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Abstract

We investigate preferences for acting stochastically in the context of social preferences. We analyze a Probabilistic Dictator Game under a broad class of models that incorporate ex-ante and ex-post social preferences as well as stochastic choice models that motivate stochastic giving beyond pro-social reasons and study their implications with lab experiments. The behavior of a majority of dictators is inconsistent with the models of ex-ante/ex-post social preferences when the parameters are selected from a meaningful range. Nevertheless, a majority of dictators are consistent with the axioms of stochastic transitivity and regularity. Furthermore, their behaviors are in line with an Additive Perturbed Utility model.

Keywords: Social preference under risk, Dictator game, Stochastic giving

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1 Introduction

Social preferences characterize the behavior of individuals who care about others' payoffs. A vast literature explores these preferences and their applications in contexts such as dictator and ultimatum games, charitable giving, public goods provision, collective action incentives, and redistribution policies (see Fehr and Charness, 2025 for a survey). While much of this literature examines deterministic environments, recent research has increasingly focused on decision-makers who may impact others' payoffs stochastically by taking risky actions or deliberately randomizing between possible strategies (see, e.g., Saito, 2013; Trautmann, 2009; Brock et al., 2013; Miao and Zhong, 2018). One approach suggests that social preferences lead to risky allocations in order to give others fair opportunities (ex-ante fairness by Saito, 2013). On the other hand, individuals may randomize between allocations because it is too costly to commit to a specific allocation (Fudenberg et al., 2015).

Motivated by these alternative approaches, in this paper, we derive the predictions of a general class of ex-ante/ex-post fairness models as well as stochastic choice models that motivate preferences for randomization beyond pro-social reasons in the context of a novel Probabilistic Dictator Game. We study these theoretical predictions in a series of laboratory experiments. While ex-ante fairness models predict either deterministic behavior or a randomization on a limited set of allocations in our setup, the subjects tend to randomize over a wide range of allocations. Nevertheless, their behaviors are consistent with the axioms of stochastic transitivity and regularity.

Consider an extension of the standard dictator game where a decision maker (dictator) has \$10 and decides how to allocate it between himself and a recipient, possibly probabilistically. We call this game the Probabilistic Dictator Game. It is applicable to situations such as a leader choosing a risky plan that will create probabilistic returns for the involved parties. In this game, the dictator could still exhibit a deterministic choice by placing 100% weight on one of the allocations and giving a fixed amount with certainty. Alternatively, he may choose a risky allocation such as giving \$5 and \$0 with 40% and 60% chances, respectively. While a selfish dictator gives \$0 and a fair-minded one aiming to equalize payoffs gives \$5 with 100% chance, the motivation for giving stochastically may not be as clear. One reason could be the dictator is concerned about ex-ante fairness (giving the other person a fair

opportunity) and on average wants to give \$2; therefore, the above randomization is a way to do that. Alternatively, he is split between being fair and selfish and employs a random utility maximization model or it might be too costly for him to commit to a fixed allocation.

Theoretically, we first characterize the predictions of the models that combine social preferences for opportunities and outcomes in a Probabilistic Dictator Game. We start with the seminal paper of Saito (2013) - Expected Inequality Aversion model (ex-ante/ex-post inequality aversion) - that combines these two motives linearly based on an inequality aversion model of Fehr and Schmidt (1999). Then we change the underlying social preference to the distributional preferences model of Charness and Rabin (2002). Next, we consider a wider class of non-parametric and possibly non-linear models of social preferences under risk and characterize the behavior of the dictator under these models, including the non-parametric exante/ex-post preference model and the quadratic welfare model of Epstein and Segal (1992). These models predict mostly deterministic giving and the type of stochastic giving they can accommodate is specific: the dictators will randomize between at most two allocations even in a domain where various ways of giving are possible. A wider range of randomization can only occur under confined parametric assumptions within these models. Our Experiment 1 investigates this sharp prediction in a simple Probabilistic Dictator Game. We observe that a majority of the dictators neither give deterministically nor randomize between two allocations. In other words, a majority of the subjects falsify the predictions of the models that combine ex-ante and ex-post social preferences.

There is a significant body of stochastic choice models and their axiomatic foundations (Strzalecki, 2025) arguing that agents may not have stable preferences (due to, for example, randomness of utility) or they may prefer randomizing between options (see, for example, Fudenberg et al., 2015). While these models are primarily written for decision problems of a single (selfish) agent, their axioms are naturally applicable to the domain of social preferences. Our Experiment 2 is designed to study the main models (or their corresponding axioms) of this literature in a Probabilistic Dictator Game. In this experiment, we elicit dictators' randomization behavior in four Probabilistic Dictator Games with nested menus of possible allocations.

Experiment 2 confirms the main message of the results we find in Experiment 1: There

is a tendency to give stochastically beyond what can be explained by the social preference models, as these models can only account for the behavior of 27-41% of subjects. Only about 17% of subjects made a deterministic choice (choosing a sure allocation), and all of them could be modeled as rational preference maximizers. A majority of subjects satisfied key rationality properties in the probabilistic choice environments: 58% satisfied Regularity, and 92% satisfied Moderate Stochastic Transitivity. In our setting, the behavior of the former group admits a Random Utility representation, while the choices of the latter align with the Moderate Utility Model of He and Natenzon (2024). Finally, we investigate the Additive Perturbed Utility model of Fudenberg et al. (2015), which combines the expected utility of consumption with a non-linear cost function. Depending on the specification, between 39% and 79% of the subjects' behavior is in line with this model.

Literature Review.

Our study relates to a class of theoretical and experimental papers that investigate social preferences with risky outcomes, particularly in the context of the dictator game (Andreoni et al., 2020; Krawczyk and Le Lec, 2010; Brock et al., 2013; Cappelen et al., 2013; Sandroni et al., 2013; Miao and Zhong, 2018). A key finding in this literature is that individuals value both ex-ante and ex-post payoffs of others.

Empirically, most experiments on the dictator game with risk components are characterized by (i) a binary-state environment where the menu of possible allocations has two elements (Krawczyk and Le Lec, 2010; Brock et al., 2013), and (ii) predetermined levels of exogenous risk (Andreoni and Bernheim, 2009; Bolton and Ockenfels, 2010; Chen and Zhong, 2025). Regarding point (i), when a dictator is making a choice in a binary-state environment and must choose between, say, keeping his entire endowment or giving the entire amount to the recipient, he might randomize between the two options because his preferred option—such as a 50-50 split—is not available. Moreover, as our theoretical analysis shows, a binary setting may not be suitable to study some predictions of the social preference models, as it does not allow subjects to exhibit preferences for randomization over a wide range of allocations. We address this problem in Experiment 1 by eliciting the dictator's behavior from a menu of eleven possible allocations. Therefore, our results fill this gap in the literature.

Regarding point (ii), Bolton and Ockenfels (2010) study the choice of a dictator who

faces a safe allocation and a risky option consisting of two given allocations with equal probabilities. Andreoni and Bernheim (2009) introduce the possibility that nature determines the allocation. In their experiments, the probability of nature deciding and the payment if nature decides are exogenously given. In contrast to these two papers, both the source and the degree of risk are endogenous and determined by the dictator in our experiments. Chen and Zhong (2025) is another relevant study with exogenous risk. In one of their experiments, people choose between keeping a randomly determined endowment, which may be high or low depending on nature, or splitting it equally with an anonymous partner. They find that individuals act more morally in uncertain environments, a pattern they attribute to quasi-magical and magical thinking, in which moral actions are believed to induce favorable outcomes from nature. Our experiments, by contrast, endogenize risk and thereby shut down these quasi-magical and magical thinking channels as the dictator can always choose to guarantee a certain outcome.

There is some recent research connecting social preferences and risk attitudes, including Zame et al. (2025) and Feldman and López Vargas (2024). Zame et al. (2025) provide conditions to deduce the dictators' preferences on lotteries with social impact from their choices in domains where outcomes are personal or riskless. Feldman and López Vargas (2024) propose and estimate a parametric model that incorporates both ex-ante and ex-post fairness motives. In our experiments, regression analyses suggest that risk attitudes have a negligible and insignificant effect on other-regarding preferences.

The theoretical research on stochastic choice has been mostly within the single-agent framework (see Strzalecki, 2025). A growing body of empirical findings documents that individuals tend to act stochastically when choosing between lotteries with private outcomes (e.g., Agranov and Ortoleva, 2017, Ellis et al., 2024). Ellis et al. (2024) find strong support for rationality for those who act deterministically, and this is consistent with what we find in our Probabilistic Dictator Games.

The following sections are organized as follows. Section 2 introduces a Probabilistic Dictator Game and studies predictions of a general class of social preferences for outcome and opportunity. Section 3 describes the design of Experiment 1, and Section 4 presents its results. Section 5 introduces Experiment 2. The results of Experiment 2 are reported in

Section 6. Additional analyses are presented in Section 7. Section 8 concludes. All the proofs, additional data analyses, and the instructions of the experiments are in the Appendices.

2 Setup and the Theoretical Predictions of Social Preference Models

A Probabilistic Dictator Game is a decision problem of an agent (dictator) who decides how to allocate an endowment, M > 0, between himself and a recipient possibly probabilistically. Let $\mathcal{A} = \{a_1, \ldots, a_n\}$, with $n \geq 2$, denote a finite menu of possible allocations where $a_i = (x_i, y_i) \in \mathbb{R}^2_+$ represents the payoffs of the dictator (x_i) and the recipient (y_i) , such that $x_i + y_i = M$ for all i. The allocations are ordered so that the dictator's payoff is increasing, i.e., $x_{i+1} > x_i$. For simplicity, we assume that $x_1 = 0$ and $x_n = M$. The dictator chooses a probability distribution $\rho = (\rho_1, \ldots, \rho_n) \in \Delta(\mathcal{A})$, where $\Delta(\mathcal{A})$ is the set of all probability measures over \mathcal{A} .

2.1 Ex-ante/Ex-post Preferences

We start the analysis by assuming that the dictator has preferences for both opportunity and outcome. This is one of the leading approaches in investigating other-regarding preferences in risky environments. The dictator's utility from choosing $\rho \in \Delta(\mathcal{A})$ is given by $\mathcal{V}(\Phi_{\text{ex-a}}(\rho), \Phi_{\text{ex-p}}(\rho))$, where $\Phi_{\text{ex-a}}$ and $\Phi_{\text{ex-p}}$ capture distributional preferences in the ex-ante and ex-post senses, respectively. Following Saito (2013), we assume that $\Phi_{\text{ex-a}}$ and $\Phi_{\text{ex-p}}$ have expected utility representations:

$$\Phi_{\text{ex-a}}(\rho) = u\left(\sum_{k} x_{k} \rho_{k}, \sum_{k} y_{k} \rho_{k}\right) = u\left(\sum_{k} x_{k} \rho_{k}, M - \sum_{k} x_{k} \rho_{k}\right),$$

$$\Phi_{\text{ex-p}}(\rho) = \sum_{k} \rho_{k} u(x_{k}, M - x_{k}),$$

where $u: \mathcal{A} \to \mathbb{R}$ is the dictator's utility function, with $u(x_i, M - x_i)$ denoting the dictator's utility from choosing allocation $(x_i, M - x_i)$ deterministically. Let ρ^* represent the optimal solution for a dictator maximizing \mathcal{V} . In the following subsections, we characterize ρ^* under various specifications of \mathcal{V} and u.

¹We use ρ_i and $\rho_{(x_i,y_i)}$ interchangeably to denote the weight of allocation (x_i,y_i) in the dictator's decision.

2.1.1 Linear combination of preferences

In this subsection, the \mathcal{V} function is assumed to be linear as in the Expected Inequality-Aversion model of Saito (2013). In this model, the dictator has distributional preferences for both opportunity (ex-ante motives) and outcome (ex-post motives). The \mathcal{V} function is a linear combination of the two motives:

$$\mathcal{V}(\Phi_{\text{ex-a}}(\rho), \Phi_{\text{ex-p}}(\rho)) = \delta\Phi_{\text{ex-a}}(\rho) + (1 - \delta)\Phi_{\text{ex-p}}(\rho), \tag{1}$$

where $\delta \in [0,1]$ measures the strength of the preference for ex-ante motives. In what follows, we characterize the dictator's optimal choice in two different specifications of the utility function, u, based on Fehr and Schmidt (1999) and Charness and Rabin (2002). Note that these two utility functions are not differentiable; we address differentiable utility functions in Subsection 2.1.2.

The distributional preferences model of Fehr and Schmidt (1999). Fehr and Schmidt (1999) propose a model of distributional preferences based on the idea that individuals are inequality averse:

$$u^{FS}(x,y) = x - \alpha \max\{y - x, 0\} - \beta \max\{x - y, 0\}.$$
 (2)

In the utility function above, parameter α captures enviousness, the (dis)utility of the dictator when his payoff is less than the recipient's payoff, and β captures guilt, the (dis)utility of the dictator when his payoff exceeds the recipient's payoff. Besides inequality aversion, the simple framework proposed by Fehr and Schmidt (1999) in (2) can also capture other types of other-regarding preferences such as altruistic preferences or spiteful preferences, depending on the values of α and β .

