G) > 0$, define the "correction mapping" in a series of steps. First, define the function f from $S(G)$ into Euclidean n -space:

$$f(s)_i = \max\{s_i + x_i - p_S(s)_i, 0\}, i = 1, \dots, n;$$

then the mapping g from $S(G)$ into the real line:

$$g(s) = \max \left\{ \sum_{i=1}^n f(s)_i, y^*(G) \right\};$$

then the mapping h from $S(G)$ into Euclidean $(n + m + 1)$ -space:

$$h(s)_i = \frac{y^*(G)}{g(s)} \cdot f(s)_i, \quad i = 1, \dots, n$$

$$h(s)_{n+j} = g_j, \quad j = 1, \dots, m$$

$$h(s)_{n+m+1} = G.$$

We must show that h is a continuous mapping into $S(G)$. Since all the component functions are continuous, so is h . Since $f(s)_i$ is constrained to be nonnegative, the first n components of h are nonnegative; i.e., the disposable incomes are nonnegative, as required. That the sum of the first n components of h is no greater than $y^*(G)$ is assured by the scaling factor $y^*(G)/g(s)$. Finally, the m government activity components of h and the collective cash balance are at those levels by which $S(G)$ is defined.

(4) Since h is a continuous mapping from $S(G)$ into $S(G)$, by Brouwer's theorem, it has a fixed point. We must now show that if s is such a fixed point, $p_S(s) = x$. If $s_i > 0$, $f(s)_i > 0$, and so $f(s)_i = s_i + x_i - p_S(s)_i$. Thus $h(s)_i = s_i$ implies

$$\frac{y^*(G)}{g(s)} (s_i + x_i - p_S(s)_i) = s_i;$$

$$x_i - p_S(s)_i = \left(\frac{g(s)}{y^*(G)} - 1 \right) s_i.$$

Since, by construction, $\frac{g(s)}{y^*(G)} \geq 1$, this implies $x_i - p_S(s)_i \geq 0$.

If $s_i = 0$, $f(s)_i = 0$, and so $x_i - p_s(s)_i \leq 0$. But, by A4, $p_s(s)_i \leq x_i$; hence $x_i - p_s(s)_i = 0$. Hence we have, for $i = 1, \dots, n$,

$$x_i - p_s(s)_i \geq 0.$$

By A3, this implies $x_i = p_s(s)_i$, $i = 1, \dots, n + m + 1$.

Q.E.D.

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