Empirical studies have documented a significant fraction of individuals exhibiting these types of preferences; for a detailed review, see Table 1 in Fehr and Charness (2025) and Figure 4 in Nunnari and Pozzi (2025). We assume that $\alpha > -1/2$ and $\alpha + \beta \geq 0$. The first condition is supported by all 144 empirical estimates of (α, β) reviewed by Nunnari and Pozzi (2025). The second condition excludes individuals with inequality-seeking preferences ($\alpha < 0$)

and $\beta < 0$) and is also strongly in line with empirical findings.² Additionally, note that when $\alpha = -1/2$, this model assigns a constant utility, $u^{FS}(x,y) = M/2$, to any allocation that favors the recipient and similarly, when $\beta = 1/2$, this model assigns a constant utility, $u^{FS}(x,y) = M/2$, to any allocation that favors the dictator in our setup. We rule out these uninteresting cases ($\alpha = -1/2$ or $\beta = 1/2$) that lack predictive power.

Next, we characterize the optimal choice of the dictator who is maximizing utility functions specified in (1) and (2). We provide two versions of this characterization depending on whether the equal division is available to the dictator, i.e., whether $(M/2, M/2) \in \mathcal{A}$. This distinction is relevant for our empirical analysis later, as the Probabilistic Dictator Games that we will use in the experiments cover both cases.

Proposition 1. Assume $a_e = (\frac{M}{2}, \frac{M}{2}) \in \mathcal{A}$. A dictator who is maximizing the utility functions specified in (1) and (2) will choose optimal ρ^* as follows:

- i. If $\delta \neq 1$,
 - (a) If $\beta < 1/2$ then $\rho^* = (0, ..., \rho_n^* = 1)$.
 - (b) If $\beta > 1/2$ then $\rho^* = (0, ..., \rho_{a_e}^* = 1, ..., 0)$.
- ii. If $\delta = 1$,
 - (a) If $\beta < 1/2$ then $\rho^* = (0, ..., \rho_n^* = 1)$.
 - (b) If $\beta > 1/2$ then ρ^* is optimal if and only if $\sum_k \rho_k^* x_k = M/2$.

Even though the optimal strategy depends on the parameters of the model in Proposition 1, note that in almost all cases the dictator plays deterministically, and the chosen allocation is either the selfish one or the fair one when it is available. Moreover, when randomizing (such as in case (ii.b)), the dictator allocates fairly in expectation. These sharp implications will be the basis for our empirical analysis later.

Case (i) of Proposition 1 states that when the dictator has ex-post concerns, he chooses deterministically and either keeps all the endowment for himself (if the guilt parameter, β , is

²The condition $\alpha + \beta \ge 0$ is satisfied in 137 out of 144 empirical estimates of (α, β) from more than 40 studies between 1999 and 2023 reviewed by Nunnari and Pozzi (2025). Individuals with inequality-seeking preferences ($\alpha < 0$ and $\beta < 0$) are rare: only 3 out of 144 empirical estimates belong to this category. See Figure 4 in Nunnari and Pozzi (2025) for details. Saito (2013)'s Expected Inequality Aversion model assumes the Fehr-Schmidt model of distributional preferences with $\alpha, \beta \ge 0$.

small) or selects the equal division (if the guilt parameter is large). In Case (ii), the dictator only has a preference for equality of opportunity, as $\delta = 1$. When β is large, any ρ that promises the dictator an expected return of M/2 is optimal. When β is small, however, the disutility associated with the guilt of having a higher private payoff is not sufficiently large to prevent the dictator from keeping all the money for himself, so he acts deterministically and selfishly.

The next proposition characterizes the strategy of the dictator when the fair allocation is not feasible.

Proposition 2. Assume $a_e = (\frac{M}{2}, \frac{M}{2}) \notin \mathcal{A}$. A dictator who is maximizing the utility functions specified in (1) and (2) will choose optimal ρ^* as follows:

- i. If $\delta \neq 1$,
 - (a) If $\beta < 1/2$ then $\rho^* = (0, ..., \rho_n^* = 1)$.
 - (b) If $\beta > 1/2$ then $\rho^* = (0, ..., \rho^*_{[e]^-} = \lambda, \rho^*_{[e]^+} = 1 \lambda, ..., 0)$ for some value $\lambda \in [0, 1]$. Here, $[e]^- = (x_{e^-}, y_{e^-})$ and $[e]^+ = (x_{e^+}, y_{e^+})$ denote the closest feasible allocations located to the left and right of a_e , respectively.
- ii. If $\delta = 1$,
 - (a) If $\beta < 1/2$ then $\rho^* = (0, ..., \rho_n^* = 1)$.
 - (b) If $\beta > 1/2$ then ρ^* is optimal if and only if $\sum_k \rho_k^* x_k = M/2$.

Overall, when the equal division is not feasible, the optimal strategy of the dictator remains largely unchanged. Specifically, Proposition 2 coincides with Proposition 1 in all cases except for case i(b). In case i(b), when the dictator has ex-post concerns and large β , Proposition 1 states that the dictator chooses the fair allocation a_e with certainty. When a_e is not feasible, Proposition 2 says that the dictator seeks to approximate the fair outcome as closely as possible, either by deterministically selecting the allocation closest to a_e or by randomizing between the two allocations nearest to a_e .³

Note that the optimal strategy of the dictator in Propositions 1-2 excludes several reasonable ways of giving, some of which will be observed in our experiments. For example, in the

³The proof of Proposition 2 provides a complete characterization of λ in case i(b) based on the values of $M, \alpha, \beta, \delta, x_{e^-}$, and x_{e^+} .

case of M = 10 with menu $\mathcal{A} = \{(0, 10), (1, 9), \dots, (9, 1), (10, 0)\}$, according to the current model, there should never be a giving strategy of (0, 10) with 1% chance and (10, 0) with 99% chance where the dictator almost always keeps the whole endowment and only in an unlikely event passes more money to the recipient.⁴ Our Experiment 1 will investigate if the dictators follow the optimal strategies characterized by these propositions or make deviations such as the aforementioned example.

Although the original version of Fehr-Schmidt preferences in (2) is based on the difference between the payoffs of dictator and recipient, it is possible for a dictator to experience (dis)utility only when he gets more (or less) than a percentage of the recipient's payoff. For example, the dictator may feel entitled to receive at least twice as much as the recipient and thus experience a (dis)utility even under an equal split. In Appendix B, we offer a general version of (2) to capture this idea by assuming that the dictator's utility from choosing allocation (x, y) is given by

$$u^{FS-general}(x,y) = x - \alpha \max\{y - \gamma x, 0\} - \beta \max\{\gamma x - y, 0\},$$

where $\gamma > 0$ represents the relative fairness from the dictator's perspective, with $\gamma = 1$ corresponding to the original model in (2). The optimal behavior of the dictator for this general version is similar to the one characterized in Propositions 1 and 2 (see Proposition B.1), and such dictators are also expected to behave mostly deterministically rather than giving the recipients a wide range of opportunities; when they do randomize, they do so in a specific way.

The distributional preferences model of Charness and Rabin (2002). Charness and Rabin (2002) propose another non-differentiable model of distributional preferences where the dictator's utility from choosing allocation (x, y) is given by

$$u^{CR}(x,y) = (1-\eta)x + \eta(\theta \min\{x,y\} + (1-\theta)(x+y)), \quad \text{with } \eta, \theta \in [0,1].$$

This model is a convex combination of the dictator's consumption utility and other-regarding utility, weighted by η . The other-regarding part is another convex combination of the payoff

⁴This behavior is actually observed in Experiment 1.

of the least-earning person and the total payoff of the group. Different from the Fehr-Schmidt model, as long as the dictator has social concerns ($\eta > 0$) and cares about the total payoff ($\theta < 1$), the dictator positively values the recipient's payoff regardless of whether his payoff is higher or lower. Consequently, the Charness-Rabin model predicts altruistic behaviors in such situations.

In general, the Charness-Rabin and Fehr-Schmidt models of distributional preferences are distinct, as they capture different psychological aspects of caring about others. In our two-player framework with efficient allocations, however, the two models are closely connected. Specifically, Proposition 3 below shows that the two models generate the same set of optimal behaviors under appropriate parameter transformation.

Proposition 3. Consider two dictators A and B such that dictator A is modeled by Charness-Rabin utility function with $(\eta, \theta) \neq (1, 0)$ and dictator B is modeled by Fehr-Schmidt utility function with (α, β) such that

$$\alpha = \frac{\eta(\theta - 1)}{2}$$
 and $\beta = \frac{\eta(\theta + 1)}{2}$.

Then the optimal strategies of the two dictators are identical.

Proposition 3 holds because when $\alpha = \frac{\eta(\theta-1)}{2}$ and $\beta = \frac{\eta(\theta+1)}{2}$, the utilities under Charness-Rabin and Fehr-Schmidt specifications are positive affine transformations of each other: $u^{CR}(x,y) = u^{FS}(x,y) - \alpha M$ for all allocations (x,y) in our setup (see the proof of Proposition 3 for details). Note that in Proposition 3 we assume that $(\eta,\theta) \neq (1,0)$. When $(\eta,\theta) = (1,0)$, the Charness-Rabin model assigns a constant utility, $u^{CR}(x,y) = M$, to any allocation in our setting. Hence, we rule out this uninteresting case that lacks predictive power.

Proposition 3, together with Propositions 1-2, implies that under the Charness-Rabin model of distributional preferences, the dictator either plays deterministically or randomizes in a specific way excluding a wide range of probabilistic behaviors.

2.1.2 Non-linear combination of preferences

In this subsection, we investigate a class of $\mathcal{V}(\Phi_{\text{ex-a}}, \Phi_{\text{ex-p}})$ and u(x, y) without committing to a specific functional form. We begin by introducing some assumptions on $\mathcal{V}(\Phi_{\text{ex-a}}, \Phi_{\text{ex-p}})$ and u(x, y).

Assumption 1. \mathcal{V} is differentiable and strictly increasing in the second argument.

Assumption 2. u is twice differentiable.

Assumption 3. u(x, M - x) is strictly concave in $x \in (0, M)$.

The differentiability of \mathcal{V} in Assumption 1 is for mathematical convenience. This assumption, together with Assumption 2 and the compactness of the domain, $\Delta(\mathcal{A})$, guarantees the existence of the dictator's optimal choice. Note that Assumption 1 also assumes the dictator enjoys a higher utility under an ex-post better outcome. Some previous studies, including Saito (2013) and Miao and Zhong (2018), impose a stronger assumption that \mathcal{V} is monotonic in both arguments. Assumption 1 is less restrictive and allows for non-monotonic preferences for opportunity. Assumptions 2-3 on the utility function are standard and satisfied in many well-known models of distributional preferences. For instance, the four leading models discussed below satisfy Assumptions 2-3.

1. Equality-efficiency trade-off preferences (Bolton and Ockenfels, 2000): The dictator cares about both his absolute payoff and his relative payoff compared to the recipient's. The dictator's utility from choosing allocation (x, y) is given by

$$u\left(x, \frac{x}{x+y}\right)$$
, with $0 \ge u_{11}, u_{22}, u_{12}$ and at least one strict inequality,

where u_{ij} is the second-order partial derivative of u with respect to the i-th and the j-th arguments.

2. **Altruistic preference** (Andreoni and Miller, 2002; Fisman et al., 2007): The dictator has altruistic preferences with Constant Elasticity of Substitution (CES):

$$u(x,y) = (\mu x^{\sigma} + (1-\mu)y^{\sigma})^{1/\sigma}$$
, where $\mu \in (0,1)$ and $-\infty < \sigma < 1$, $\sigma \neq 0$.

3. **Emotional state model** (Cox et al., 2007): The dictator's utility from selecting allocation (x, y) is given by

$$u(x,y) = \begin{cases} (x^{\kappa} + \tau y^{\kappa})/\kappa & \text{if } \kappa < 1, \kappa \neq 0, \\ xy^{\tau} & \text{otherwise,} \end{cases}$$

where parameter $\tau \in (0,1]$ measures the dictator's emotional state.

4. Strict egalitarian model (Cappelen et al., 2007): The dictator has an egalitarian view about fairness: He values his private payoff and experiences a cost when acting unfairly. The cost function is assumed to be quadratic, and the dictator's utility from selecting allocation (x, y) is given by

$$u(x,y) = \lambda x - \zeta \frac{(x-F)^2}{2(x+y)}, \text{ where } \lambda, \zeta > 0,$$

with F = (x + y)/2 being the fair amount.

Provided that Assumptions 1-3 hold, Proposition 4 below establishes that the dictator's optimal behavior is either deterministic or, if stochastic, it involves randomization between at most two allocations. Moreover, if the optimal choice is stochastic, then the dictator must randomize between two consecutive allocations.

Proposition 4. For \mathcal{V} and u satisfying Assumptions 1-3, if ρ^* is the optimal strategy of the dictator then either ρ^* is deterministic or there exist two distinct and consecutive allocations (x_i, y_i) and (x_{i+1}, y_{i+1}) such that $\rho_i^* + \rho_{i+1}^* = 1$.

Proposition 4 provides a testable prediction on the dictator's optimal behavior without committing to a particular utility function. We will study its predictions in our Experiment 1. Note that the result predicts very little randomization, if any. Furthermore, it excludes certain behaviors; for example, in the case of M = 10 with menu $\mathcal{A} = \{(0, 10), (1, 9), \dots, (9, 1), (10, 0)\}$, it excludes randomizing between (8,2) and (10,0) as they are not consecutive ways of giving in the presence of allocation (9,1). The proof of Proposition 4 uses a standard result in constrained optimization that the gradient of the Lagrangian is zero at any interior solution. The concavity of the utility function (Assumption 3) effectively prevents the dictator from randomizing between two non-consecutive allocations.

We conclude this section by discussing the assumption of the concave utility function (Assumption 3). If u(x, M - x) is strictly convex rather than concave, Proposition 4 is inapplicable. The convexity of u(x, M - x) can arise when the dictator exhibits altruistic or spiteful social preferences.⁵ For example, under altruism, the dictator may adopt an $\frac{5}{4}$ Many studies have documented a nontrivial fraction of individuals with altruistic or spite-

exponential utility function, such as $u(x,y) = e^{ax} + e^{by}$, where $a,b \in \mathbb{R}$. Under spitefulness, the dictator negatively values the recipient's payoff and may have a utility function like $u(x,y) = x^2 - y$. In these cases, the first result of Proposition 4 remains unchanged: the dictator either plays deterministically or randomizes between at most two allocations. The two allocations in the latter case, however, are not necessarily consecutive. This result is stated in Remark 1 below.

Remark 1. Suppose \mathcal{V} and u satisfy Assumptions 1-2. Suppose u(x, M-x) is strictly convex in $x \in (0, M)$. For any optimal ρ^* , either ρ^* is deterministic or there exist distinct i and j (not necessarily consecutive) such that $\rho_i^* + \rho_j^* = 1$.

2.2 Quadratic Social Welfare

In this subsection, we draw on an alternative approach which deviates from the ex-ante/expost preferences framework of Subsection 2.1. Assume that a dictator wants to maximize a combination of his own utility and the receiver's utility. Following the model of Epstein and Segal (1992), the dictator chooses $\rho \in \Delta(\mathcal{A})$ to maximize a quadratic objective function $\mathbf{V} \colon \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ given by

$$\mathbf{V}(U_D(\rho), U_R(\rho)) = c_1 U_D^2(\rho) + c_2 U_R^2(\rho) + c_3 U_D(\rho) U_R(\rho) + c_4 U_D(\rho) + c_5 U_R(\rho).$$
 (3)

In equation (3), $U_D(\rho)$ and $U_R(\rho)$ are the private utilities of the dictator and the recipient, respectively. The coefficients c_i (for $i=1,\ldots,5$) are constants that measure the degree to which the dictator cares about his own utility and the recipient's utility. Following Epstein and Segal (1992), we assume that at least one of c_1, c_2, c_3 is nonzero, so that \mathbf{V} is a second-degree polynomial over its domain. This quadratic functional form allows the dictator to exhibit concerns for ex-ante fairness. Moreover, \mathbf{V} is assumed to be strictly increasing in both arguments (Epstein and Segal, 1992).

In Epstein and Segal (1992), the private utilities of the dictator and receiver admit ex-

ful/competitive social preferences. See Table 1 in Fehr and Charness (2025) for details.

 $^{^6}$ Epstein and Segal (1992) also assume that **V** is strictly quasi-concave, but this additional assumption is not needed for our analysis.

pected utility representations and are given by:

$$U_D(\rho) = \sum_i \rho_i u_D(x_i)$$
 and $U_R(\rho) = \sum_i \rho_i u_R(y_i),$ (4)

where $u_D \colon \mathbb{R}_+ \to \mathbb{R}$ and $u_R \colon \mathbb{R}_+ \to \mathbb{R}$ are Neumann-Morgenstern utility functions of the dictator and the receiver, respectively. We impose the following assumptions on u_D and u_R .

Assumption 4. $u_D(x)$ and $u_R(x)$ are strictly increasing and strictly concave in $x \in [0, M]$, and twice differentiable on [0, M].

Assumption 4 states that both the dictator and receiver prefer higher payoffs and exhibit diminishing marginal utility. Under these standard assumptions, Proposition 5 predicts that the dictator either plays deterministically or randomizes between two consecutive allocations at the optimum.⁷

Proposition 5. Suppose the dictator maximizes **V** in (3), where the dictator's and receiver's private utilities are given in (4). Suppose Assumption 4 is satisfied. If ρ^* is an optimal strategy then either ρ^* is deterministic or there exist two distinct and consecutive allocations (x_i, y_i) and (x_{i+1}, y_{i+1}) such that $\rho_i^* + \rho_{i+1}^* = 1$.

Note that the predictions on the dictator's optimal behavior in Proposition 5 are in line with those in Proposition 4. Consequently, the non-parametric ex-ante/ex-post preference and quadratic preference approaches yield the same class of optimal strategies despite being grounded in distinct conceptual foundations and motivations. This observation highlights the robustness of the limited-randomization prediction for the dictator's optimal behavior, which we will study in Experiments 1 and 2.

To conclude this section, Figure 1 summarizes the theoretical predictions when the fair allocation is available. Note that the deterministic behavior of selecting either selfish or fair allocations is a common prediction of all the models discussed.

⁷Proposition 5 continues to hold when the dictator's and receiver's utility functions exhibit other-regarding preferences; in this case, some mild conditions on u_D and u_R analogous to Assumption 4 are required.

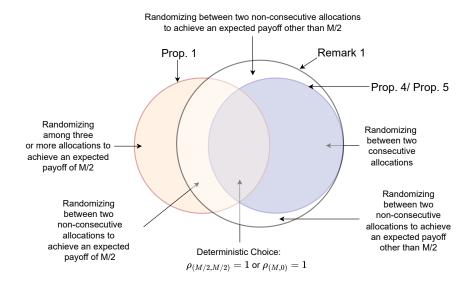


Figure 1: Theoretical Predictions in a Probabilistic Dictator Game

3 Experiment 1

We conducted an in-person laboratory experiment to empirically investigate the predictions of leading theories in social preferences for the Probabilistic Dictator Game, as stated in Propositions 1-5 and Remark 1. The design closely follows the theoretical setup. In Experiment 1, the dictator receives an endowment of M = \$10 and decides how to allocate this amount between himself and an anonymous receiver. Unlike the standard dictator game, where the dictator chooses an allocation of (X, 10-X) with $X \in [0, 10]$ denoting the amount he keeps for himself, our probabilistic dictator game presents the dictator with eleven feasible allocations:

$$\mathcal{A} = \{(0,10), (1,9), (2,8), \dots, (8,2), (9,1), (10,0)\}.$$

The dictator selects a distribution (a lottery) over these allocations. For each allocation, the dictator chooses an associated weight, which must be an integer from the set $\{0, 1, ..., 99, 100\}$ (see Figure 2 for a screenshot). The weight indicates the chance that the corresponding allocation will be chosen for payment and the weights over all allocations must sum to 100. Experiment 1 allows us to study the extent to which individuals exhibit a preference for giving an opportunity to others and whether their behaviors align with the predictions from leading theories of social preferences under risk studied in Section 2.

| | | | Remaining time [seconds]: 216 | |
|--|---|---|---------------------------------|--|
| | | | | |
| | YOU ARE A DECIDE | ER. MAKE YOUR CHOICE. | | |
| Please enter a number for | each row to indicate the weight of corresponding allo | cation. Make sure the weights are from the set {0,1,2,3,. | .,97,98,99,100} and sum to 100. | |
| | | | | |
| | Allocation | Weight (%) | | |
| | Keep \$0 and give \$10 to the recipient | | | |
| | Keep \$1 and give \$9 to the recipient | | | |
| | Keep \$2 and give \$8 to the recipient | | | |
| | Keep \$3 and give \$7 to the recipient | | | |
| | Keep \$4 and give \$6 to the recipient | | | |
| | Keep \$5 and give \$5 to the recipient | | | |
| | Keep \$6 and give \$4 to the recipient | | | |
| | Keep \$7 and give \$3 to the recipient | | | |
| | Keep \$8 and give \$2 to the recipient | | | |
| | Keep \$9 and give \$1 to the recipient | | | |
| | Keep \$10 and give \$0 to the recipient | | | |
| Please also copy your choice onto the paper given to you before clicking 'Continue.' The paper will be shown to the receiver within your group. We will verify the accuracy of your choices, so make sure you entered the weights correctly. | | | | |
| Click Continue to go to the result page: | | | | |
| | | | | |

Figure 2: Experimental Interface in Experiment 1

We used z-Tree to program the experiment (Fischbacher, 2007). Participants were recruited through ORSEE (Greiner, 2015) and randomly matched into pairs. In each pair, one participant assumed the role of dictator while the other acted as the receiver. Dictators and receivers were simultaneously placed in two separate rooms, with no interaction between them before or during the session. Neither dictators nor receivers knew the identity of their matched counterparts, but participants were assured that their counterparts were real people. An experimenter remained in each of the two rooms throughout the experiment. Dictators were first provided with instructions, which were integrated into the experimental interface (see the Appendix for screenshots and complete instructions). After reading the instructions, dictators completed a two-question quiz designed to check their understanding of the instructions. The dictators had to answer both questions correctly before proceeding with the experiment. Before deciding how to split the \$10 endowment, dictators were instructed to copy their choices onto a piece of paper once they made a decision. They were informed that this paper would later be shown to the receiver matched with them. Finally, the dictators completed an incentivized multiple-price-list question to measure their risk attitudes and an

unincentivized questionnaire about demographic information and charity-related activities.⁸

160 undergraduate students at the University of Maryland, College Park, participated in six experimental sessions in October and November 2024. A typical experimental session lasted approximately 40 minutes. Each subject received a \$10 show-up fee. On average, a dictator earned \$17.84, and a receiver earned \$13.78 (see Appendix A.2 for the distribution of expected payoffs). Payments were made privately in cash at the end of the experiment. Demographics of 80 participants playing the dictator role are presented in Appendix A.1.

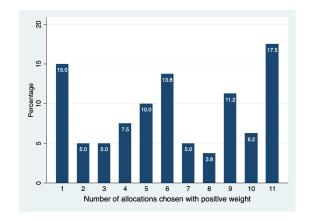
4 Results of Experiment 1

Recall that the main prediction of the theoretical analysis was about the support of dictators' choices: They were expected to act either deterministically or randomize on a limited set of allocations (mostly between two allocations) for all of the utility functions we studied earlier. To address these predictions, we first investigate the cardinality of the support of choices, defined as the number of allocations to which the dictators assigned positive weights. We will take a closer look at the nature of randomization that the dictators picked and classify them based on the trends in their preferences for randomization in Section 7.

Support of Choice. Figure 3 shows that the majority of dictators randomized among multiple allocations; only 15% of the participants had deterministic behaviors. Among those randomizing dictators, most assigned positive probabilities to three or more allocations (80% of the participants). Particularly, 17.5% of subjects selected positive weights in all eleven allocations.

Table 1 presents the five most common sets of allocations chosen with positive probabilities. These five sets of allocations cover exactly half of the participants. Overall, Table 1 is in line with Figure 3 and it shows that subjects tend to randomize among several allocations. Specifically, out of the five most common sets of allocations, only one is a singleton containing the equal division of (5,5). For the remaining four collections, the dictators tend to assign positive weights to allocations that ensure at least \$5 to themselves or to the allocations that are not fully selfish or altruistic.

⁸See Appendices A.1 and A.3 for results related to demographics.



| Support of Choice | % |
|---|--------|
| All allocations (x_i, y_i) | 17.50% |
| Fair allocation only | 10.00% |
| All (x_i, y_i) such that $x_i \geq 5$ | 10.00% |
| All (x_i, y_i) except $(0,10)$ | 6.25% |
| All (x_i, y_i) except (0.10) and (10.0) | 6.25% |

Figure 3: Support of choice in Experiment 1

Table 1: Most common sets of allocations

Consistency with Ex-Ante/Ex-Post Preferences and Quadratic Social Preferences. Figure 4 presents the percentages of dictators whose behavior can be explained by a utility maximization studied in Section 2. Overall, this figure provides only limited support for these models. First, 13.75% (11 dictators) made deterministic choices by being either fully selfish or fully fair. These dictators are consistent with all the models (Propositions 1, 4, 5 and Remark 1). Those who randomized between exactly two allocations were rare. No dictator randomized between two consecutive allocations. Only 1.25% (1 dictator) randomized between two non-consecutive allocations to achieve a fair outcome in expectation, i.e. an expected payoff of \$5 for both the dictator and the receiver, a behavior predicted by Proposition 1 and Remark 1. 3.75% (3 dictators) randomized between two non-consecutive allocations yielding an unfair expected payoff, also consistent with Remark 1. Finally, 15% (12 dictators) are consistent only with Proposition 1, as they randomized among three or more allocations to obtain an expected payoff of \$5. These 12 dictators' behavior can be rationalized with $\delta = 1$ according to Proposition 1; hence, they must have a preference for equality of opportunity only.

To sum up, 30% of dictators are consistent with Saito (2013) and an additional 3.75% can be explained by the other models discussed in Section 2. Consequently, 66.25% of the dictators in Experiment 1 remain unexplained by these models.

 $^{^{9}}$ One additional dictator chose (0, 10) deterministically, which is not predicted by any of the previous utility specifications.

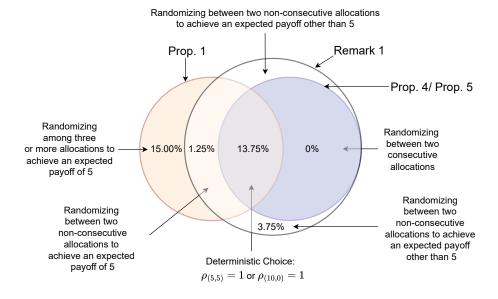


Figure 4: Percentages of participants in Experiment 1 consistent with ex-ante/ex-post preferences and quadratic preferences

The degree of randomization. The previous analysis indicates that the dictators randomized more than predicted by the social preference models of Section 2. To measure the randomness of dictators' choices in Experiment 1, we analyze the Shannon entropy associated with each choice distribution.¹⁰ Shannon entropy of a distribution ρ is given by

$$E_{\text{Shannon}}(\rho) = -\sum_{i=1}^{11} \rho_{(x_i, y_i)} \log(\rho_{(x_i, y_i)}).$$

Shannon entropy is nonnegative and equal to zero only if ρ is deterministic. A higher entropy value represents a greater randomness in the choice distribution, indicating that the choice probabilities are more dispersed or more evenly distributed among allocations. Conversely, a lower entropy value indicates a greater similarity to deterministic choices. Figure 5 presents the histogram and density of $E_{\text{Shannon}}(\rho)$.¹¹ Overall, Figure 5 indicates that dictators tend to act stochastically and heterogeneously enjoy giving a wide range of opportunities to the recipients, as evidenced by the dispersed distributions of entropy values.

 $^{^{10}}$ In Appendix A.5, we repeat the analysis using Rényi entropy and the results are robust to this alternative measure of randomness.

¹¹We follow conventional practice when computing Shannon entropy by assuming that $\rho_{(x_i,y_i)}\log(\rho_{(x_i,y_i)})=0$ when $\rho_{(x_i,y_i)}=0$.

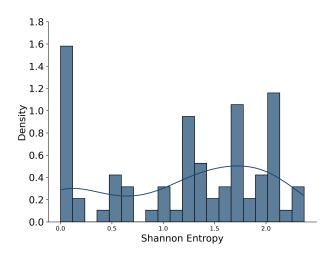


Figure 5: Shannon entropy of behavior in Experiment 1

Notes: The width of the bins for the histogram is 0.1. The line indicates the estimated Gaussian kernel, which uses Scott's rule for bandwidth selection. Given our design in Experiment 1, the maximum Shannon entropy is approximately equal to 2.4 (using natural logarithm).

5 Experiment 2

Motivated by the tendency to choose stochastically that we observed in Experiment 1, we next examine the behavior in Probabilistic Dictator Game from the perspective of stochastic choice models and their axiomatic foundations (Strzalecki, 2025). Although this literature is primarily developed for decision-making by a single, self-interested agent, the stochastic choice models are applicable to other-regarding preferences. We conduct Experiment 2 to investigate key models from this literature and offer new insights into the properties of randomization behaviors in the social preference domain.

Experiment 2 follows a design similar to that of Experiment 1. Each dictator is given \$10 and a list of feasible ways to split the money between himself and an anonymous receiver. The dictator's task involves assigning a weight to each feasible allocation, using the same procedure described in Experiment 1. Compared to Experiment 1, we reduce the number of feasible allocations in a game but increase the number of games to be able to study the axioms involving behaviors on different menus of allocations. As Experiment 1 demonstrates that the equal division of (5,5), selfish allocation of (10,0), and fully altruistic giving of (0,10) play significant roles in subjects' behavior, we restrict the set of allocations in Experiment 2 to these three. Each dictator in Experiment 2 completed four decision problems: three

involving binary menus and one involving a tripleton menu. We use A_i to indicate the menu in the decision problem $i \in \{1, 2, 3, 4\}$:

$$\mathcal{A}_1 = \{(0,10), (5,5), (10,0)\}; \mathcal{A}_2 = \{(5,5), (10,0)\}; \mathcal{A}_3 = \{(0,10), (10,0)\}; \mathcal{A}_4 = \{(0,10), (5,5)\}.$$

To minimize strategic behavior across questions, the dictators are not informed in advance about the set and order of questions they might encounter during the experiment. We use eight orders of questions, with an equal number of dictators assigned to each order within an experimental session.¹² The dictators make their allocations for a menu of options on the computer screen and copy them on paper as well. After each decision problem, an experimenter collects the paper copy before allowing the experiment to proceed to the next decision problem so that the dictators cannot possibly alter the distributions they pick. Only the dictator's choice in the decision problem selected for payment is later shown to the receiver. The decision selected for payment is chosen randomly by the computer from the four decision problems with equal probability.

As in Experiment 1, we programmed Experiment 2 in z-Tree (Fischbacher, 2007) and recruited participants using ORSEE (Greiner, 2015). Subjects of the two experiments were distinct. 208 undergraduate students at the University of Maryland, College Park, participated in seven experimental sessions in October and November 2024. In each session, half of the participants were the dictators and the other half were the receivers. A typical session lasted approximately 45 minutes. On average, a dictator earned \$18.27, while a receiver earned \$13.51 (see Appendix A.2 for the dictators' payoff distributions for each menu). Demographics of 104 participants playing the dictator role are presented in Appendix A.1.

6 Results of Experiment 2

6.1 Behavior in Experiment 2

Similar to our analysis for Experiment 1, below we first report the trends in dictators' behaviors and then investigate the predictions of social preference and stochastic choice models.

Support of choice. Figure 6 presents the support of choices in Experiment 2. In the tripleton menu, A_1 (first panel in Figure 6), about 25% of participants exhibited deterministic

¹²The sequence of decision problems is randomized at the individual level; see Appendix D for details.

behaviors, mainly selecting allocation (10,0), followed by (5,5). 44.2% of participants chose all three allocations with positive weights. 30.8% (32 subjects) randomized on two allocations and 31 of them mixed between (5,5) and (10,0), while only one person mixed between (0,10) and (10,0). No participant assigned positive weights exclusively to (5,5) and (0,10) on \mathcal{A}_1 .

On binary menus, the fraction of randomizing dictators is the highest in the menu $\mathcal{A}_2 = \{(5,5),(10,0)\}$, followed by $\mathcal{A}_3 = \{(0,10),(10,0)\}$, and the lowest in $\mathcal{A}_4 = \{(0,10),(5,5)\}$, as the tallest gray bar is observed in panel 2, followed by panel 3, and then panel 4 of Figure 6. This is intuitive as \mathcal{A}_2 guarantees a positive payoff to the dictator independent of the realization of the randomization, but the other two menus expose the dictators to the possibility of keeping \$0 if they randomize and the dictators may not like that possibility.

Figure 6 also shows the choices of deterministic dictators in the first bar of each panel. On $\mathcal{A}_3 = \{(0,10),(10,0)\}$ and $\mathcal{A}_4 = \{(0,10),(5,5)\}$, participants overwhelmingly selected the allocation yielding a higher private payoff. For menu $\mathcal{A}_2 = \{(5,5),(10,0)\}$, the fractions of subjects selecting (5,5) and (10,0) deterministically only differ slightly (18.3% and 13.5%, respectively).

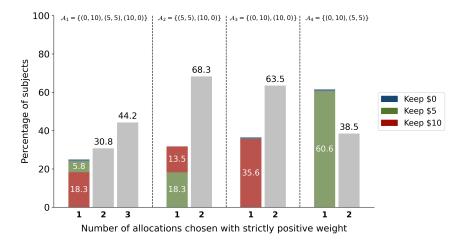


Figure 6: Support of choice in Experiment 2

While Figure 6 illustrates variation in randomization across menus, the shift in the type of randomization used by dictators as the menu changes suggests deliberate decision-making rather than automatic use of the experimental tool. As discussed above, dictators randomize more frequently in A_2 , where both available allocations guarantee them a positive payoff, than in A_4 , where randomization may result in a zero payoff. Moreover, in a post-experiment

survey, many participants explained that they chose to spread out the chances in order to reduce control over the outcome, either to give the receiver some "chance" or some "money" or both. Specifically, Figure 7 presents an analysis of the relative frequency of each word in the participants' open-ended responses regarding their reasoning for randomization. The high frequency of the words "chance" and "money" in this figure indicates subjects' intentional delegation of the giving decision to nature.



Figure 7: Word frequencies describing views on randomization in Experiments 1 and 2 *Notes:* The relative size of each word reflects its frequency in the participants' responses to two post-experiment surveys in which they were asked about their underlying reasons for randomization in Experiments 1 and 2. See Appendix A.4 for details of the analysis.

Distribution of Choice. Figure 8 presents choice probabilities in Experiment 2. The triangular simplex represents choice probabilities in the tripleton menu. The three vertices of the simplex are labeled \$0, \$5, and \$10, denoting the amounts dictators kept for themselves. Each vertex of the simplex corresponds to a deterministic behavior, assigning a probability of 1 to the allocation associated with that vertex. A point on the edge of the simplex indicates a choice mixture between two allocations associated with the vertices of that edge, while an interior point represents a probabilistic choice involving all three allocations. The size of each circle reflects the frequency of the corresponding choice in the data.

Three outer lines parallel to the edges of the triangle simplex represent choice probabilities in binary menus, \mathcal{A}_2 , \mathcal{A}_3 , and \mathcal{A}_4 . For instance, the line parallel to the edge connecting the vertices \$0 and \$5 represents the menu $\mathcal{A}_4 = \{(0,10),(5,5)\}$. The two endpoints of each parallel line correspond to deterministic behaviors.

Overall, in tripleton menu A_1 , Figure 8 shows that almost all observations (96.2%) lie within the sub-triangle formed by two blue dashed lines and the edge connecting \$5 and \$10

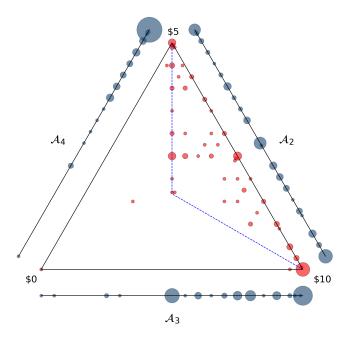


Figure 8: Choice probabilities in Experiment 2

vertices. The two blue dashed lines link vertices \$5 and \$10 with the simplex's centroid, i.e., the point at which the choice attaches equal probability to three allocations. This indicates that the dictators' randomization is skewed towards keeping positive amounts.

In all binary menus except $A_2 = \{(5,5), (10,0)\}$, the choices are also skewed toward the allocation giving dictators a higher private payoff. This trend is visible on the left and bottom outer lines (representing menus A_4 and A_3 , respectively), where the gray data points concentrate toward the vertex offering the dictator a higher payoff. On A_2 , the distribution of choice tends to be more dispersed. Notably, among 56 subjects whose behaviors lie on the right edge of the triangle, 42 exhibited identical choice in menus A_1 and A_2 .

6.2 Consistency with Models

In this section, we evaluate the extent to which the dictators' behaviors are rational and examine whether their choices are consistent with various models. We start with the social-preference models of ex-ante and ex-post preferences and the quadratic social welfare model that we discussed for Experiment 1. Then we continue with some relevant stochastic choice models.

As there are four decision problems with different menus in Experiment 2, throughout this section, we extend the notation for the dictator's choice to indicate the menu on which the choice is made. Let \mathcal{A} be a menu of allocations. Let $\rho \in \Delta(\mathcal{A})$ denote the choice of a dictator where $\rho_a(\mathcal{A})$ is the weight assigned to allocation a on menu \mathcal{A} .

6.2.1 Social Preferences

We analyze the choice of a dictator on the menus used in Experiment 2 under the parametric model of Saito (2013) incorporating ex-ante and ex-post preferences based on the Fehr-Schmidt fairness model in Section 2. Proposition 6 below characterizes the dictator's optimal behaviors in two questions with menus $A_2 = \{(5,5), (10,0)\}$ and $A_4 = \{(0,10), (5,5)\}$. Note that optimal behaviors in questions with menus A_1 and A_3 can be obtained directly from Propositions 1 and 2, respectively.

Proposition 6. Suppose a dictator in Experiment 2 maximizes the utility function described by (1) and (2). Then

(i)
$$\rho_{(5,5)}^*(\mathcal{A}_4) = 1;$$

(ii)
$$\rho_{(5,5)}^*(\mathcal{A}_2) = 1$$
 if $\beta > 1/2$ and $\rho_{(10,0)}^*(\mathcal{A}_2) = 1$ if $\beta < 1/2$.

Part (i) of Proposition 6 characterizes the optimal decision in menu \mathcal{A}_4 . It states that the dictator's choice is deterministic, and the dictator must select allocation (5,5) with certainty, regardless of the values of (δ, α, β) . To elucidate this result, note that in menu $\mathcal{A}_4 = \{(0,10),(5,5)\}$, any mixture of the two feasible allocations yields an expected allocation in which the dictator's share is (weakly) smaller than that of the recipient. Consequently, the dictator's utility does not depend on parameters δ and β . Furthermore, the condition $\alpha > -1/2$ implies that the marginal (dis)utility of envy when having a smaller private payoff is relatively small, making choosing (5,5) with certainty the optimal strategy.

Part (ii) of Proposition 6 states the optimal choice in menu A_2 . In this menu, the optimal behavior is also deterministic, but the chosen allocation can be either fair or selfish. The former happens when the marginal disutility from the guilt of having a higher private payoff is sufficiently large ($\beta > 1/2$), which effectively prevents the dictator from keeping all the money for himself.

Based on the testable predictions in Propositions 1, 2, and 6, 73.07% (76 dictators) exhibited behaviors inconsistent with the ex-ante and ex-post preferences framework. The remaining 26.93% (28 dictators) have behavior that can be explained by this model for some parameters. Hence, the results of Experiment 2 align with those of Experiment 1 (see Figure 4) and further reinforce the limited support for the ex-ante and ex-post preferences framework in our experiments.

Recall that a dictator who maximizes a quadratic social welfare function should either choose deterministically or randomize between at most two consecutive alternatives according to Proposition 5. Therefore, it is mechanically easier to be consistent with this model in Experiment 2 than it was in Experiment 1, since the former has at most three alternatives in a menu and the latter has eleven. Nevertheless, a majority acted inconsistently with the quadratic social welfare model in Experiment 2 by randomizing between all three alternatives on \mathcal{A}_1 . Only 41.35% (43 dictators) behaved consistently by mixing between two alternatives on \mathcal{A}_1 and using the same weights on those two alternatives in the corresponding binary menu. For example, one such dictator chose (10,0) and (5,5) each with a 50% chance on \mathcal{A}_1 , and employed the identical 50/50 randomization on $\mathcal{A}_2 = \{(5,5), (10,0)\}$.

6.2.2 Rationalizability

The limited support for the implications of social preference models documented in the previous subsection does not necessarily mean that the participants acted irrationally. In this subsection, we show instead that the majority of participants behaved rationally, with their choices satisfying several well-known axioms of rationality.

We start our analysis with deterministic subjects for whom rationality corresponds to preference maximization. In Experiment 2, 18 out of 104 subjects exhibited deterministic behaviors in all four decision problems. 17 of them behaved in a way consistent with maximizing a linear order of (10,0) being the best allocation followed by (5,5) and then by (0,10). The remaining one was extremely altruistic (choosing (0,10) with certainty when it is available). Hence, all the deterministic dictators' choices were in line with a maximization of a complete and transitive preference relation.

For the dictators who chose stochastically, the Regularity axiom can be seen as a measure of stochastic rationality. Regularity states that the probability of selecting an allocation cannot increase when a new allocation is added to the menu. The underlying intuition is that in a menu with more allocations, an allocation must compete with more alternatives to be chosen by the decision maker. Regularity imposes a consistency constraint on decision-making, ensuring that choices follow a predictable pattern in response to changes in the decision problems.

Regularity. For any
$$a, b \in \mathcal{A}$$
 and $a \neq b$, $\rho_a(\mathcal{A}) \leq \rho_a(\mathcal{A} \setminus \{b\})$.

In Experiment 2's design, since the cardinality of a menu is at most three, the Regularity axiom fully characterizes the Random Utility Model (Strzalecki, 2025), which is the leading rational framework for analyzing stochastic behavior. 57.69% (60 dictators) satisfied Regularity in Experiment 2.

Moderate Stochastic Transitivity is one of the natural probabilistic generalizations of the standard transitivity property. In deterministic choice theory, transitivity requires that if the individual prefers allocations a over b and b over c, they should prefer a over c. The stochastic version adapts this logic to a probabilistic environment. It ensures that the choices reflect a coherent preference ordering and follow a predictable pattern in response to changes in decision problems.

Moderate Stochastic Transitivity. For any allocations a, b, c and binary menus created by them

$$\min\{\rho_a(\{a,b\}), \rho_b(\{b,c\})\} \ge 1/2 \text{ implies } \begin{cases} \rho_a(\{a,c\}) > \min\{\rho_a(\{a,b\}), \rho_b(\{b,c\})\} \text{ or } \\ \rho_a(\{a,c\}) = \rho_a(\{a,b\}) = \rho_b(\{b,c\}). \end{cases}$$

Moderate Stochastic Transitivity is both necessary and sufficient for the moderate utility model studied in He and Natenzon (2024). In Experiment 2, 92.31% (96 participants) exhibited choices consistent with Moderate Stochastic Transitivity. Furthermore, 55.77% (58 dictators) satisfied both Regularity and Moderate Stochastic Transitivity. Appendix A.6 and Table 6 provide additional results for weaker and stronger versions of Regularity and stochastic transitivity.

6.2.3 Additive Perturbed Utility Model

In this subsection, we investigate whether the dictator's behavior aligns with the Additive Perturbed Utility (APU) model studied in Fudenberg et al. (2015). This model provides a simple framework for explaining stochasticity in individual choices and is relevant to our experiments, as the dictator's behavior is largely stochastic. The model also satisfies both the Regularity and Moderate Stochastic Transitivity axioms.

Given menu \mathcal{A} , Fudenberg et al. (2015) model an individual whose utility from choosing $\rho \in \Delta(\mathcal{A})$ is the expected consumption utility minus a cost associated with randomization. The individual solves the following maximization problem:

$$\max_{\rho \in \Delta(\mathcal{A})} \left[\sum_{a \in \mathcal{A}} u(a) \rho_a(\mathcal{A}) - \sum_{a \in \mathcal{A}} c(\rho_a(\mathcal{A})) \right]. \tag{APU}$$

In the objective function above, u(a) is the individual's utility from choosing allocation a. The cost function $c: [0,1] \to \mathbb{R} \cup \{\infty\}$ is assumed to be strictly convex and continuously differentiable in (0,1). As the cost function is strictly convex, stochastic behaviors benefit the dictator not due to preference for opportunity, as that was the case for ex-ante/expost preference models studied in Section 2. Instead, APU can be interpreted as a linear framework combining the dictator's ex-post motive, $\sum_{a\in\mathcal{A}} u(a)\rho_a(\mathcal{A})$, and the cost of opting for randomization, $\sum_{a\in\mathcal{A}} c(\rho_a(\mathcal{A}))$.

Since Regularity is a necessary condition for APU (Fudenberg et al., 2015), the 44 dictators who violated Regularity also violated the APU model. Among the 60 dictators who satisfied Regularity, 41 of them behaved in line with APU.

APU with allocation- or menu-dependent costs. The baseline APU model assumes that the cost term depends solely on choice probabilities and remains invariant to changes in menus or allocations. Fudenberg et al. (2015) offer two other versions of APU where one has the cost of randomization depending on the chosen allocation (a-APU) and another has the cost depending on the menu (m-APU). In our context, one might expect the act of randomization to be less costly in menu \mathcal{A}_2 than in menus \mathcal{A}_3 or \mathcal{A}_4 , as any randomization in \mathcal{A}_2 still guarantees a payoff of at least 5, whereas randomization in either \mathcal{A}_3 or \mathcal{A}_4 carries the risk of receiving a zero payoff. By allowing for such generalizations of the cost function,

APU may offer greater explanatory power in Experiment 2.

To investigate m-APU and a-APU, we check two properties called Item Acyclicity and Menu Acyclicity, which respectively characterize these two models (see Fudenberg et al. (2015) for details). a-APU can rationalize the choices of 43.27% (45 dictators), including 41 dictators whose behavior is also consistent with APU. This indicates that introducing allocation-dependent costs yields only modest improvements in the model's fit to the data. In contrast, incorporating menu-dependent costs significantly enhances explanatory power, as the m-APU model can explain the behavior of 78.85% (82 dictators). The high explanatory power of m-APU results from its ability to accommodate violations of Regularity, whereas a-APU cannot do so, as it necessarily satisfies Regularity (Fudenberg et al., 2015).

To conclude this section, Table 2 summarizes the percentage (number) of participants consistent with various models and measures of rationality discussed above. Appendix A.6 presents additional results on other rationality measures, including different versions of Regularity and other forms of stochastic transitivity.

| Models and/or Measures of Rationality | % (#) satisfying |
|---|------------------|
| Expected Inequality Aversion | 26.92% (28) |
| Quadratic Preference | 41.35% (43) |
| Preference maximization | 17.31% (18) |
| Regularity | 57.69% (60) |
| Moderate Stochastic Transitivity | 92.31% (96) |
| Additive Perturbed Utility | 39.42% (41) |
| Additive Perturbed Utility with allocation-dependent cost | 43.27% (45) |
| Additive Perturbed Utility with menu-dependent cost | 78.85% (82) |

Table 2: The percentage (number) of participants consistent with various models and measures of rationality in Experiment 2

¹³Chambers et al. (2025) offers a particular case of a-APU with a quadratic cost function, but only 2.8% (3 dictators) satisfy the main axiom of that model (Strict Regularity).

7 Further Analyses

7.1 Classification of Behaviors in Experiment 1

There are some trends in the way the dictators randomized among available allocations in Experiment 1. Based on the allocation that received the highest weight, we classify dictators into three groups: (im)pure selfish behavior, (im)pure fair behavior, and other behavior.

(Im)pure selfish behavior. This group includes 33.75% (27 dictators) who assigned the highest weight to the selfish allocation (10,0), thereby reflecting their tendency to behave selfishly. Among the participants within this group, approximately half (12 subjects) assigned a weight of 50% or more to allocation (10,0), including three subjects choosing (10,0) with certainty. The behavior of a typical participant in this group is illustrated in the first graph of Figure 9.

(Im)pure fair behavior. This group contains 48.75% (39 dictators) who assigned the highest weight to the equal division allocation (5,5), indicating a tendency to behave altruistically and fairly.¹⁴ Within this group, 8 dictators implemented the fair outcome (5,5) with certainty. Another 10 dictators, while assigning positive weights to allocations other than (5,5), adhered to the equality-of-opportunity principle by choosing weights symmetrically around (5,5), i.e., they assigned identical weights to allocations (x,y) and (y,x). Typical behavior of these dictators is illustrated in the second graph of Figure 9. In contrast, 15 dictators within this group tended to favor allocations that provide them with a higher payoff. For these dictators, the distribution of weights is asymmetric around (5,5), positively skewed, and the weight assigned to allocation (x,y) is generally higher than that of allocation (y,x) when x > y. Hence, even intending to be fair and altruistic, these subjects might be tempted to act selfishly, leading them to assign higher probabilities to allocations giving themselves a higher payoff. Typical behavior of these dictators is illustrated in the third graph of Figure 9. The remaining 6 dictators in this group tended to favor allocations that provide the receiver with a (weakly) higher payoff.

Other behavior. This group includes the remaining 17.50% (14 dictators) who assigned the highest weight to neither the selfish allocation nor the fair allocation. The majority (12

¹⁴Two subjects assigned the highest weight to both allocations (10,0) and (5,5). Since they are already included in the (im)pure selfish behavior group, they are excluded from the (im)pure fair behavior group to avoid double counting.

dictators) selected the highest weight for an allocation that provides them with a higher payoff, i.e., allocations (9,1), (8,2), (7,3), or (6,4). The behavior of a typical participant in this group is illustrated in the last graph of Figure 9.

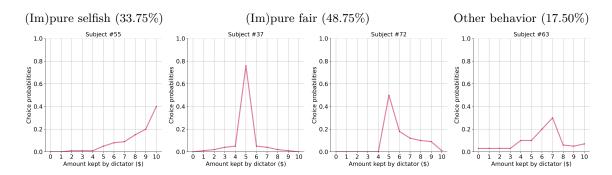


Figure 9: Classification of the behavior in Experiment 1

In Appendix A.3 we analyze the impact of demographic variables including gender, race, and age, involvement in prosocial activities, and risk attitude on the subjects' behavior in this probabilistic dictator game. We find that male subjects are more likely to assign the highest weight to the selfish allocation and female subjects are more likely to assign the highest weight to the fair allocation (see Table 4). This result is consistent with a common finding in the standard dictator game literature where women are often found to be more generous than men (Engel, 2011). The female subjects also randomize on a larger set of allocations than the males (see Table 5). Notably, risk attitude of the subjects does not play a significant role in determining their behavior.

7.2 Comparison to the deterministic dictator game

We compare the behaviors of dictators in our Probabilistic Dictator Game in Experiment 1 with the previous findings of the standard (deterministic) dictator games in the literature. Specifically, we measure how far the behaviors in our probabilistic design deviate from the aggregate behaviors reported in Engel (2011) (Figure 2 of this meta-study).¹⁵ For each subject, we calculate the distance between their choice and the aggregate distribution using Kullback-

 $^{^{15}}$ This aggregate distribution includes data from 20,813 individuals. Of these observations, 99.1% exhibit no uncertainty regarding the dictator's behavior; hence, the aggregate distribution can be interpreted as representing behaviors under deterministic dictator games.

Leibler divergence, which is based on Shannon entropy. Let ρ be a choice distribution of a dictator in our experiment and $\tilde{\rho}$ be the aggregate distribution of the dictators' behavior in Engel (2011). The Kullback-Leibler divergence between ρ and $\tilde{\rho}$ is given by:

$$d_{KL}(\rho, \widetilde{\rho}) = \sum_{i=1}^{11} \rho_{(x_i, y_i)} \log \left(\frac{\rho_{(x_i, y_i)}}{\widetilde{\rho}_{(x_i, y_i)}} \right).$$

Note that $d_{KL}(\rho, \tilde{\rho})$ is nonnegative and equal to zero only if ρ and $\tilde{\rho}$ are identical. Higher values of $d_{KL}(\rho, \tilde{\rho})$ indicate greater differences between the two distributions. Figure 10 reports the histogram and density of $d_{KL}(\rho, \tilde{\rho})$.¹⁷ Overall, the individuals' behaviors in the Probabilistic Dictator Game deviate from those in the deterministic version, as there is no mass concentration of $d_{KL}(\rho, \tilde{\rho})$ around zero.

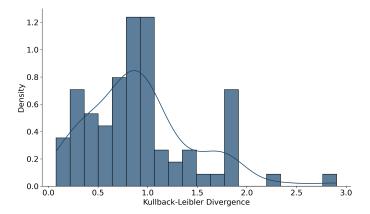


Figure 10: Behaviors in Experiment 1 compared to the deterministic dictator game

Notes: The width of the bins for the histogram is 0.1. The line indicates the estimated Gaussian kernel,
which uses Scott's rule for bandwidth selection.

Simulation Evidence. The previous analysis suggests that individual-level behavior in our Probabilistic Dictator Game departs from aggregate-level behavior observed in the standard dictator game. To study whether the aggregate realized outcomes from our game align with those of the deterministic dictator game, we conduct a Monte Carlo simulation. For each of the 80 dictators in Experiment 1, we first draw one realized outcome from their observed choice distribution and treat the draw as a dictator's decision in the standard dictator game.

¹⁶In Appendix A.5 we repeat the analysis using Rényi divergence and the results are robust to this alternative measure.

¹⁷ Following standard convention, we assume that if $\rho_{(x_i,y_i)} = 0$, then $\rho_{(x_i,y_i)} \log(\rho_{(x_i,y_i)}/\widetilde{\rho}_{(x_i,y_i)}) = 0$ when computing the Kullback-Leibler divergence.

Aggregating these draws across all 80 dictators yields one simulated aggregate behavior in our game. We then compare this distribution to the aggregate behavior reported in the meta-analysis by Engel (2011) (Figure 2 in this paper) using a one-sample, non-parametric Kolmogorov-Smirnov test. Adopting a conservative approach, we set the significance level for the Kolmogorov-Smirnov test at 0.005. This means we reject the null hypothesis that the simulated aggregate distribution of realized outcomes from our game is drawn from the aggregate behavior in the deterministic dictator game if the p-value from the Kolmogorov-Smirnov test is less than 0.005. We repeat the simulation and comparison procedure 10,000,000 times. Across these simulations, the Kolmogorov-Smirnov test rejected the null hypothesis in 97.9% of the cases. Using alternative tests produces a similar finding. For example, the Chi-squared goodness-of-fit test rejected the null hypothesis in 91% of the simulations. This finding is in line with the preceding analysis using Kullback-Leibler divergence and provides additional strong evidence that behavior in our Probabilistic Dictator Game differs from its deterministic counterpart.

8 Conclusion

Combining ex-ante and ex-post motives has been the primary approach in investigating other-regarding preferences under risk. Using a novel design called the Probabilistic Dictator Game, we present experimental evidence indicating that none of the leading models incorporating the ex-ante and ex-post motives can fully explain dictators' decisions. By designing an experiment with a set of nested decision problems, we also identify trends in other-regarding behaviors. A majority of the observed randomization in social preferences is aligned with models from the stochastic choice literature, suggesting a promising path for future research: extending models of stochastic choice to investigate other-regarding preferences.

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APPENDIX Appendix A Additional Results A.1 Demographics of Participants

| | Experiment 1 (N=80) | | Experiment 2 (N=104) | |
|--|---------------------|-------------|----------------------|-------------|
| | #Subjects | Percentages | #Subjects | Percentages |
| Gender | | | | |
| Female | 43 | 53.75% | 50 | 48.08% |
| Male | 36 | 45.00% | 50 | 48.08% |
| Non binary | 0 | 0.00% | 3 | 2.88% |
| Preferred not to say | 1 | 1.25% | 1 | 0.96% |
| Major | | | | |
| Arts and communication | 4 | 5.00% | 1 | 0.96% |
| Business and economics | 14 | 17.50% | 23 | 22.12% |
| Health and medical | 3 | 3.75% | 6 | 5.77% |
| Science, technology, engineering, and math | 39 | 48.75% | 47 | 45.19% |
| Social sciences and humanities | 11 | 13.75% | 21 | 20.19% |
| Undeclared | 2 | 2.50% | 2 | 1.92% |
| Other majors | 7 | 8.75% | 4 | 3.85% |
| Year in college | | | | |
| Freshman | 24 | 30.00% | 27 | 25.96% |
| Sophomore/Junior | 42 | 52.50% | 55 | 52.88% |
| Senior | 14 | 17.50% | 22 | 21.15% |
| Race | | | | |
| White | 38 | 47.50% | 36 | 34.62% |
| Asian | 17 | 21.25% | 25 | 24.04% |
| White and Asian | 4 | 5.00% | 3 | 2.88% |
| Preferred not to say | 6 | 7.50% | 6 | 5.77% |
| Other races | 15 | 18.75% | 34 | 32.69% |

Table 3: Demographics of dictators

Notes: This table presents the demographics of dictators in Experiments 1 and 2. The mean ages of participants playing a dictator role in Experiments 1 and 2 are 19.5 and 19.9, respectively. The race question is open-ended; the options listed in the table reflect the actual responses of participants.

A.2 Expected Payoff of the Dictator

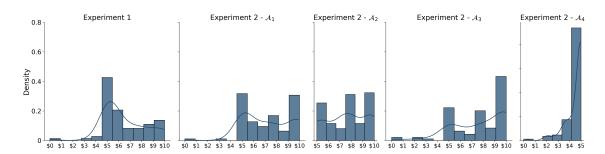


Figure 11: Expected Payoff of Dictators in Experiments 1 and 2

Notes: This figure presents the histograms and densities of the expected payoff of the dictators in Experiments 1 and 2. The five lines are estimated Gaussian kernels, which use Scott's rule for bandwidth selection. For each dictator, we compute the expected payoff as follows. Let the menu be $\mathcal{A} = \{(x_1, y_1), \ldots, (x_n, y_n)\}$ and let the dictator's choice be $\rho = (\rho_1, \ldots, \rho_n)$. Then the expected payoff of the dictator is $\sum_{i=1}^n \rho_i x_i$. In Experiment 2, the menus are $\mathcal{A}_1 = \{(0, 10), (5, 5), (10, 0)\}$, $\mathcal{A}_2 = \{(5, 5), (10, 0)\}$, $\mathcal{A}_3 = \{(0, 10), (10, 0)\}$, and $\mathcal{A}_4 = \{(0, 10), (5, 5)\}$, respectively.

A.3 Demographics and Dictator's Behavior in Experiment 1

In this section, we present the regression results relating demographics to the dictator's behavior in Experiment 1, focusing on the degrees of selfishness, fairness, and randomization.

Demographics and Selfishness/Fairness. We employ two metrics to measure selfishness and fairness in the dictator's behavior. Selfishness is indicated when the selfish allocation of (10,0) is assigned the highest weight, and fairness is indicated when the fair allocation of (5,5) is assigned the highest weight. Table 4 reports the regression results. Overall, Table 4 shows that men are more selfish than women (Panel A). Correspondingly, women are fairer than men (Panel B). There is no statistically significant relationship between risk attitudes and either selfish or fair behaviors. Note that these results remain largely robust when the weights assigned to the (10,0) or (5,5) allocations are used as dependent variables.

| | A. Is weigh | t of (10,0) the highest? | B. Is weight | of $(5,5)$ the highest? |
|-----------------|-------------|--------------------------|--------------|-------------------------|
| | (1) | (2) | (3) | (4) |
| Age | -0.198 | -0.279 | 0.445** | 0.571** |
| | (0.20) | (0.23) | (0.22) | (0.24) |
| Gender | 1.299** | 1.165* | -1.364*** | -0.972* |
| | (0.54) | (0.61) | (0.52) | (0.57) |
| Race | 0.561 | 0.925 | -1.301** | -1.460** |
| | (0.55) | (0.61) | (0.54) | (0.58) |
| Major | 0.029 | -0.079 | -0.522 | -0.266 |
| | (0.60) | (0.65) | (0.59) | (0.59) |
| Social Activity | 0.283 | -0.127 | -0.133 | 0.064 |
| | (0.31) | (0.33) | (0.28) | (0.33) |
| Risk Attitude | | 0.340 | | -0.216 |
| | | (0.22) | | (0.18) |
| Constant | 1.795 | 1.717 | -6.816 | -8.650* |
| | (4.05) | (4.53) | (4.30) | (4.71) |
| # observations | 79 | 68 | 79 | 68 |
| log-likelihood | -46.798 | -38.854 | -47.084 | -39.580 |

Table 4: Relationship between demographic factors and selfishness/fairness in Experiment 1.

Notes: $^*p < 0.1,^{**}p < 0.05,^{***}p < 0.01$. Standard errors are in parentheses. Regressions are estimated using robust standard errors. The dependent variables in Panels A and B are binary, reflecting whether the weight the dictator assigned to the selfish allocation and fair allocation is the highest, respectively. These binary dependent variables equal 1 if the associated weights are the highest and 0 otherwise. Independent variables include:

- Age: Age of the participant.
- Gender: Gender of the participant (1 = male; 0 = female). One subject did not reveal her gender, so we excluded her from the analysis.
- Race: Race as self-reported by the participant (1 = white; 0 = others).
- Major: Major of the participant's study (1 = STEM or business/economics major, including marketing, finance, accounting, etc.; 0 = others).
- Social Activity: The degree to which subjects are involved in social and other-regarding activities. This variable is the sum of three dummy variables: Community Involvement Whether the subject is involved in a student or social organization (1 = yes, 0 = no); Charitable Donation When the subject last donated to or volunteered for a nonprofit or charity organization, i.e., 501(c) organizations (1 = less than 6 months ago; 0 = more than 6 months ago or never); Street Giving When the subject last gave something to someone on the street (1 = less than 6 months ago; 0 = more than 6 months ago or never).
- Risk Attitude: In the multiple-price list (MPL) question used to elicit risk attitudes (see Figure 18 for details), a subject is presented with a binary choice between Option A, which is a fixed lottery paying \$3 or \$0 with equal probability, and Option B, which is a sure amount ranging from \$0.25 to \$2.75. We restrict our analysis to participants who exhibited normal and standard behavior: the participants chose Option A when the fixed dollar amount in Option B does not exceed a threshold and chose B afterward. The Risk Attitude variable is the point at which participants switched from choosing A to choosing B in the MPL question. It takes values in $\{2, \ldots, 11\}$, with higher values indicating lower risk aversion.

Demographics and Randomization. We measure the degree of randomization in the dictator's behavior by quantifying the size of the support of the dictator's weight distribution. This metric, defined as the number of allocations assigned a strictly positive weight, served as the dependent variable in our regression analysis. Table 5 reports the regression results. Overall, Table 5 indicates that randomization is more prevalent among women. Risk attitude is not a statistically significant predictor of randomization behaviors.

| | # of allocati | ons with positive weights |
|-----------------|---------------|---------------------------|
| | (1) | (2) |
| Age | -0.203 | 0.096 |
| | (0.30) | (0.32) |
| Gender | -2.528*** | -2.128** |
| | (0.81) | (0.89) |
| Race | -0.649 | -0.285 |
| | (0.78) | (0.87) |
| Major | 0.375 | 0.025 |
| | (0.88) | (0.93) |
| Social Activity | 0.431 | 0.229 |
| | (0.39) | (0.44) |
| Risk Attitude | | -0.099 |
| | | (0.30) |
| Constant | 10.661* | 5.290 |
| | (5.94) | (6.45) |
| # observations | 79 | 68 |
| log-likelihood | -203.301 | -172.489 |

Table 5: Relationship between demographic factors and the degree of randomization in the dictator's choice in Experiment 1.

Notes: ${}^*p < 0.1, {}^{**}p < 0.05, {}^{***}p < 0.01$. Standard errors are in parentheses. Regressions are estimated using robust standard errors. See Table 4 for definitions of independent variables.

Overall, Tables 4-5 indicate that among demographic factors, gender plays a significant role in the dictator's behavior, whereas risk attitude plays a minimal or insignificant role.

A.4 Views on Randomization

In the questionnaire at the end of both Experiments 1 and 2, we asked participants to explain why they chose to randomize if they did so. To understand their underlying reasons, we conduct a simple analysis of the frequency of words in the participants' open-ended responses. This analysis helps identify patterns in the participants' reasoning. Figure 7 in the main body of the paper presents the results.

We exclude a set of specific words when analyzing word frequency: {allocation, allocations, feel, positive, positive weight, positive weights, one, question, receiver, rather, selected, think, thought, two, yes, want, wanted, weight, and weights}. These words are not generally related to the reasons for choosing a weight distribution; rather, they are either linked to the phrasing of the questions (allocation, positive weight, one, two, yes, weight, receiver, etc.) or function as linking terms (chose, feel, rather, selected, think, thought, want, wanted). We also group together some words that share the same meaning or inflected form (give/gave/giving; keep/kept; recipient/receiver; equally/equal; etc.) and correct obvious misspellings in the participants' responses before conducting the analysis.

Below we also report four responses that are quite typical in Experiment 1. These responses are generally in line with findings reported in Figure 7.

- Subject ID #17: "I chose many allocations with positive weights. I chose 8 with postive weights. I wanted to spread out the chances of what would happen because I didn't want myself to have all the power in deciding who got how much money I didn't think that would be fair."
- Subject ID #20: "I chose 6 allocations. I chose this many to spread out the chances, and make it more luck-based, but so that either way both people would get something."
- Subject ID #27: "I gave them a chance to possibly get some of the money but the biggest allocation was to me."
- Subject ID #43: "I chose 5 allocations rather than just 1 because I didn't want to make the full decision myself of which one to choose. I let chance play some role in how much money each participant got because that felt more fair, while also letting me bias the weights in my favor because I did want the money."

A.5 Rényi entropy and Rényi divergence

We repeat the analyses of Experiment 1 in Section 7 using Rényi entropy and Rényi divergence. Given menu $\mathcal{A} = \{(x_1, y_1), \dots, (x_n, y_n)\}$, Rényi entropy of a choice distribution $\rho = (\rho_1, \dots, \rho_n) \in \Delta(\mathcal{A})$ is given by:

$$E_{\tau,\text{R\'enyi}}(\rho) = \frac{1}{1-\tau} \log \left(\sum_{i=1}^{n} \rho_{(x_i,y_i)}^{\tau} \right) \quad \text{for some } \tau > 0, \, \tau \neq 1.$$

In $E_{\tau,\text{Rényi}}(\rho)$, $\tau > 0$ is the order of the entropy and measures how much weight is given to different probabilities in the distribution. Specifically, τ determines the emphasis on rare and frequent events in the entropy calculation. In general, the Rényi entropy with $\tau > 1$ is more robust and less sensitive to outliers in ρ . The Shannon entropy analyzed in Section 7 is the limiting case of Rényi entropy when $\tau \to 1$. Figure 12a presents the histogram and density of $E_{2,\text{Rényi}}(\rho)$ for observed dictator's choice, ρ , in our Experiment 1. We set $\tau = 2$ in Rényi entropy to enhance robustness to outliers. Overall, Figure 12a is similar to Figure 5 when using Shannon entropy, suggesting that the result in Figure 5 is robust to alternative measures of randomness.

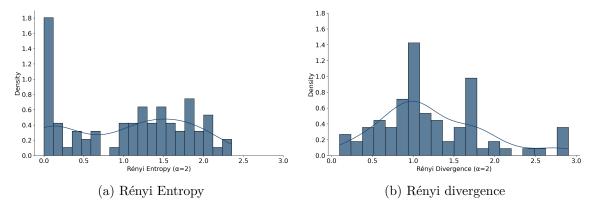


Figure 12: Rényi Entropy and Rényi Divergence in Experiment 1

Notes: The width of the bins for the histogram is 0.1. The lines indicate the estimated Gaussian kernels, which use Scott's rule for bandwidth selection. Given our design in Experiment 1, the maximum Rényi entropy ($\tau = 2$) is approximately equal to 2.4 (using natural logarithm).

In Figure 12b, we measure how far the behaviors in our Experiment 1 deviate from the aggregate behaviors reported in Engel (2011) using Rényi divergence, which is based on Rényi entropy. For each subject, we calculate the Rényi divergence between their choice and the aggregate distribution. Let ρ be a choice distribution of a dictator in our experiment and $\tilde{\rho}$ the aggregate distribution of the dictator's behavior in Engel (2011) (Figure 2 in the paper). The Rényi divergence between ρ and $\tilde{\rho}$ is given by:

$$d_{\tau,\text{R\'enyi}}(\rho,\widetilde{\rho}) = \frac{1}{\tau - 1} \log \left(\sum_{i=1}^{n} \rho_{(x_i,y_i)}^{\tau} \widetilde{\rho}_{(x_i,y_i)}^{1-\tau} \right).$$

Note that $d_{\tau,\text{Rényi}}(\rho,\tilde{\rho})$ is nonnegative and equals zero only if ρ and $\tilde{\rho}$ are identical. Figure 12b reports the histogram and density of $d_{2,\text{Rényi}}(\rho,\tilde{\rho})$. We again set $\tau=2$ to enhance

robustness to outliers. Overall, Figure 12b is similar to Figure 10 when using Kullback-Leibler divergence, suggesting that the result in Figure 10 is robust to alternative measures of divergence.

A.6 Additional Rationality Properties in Experiment 2

Other versions of Regularity. Given that only 57.69% of subjects in Experiment 2 satisfy the Regularity condition (see Section 6.2.2 in the main body of the paper), we provide additional results on whether the dictator's behavior in Experiment 2 satisfies two weaker versions of Regularity in the literature.

Filiz-Ozbay and Masatlioglu (2023) proposes a weak regularity condition that allows for regularity violations. In their property, the probability of choosing an alternative in a bigger menu cannot exceed the maximum probability of choosing it from any relevant binary menus; hence, the violations of regularity cannot happen with respect to every binary comparison. This property is relevant to our study as subjects faced three binary menus in Experiment 2.

Weak Regularity-1. For any
$$a, b \in \mathcal{A}$$
 and $a \neq b$, $\rho_a(\mathcal{A}) \leq \max_{b \in \mathcal{A} \setminus \{a\}} \rho_a(\{a, b\})$.

Echenique and Saito (2019) develop the following weak regularity condition, which applies to the alternatives with zero choice probabilities. Suppose an option is not chosen in a binary choice set. The property then requires that the option be selected with zero probability in all bigger menus containing the binary menu. This property is also relevant to our study as several participants never chose some allocations.

Weak Regularity-2. For any
$$a, b \in \mathcal{A}$$
 and $a \neq b$, $\rho_a(\{a, b\}) = 0$ implies $\rho_a(\mathcal{A}) = 0$.

In Experiment 2, the two weaker versions of Regularity are overwhelmingly supported. Specifically, Weak Regularity-1 and Weak Regularity-2 are supported by 96.15% (100 dictators) and 87.50% (91 dictators), respectively.

Stochastic Transitivity Axioms. We investigate whether the dictator's behavior in Experiment 2 satisfies other versions of stochastic transitivity in the literature. Besides Moderate Stochastic Transitivity, Weak Stochastic Transitivity and Strong Stochastic Transitivity are also natural probabilistic generalizations of the standard transitivity axiom. Weak Stochastic Transitivity is the weaker version of the other two. In single-individual domains, empirical studies documented robust violations of the strong version but few violations of the weak one (Rieskamp et al., 2006; Mellers and Biagini, 1994).

Weak Stochastic Transitivity. For any allocations a, b, c and binary menus created by them

$$\min\{\rho_a(\{a,b\}), \rho_b(\{b,c\})\} \ge 1/2 \Rightarrow \rho_a(\{a,c\}) \ge 1/2.$$

Strong Stochastic Transitivity. For any allocations a, b, c and binary menus created by them

$$\min\{\rho_a(\{a,b\}), \rho_b(\{b,c\})\} \ge 1/2 \Rightarrow \rho_a(\{a,c\}) \ge \max\{\rho_a(\{a,b\}), \rho_b(\{b,c\})\}.$$

In Experiment 2, 97.12% (101 dictators) satisfied Weak Stochastic Transitivity, and 74.04% (77 dictators) satisfied Strong Stochastic Transitivity. Table 6 summarizes the number of subjects satisfying different rationality properties.

| | | Stochastic Transitivity | | | Overall |
|------------|-------------------|-------------------------|----------|--------|---------|
| | | Weak | Moderate | Strong | Overan |
| ity | Weak Regularity-1 | 98 | 93 | 74 | 100 |
| Regularity | Weak Regularity-2 | 89 | 84 | 66 | 91 |
| ${ m Reg}$ | Regularity | 59 | 58 | 51 | 60 |
| | Overall | 101 | 96 | 77 | |

Table 6: The number of participants consistent with various properties of rationality in Experiment 2 $\,$

Notes: There are 104 subjects in Experiment 2.

Appendix B Proofs

Proof of Propositions 1-2: Suppose the dictator's utility from choosing allocation (x, y) is given by

$$u^{FS-general}(x,y) = x - \alpha \max\{y - \gamma x, 0\} - \beta \max\{\gamma x - y, 0\}, \tag{5}$$

where $\gamma>0$. Fehr-Schmidt utility function corresponds to the case when $\gamma=1$. Proposition B.1 below provides a complete characterization of the dictator's optimal choice under this more general utility function. Propositions 1-2 in the main body are a special case of Proposition B.1 below with $\gamma=1$. In Proposition B.1, we assume that $\alpha+\beta\geq 0$ and $1+\alpha(\gamma+1)>0$ and $\beta\neq 1/(\gamma+1)$. This generalizes the assumptions that $\alpha+\beta\geq 0$ and $1+2\alpha>0$ and $\beta\neq 1/2$ in the main body. Note that under the utility function specified in (5), the "fair allocation," i.e., the allocation that results in no guilt or envy, is $(\frac{M}{\gamma+1},\frac{M\gamma}{\gamma+1})$.

Proposition B.1. A dictator who is maximizing the utility functions specified in (1) and (5) will choose ρ^* as follows:

i. If $\delta \neq 1$,

(a) If allocation
$$a_e = (\frac{M}{\gamma+1}, \frac{M\gamma}{\gamma+1}) \in \mathcal{A}$$
 then $\rho^* = \begin{cases} (0, 0, ..., \rho_n^* = 1) & \text{if } \beta < 1/(\gamma+1), \\ (0, ..., \rho_{a_n}^* = 1, ..., 0) & \text{if } \beta > 1/(\gamma+1). \end{cases}$

(b) If allocation $a_e = (\frac{M}{\gamma+1}, \frac{M\gamma}{\gamma+1}) \notin \mathcal{A}$, then ρ^* takes one of the following forms:

$$\rho^* = \begin{cases} (0,...,\rho_n^* = 1) & \text{if } \beta < 1/(\gamma + 1), \\ (0,...,\rho_{[e]^-}^* = \lambda,\rho_{[e]^+}^* = 1 - \lambda,...,0), \text{ with some } \lambda \in [0,1] & \text{if } \beta > 1/(\gamma + 1), \end{cases}$$

where $[e]^- = (x_{e^-}, y_{e^-})$ and $[e]^+ = (x_{e^+}, y_{e^+})$ denote the closest feasible allocations located to the left and right of a_e , respectively.

ii. If $\delta = 1$,

- (a) If $\beta < 1/(\gamma + 1)$ then $\rho^* = (0, ..., \rho_n^* = 1)$
- (b) If $\beta > 1/(\gamma + 1)$ then ρ^* is optimal if and only if $\sum_{k=1}^n \rho_k^* x_k = M/(\gamma + 1)$.

Proof. Let $\Delta^l(\mathcal{A}) = \{\rho \colon \rho \in \Delta(\mathcal{A}) \text{ and } \sum_{k=1}^n \rho_k x_k \leq M/(\gamma+1)\}$ and $\Delta^h(\mathcal{A}) = \{\rho \colon \rho \in \Delta(\mathcal{A}) \text{ and } \sum_{k=1}^n \rho_k x_k \geq M/(\gamma+1)\}$. Note that $\Delta^l(\mathcal{A})$ and $\Delta^h(\mathcal{A})$ are compact sets because they are subsets of \mathbb{R}^n and they are both closed and bounded (Heine-Borel theorem). We show below that we can express

$$V(\rho) = \begin{cases} V^l(\rho) & \text{if } \rho \in \Delta^l(\mathcal{A}) \\ V^h(\rho) & \text{if } \rho \in \Delta^h(\mathcal{A}), \end{cases}$$

where $V^l(\rho)$ and $V^h(\rho)$ are continuous and linear functions of ρ . This is useful because since $\Delta^l(\mathcal{A})$ and $\Delta^h(\mathcal{A})$ are compact sets and $V^l(\rho)$ and $V^h(\rho)$ are continuous and linear functions of ρ , the maximizers of $V^l(\rho)$ and $V^h(\rho)$ exist.

Let $\rho \in \Delta^l(\mathcal{A})$. Using $x_k + y_k = M$ for all k, some algebra and the fact that

$$\sum_{k:x_k < M/(\gamma+1)} \rho_k(M - (\gamma+1)x_k) - \sum_{k:x_k \ge M/(\gamma+1)} \rho_k((\gamma+1)x_k - M) = M - (\gamma+1)\sum_{k=1}^n \rho_k x_k,$$

we can write:

$$\begin{split} V(\rho) &= \delta u(E_{\rho}) + (1 - \delta)E_{\rho}(u) \\ &= \delta \left[\sum_{k=1}^{n} \rho_{k} x_{k} - \alpha \left(M - (\gamma + 1) \sum_{k=1}^{n} \rho_{k} x_{k} \right) \right] + \\ &+ (1 - \delta) \left[\sum_{k=1}^{n} \rho_{k} x_{k} - \alpha \sum_{k: x_{k} < M/(\gamma + 1)} \rho_{k} (M - (\gamma + 1) x_{k}) - \beta \sum_{k: x_{k} \ge M/(\gamma + 1)} \rho_{k} ((\gamma + 1) x_{k} - M) \right] \\ &= (1 + \delta \alpha (\gamma + 1)) \sum_{k=1}^{n} \rho_{k} x_{k} - \delta \alpha M \\ &- (1 - \delta) \left[\alpha \sum_{k: x_{k} < M/(\gamma + 1)} \rho_{k} (M - (\gamma + 1) x_{k}) + \beta \sum_{k: x_{k} \ge M/(\gamma + 1)} \rho_{k} ((\gamma + 1) x_{k} - M) \right] \\ &= (1 + \delta \alpha (\gamma + 1) - (1 - \delta) \beta (\gamma + 1)) \sum_{k=1}^{n} \rho_{k} x_{k} - \delta \alpha M + (1 - \delta) \beta M \\ &- (1 - \delta) (\alpha + \beta) \sum_{k: x_{k} < M/(\gamma + 1)} \rho_{k} (M - (\gamma + 1) x_{k}), \end{split}$$

We call the right-hand side of the last equation $V^{l}(\rho)$.

Next, let $\rho \in \Delta^h(\mathcal{A})$. Similarly, we can write

$$V(\rho) = \delta u(E_{\rho}) + (1 - \delta)E_{\rho}(u)$$

$$= \delta \left[\sum_{k=1}^{n} \rho_{k} x_{k} - \beta \left((\gamma + 1) \sum_{k=1}^{n} \rho_{k} x_{k} - M \right) \right] +$$

$$+ (1 - \delta) \left[\sum_{k=1}^{n} \rho_{k} x_{k} - \alpha \sum_{k: x_{k} < M/(\gamma + 1)} \rho_{k} (M - (\gamma + 1) x_{k}) - \beta \sum_{k: x_{k} \ge M/(\gamma + 1)} \rho_{k} ((\gamma + 1) x_{k} - M) \right]$$

$$= (1 - \delta \beta (\gamma + 1)) \sum_{k=1}^{n} \rho_{k} x_{k} + \delta \beta M$$

$$- (1 - \delta) \left[\alpha \sum_{k: x_{k} < M/(\gamma + 1)} \rho_{k} (M - (\gamma + 1) x_{k}) + \beta \sum_{k: x_{k} \ge M/(\gamma + 1)} \rho_{k} ((\gamma + 1) x_{k} - M) \right]$$

$$= (1 - \beta (\gamma + 1)) \sum_{k=1}^{n} \rho_{k} x_{k} + \beta M - (1 - \delta) (\alpha + \beta) \sum_{k: x_{k} < M/(\gamma + 1)} \rho_{k} (M - (\gamma + 1) x_{k}).$$

We call the right-hand side of the last equation $V^h(\rho)$. The dictator's optimal choice, ρ^* , satisfies

$$\rho^* \in \operatorname*{argmax}_{\rho \in \Delta^l(\mathcal{A})} V^l(\rho) \cup \operatorname*{argmax}_{\rho \in \Delta^h(\mathcal{A})} V^h(\rho).$$

Next, we characterize the optimal strategy for $\delta = 1$ (i.e., part (ii) of Proposition B.1.)

Part (ii) of Proposition B.1: When $\delta = 1$, the dictator's utility from choosing ρ can be simplified to

$$V(\rho) = \begin{cases} V^l(\rho) = (1 + \alpha(\gamma + 1)) \sum_{k=1}^n \rho_k x_k - \alpha M & \text{if } \rho \in \Delta^l(\mathcal{A}), \\ V^h(\rho) = (1 - \beta(\gamma + 1)) \sum_{k=1}^n \rho_k x_k + \beta M & \text{if } \rho \in \Delta^h(\mathcal{A}). \end{cases}$$
(6)

Note that $V^l(\rho)$ is an increasing function of $\sum_{k=1}^n \rho_k x_k$ as $1 + \alpha(\gamma + 1) > 0$ by assumption. Also, by definition, $\rho \in \Delta^l(\mathcal{A})$ implies $\sum_{k=1}^n \rho_k x_k \leq M/(\gamma + 1)$. Hence, $V^l(\rho) \leq (1 + \alpha(\gamma + 1))M/(\gamma + 1) - \alpha M = M/(\gamma + 1)$, with equality when $\sum_{k=1}^n \rho_k x_k = M/(\gamma + 1)$.

For $V^h(\rho)$, first, suppose $\beta < 1/(\gamma+1)$. Then $V^h(\rho)$ is an increasing function of $\sum_{k=1}^n \rho_k x_k$. Using $\sum_{k=1}^n \rho_k x_k \leq M$, we have $V^h(\rho) \leq (1-\beta(\gamma+1))M+\beta M=M(1-\beta\gamma)$, with equality when $\rho_n=1$. Note that $M(1-\beta\gamma)>M/(\gamma+1)$ as $\beta < 1/(\gamma+1)$. It follows that $\max_{\rho \in \Delta^l(\mathcal{A})} V^l(\rho) < \max_{\rho \in \Delta^h(\mathcal{A})} V^h(\rho)$ and $V(\rho)$ achieves its maximum when $\rho_n=1$. Therefore, for the case described in part (ii)(a) of Proposition B.1, the optimal strategy is to act selfishly with $\rho^*=(0,\ldots,\rho_n^*=1)$.

Now, suppose $\beta > 1/(\gamma+1)$. Then $V^h(\rho)$ is a decreasing function of $\sum_{k=1}^n \rho_k x_k$. By definition, $\rho \in \Delta^h(\mathcal{A})$ implies $\sum_{k=1}^n \rho_k x_k \geq M/(\gamma+1)$. Using this, we have $V^h(\rho) \leq (1-\beta(\gamma+1))M/(\gamma+1)+\beta M=M/(\gamma+1)$, with equality when $\sum_{k=1}^n \rho_k x_k = M/(\gamma+1)$. Hence, $\max_{\rho \in \Delta^l(\mathcal{A})} V^l(\rho) = \max_{\rho \in \Delta^h(\mathcal{A})} V^h(\rho)$ and $V(\rho)$ achieves its maximum value of $M/(\gamma+1)$ when $\sum_{k=1}^n \rho_k x_k = M/(\gamma+1)$. Consequently, the optimal strategy is any lottery that gives the dictator $M/(\gamma+1)$ in expectation. This completes our proof of part (ii) of Proposition B.1.

Part (i) of Proposition B.1: For any choice ρ of the dictator, let $S(\rho) = \{k \in \mathbb{N} : \rho_k > 0\} = \{n_1, n_2, \dots, n_{|S(\rho)|}\}$ be the support of ρ , where $|S(\rho)|$ is the cardinality of $S(\rho)$ and $n_1 < n_2 < \dots < n_{|S(\rho)|}$. We suppose that $|S(\rho)| \ge 2$ (i.e., the strategy is stochastic) and will deal with the deterministic case of $|S(\rho)| = 1$ later. We begin with the following Lemma, which argues that any optimal strategy that is stochastic must have elements smaller and larger than the fair amount and its support has to contain only two allocations that are consecutive.

Lemma B.1. Suppose ρ is optimal and $|S(\rho)| \geq 2$. Then

- (i) $x_{n_1} < M/(\gamma + 1) < x_{n_{|S(\rho)|}}$;
- (ii) $n_{|S(\rho)|} n_1 = 1$.

Proof. For part (i), the proof is by contradiction. Suppose ρ^* is optimal but $x_{n_{|S(\rho^*)|}} \leq M/(\gamma+1)$. Then $x_{n_j} < M/(\gamma+1)$ for all $j < |S(\rho^*)|$ and $\sum_k \rho_k^* x_k < M/(\gamma+1)$. Hence, the optimal choice ρ^* must be a maximizer of $V^l(\rho)$. Next, we write the corresponding Lagrangian of this maximization

$$\mathcal{L}(\rho) = (1 + \delta\alpha(\gamma + 1)) \sum_{k=1}^{n} \rho_k x_k - \delta\alpha M +$$

$$-(1 - \delta) \left[\alpha \sum_{k: x_k \le M/(\gamma + 1)} \rho_k (M - (\gamma + 1)x_k) + \beta \sum_{k: x_k > M/(\gamma + 1)} \rho_k ((\gamma + 1)x_k - M) \right] +$$

$$+\sum_{k=1}^{n}\omega_{k}\rho_{k}+\sum_{k=1}^{n}\psi_{k}(1-\rho_{k})+\lambda_{1}\left(1-\sum_{k=1}^{n}\rho_{k}\right)+\lambda_{2}\left(M/(\gamma+1)-\sum_{k=1}^{n}\rho_{k}x_{k}\right),$$

where $\omega_k \geq 0$, $\psi_k \geq 0$, λ_1 , and $\lambda_2 \geq 0$ are the Lagrange multipliers associated to conditions $\rho_k \geq 0$, $1 - \rho_k \geq 0$, $1 - \sum_{k=1}^n \rho_k = 0$, and $M/(\gamma + 1) - \sum_{k=1}^n \rho_k x_k \geq 0$, respectively. As ρ^* is a maximizer of $V^l(\rho)$, for all $k \in S(\rho^*)$ we must have

$$0 = \frac{\partial \mathcal{L}(\rho)}{\partial \rho_k}|_{\rho = \rho^*} = (1 + \delta \alpha (\gamma + 1)) x_k - (1 - \delta) Z_k + \omega_k - \psi_k - \lambda_1 - \lambda_2 x_k,$$

where $Z_k = \alpha(M - (\gamma + 1)x_k)$ for all $k \in S(\rho^*)$ because of our assumption that $x_{n_{|S(\rho^*)|}} \leq M/(\gamma + 1)$. The complementary slackness requires $\omega_k \rho_k^* = 0$ and $\psi_k(\rho_k^* - 1) = 0$ for all $k \in S(\rho^*)$ and $\lambda_1(1 - \sum_{k=1}^n \rho_k^*) = \lambda_2(\sum_{k=1}^n \rho_k^* x_k - M/(\gamma + 1)) = 0$. As we show $\sum_{k=1}^n \rho_k^* x_k < M/(\gamma + 1)$ earlier, λ_2 must be equal to 0. Also, $\rho_k^* \in (0, 1)$ for all $k \in S(\rho^*)$ implies $\omega_k = \psi_k = 0$ for all $k \in S(\rho^*)$. Then the first-order condition simplifies to

$$(1 + \delta\alpha(\gamma + 1))x_k - (1 - \delta)Z_k - \lambda_1 = 0$$
 for all $k \in S(\rho^*)$.

Consequently, for any pair $(k, k') \in S(\rho^*) \times S(\rho^*)$ we have:

$$(1 + \delta\alpha(\gamma + 1))x_k - (1 - \delta)Z_k = (1 + \delta\alpha(\gamma + 1))x_{k'} - (1 - \delta)Z_{k'}$$

Particularly, the above equation holds when $k = n_1$ and $k' = n_{|S(\rho^*)|}$. Hence,

$$(1 + \delta\alpha(\gamma + 1))x_{n_{1}} - (1 - \delta)Z_{n_{1}} = (1 + \delta\alpha(\gamma + 1))x_{n_{|S(\rho^{*})|}} - (1 - \delta)Z_{n_{|S(\rho^{*})|}}$$

$$(1 + \delta\alpha(\gamma + 1))x_{n_{1}} - (1 - \delta)\alpha(M - (\gamma + 1)x_{n_{1}}) = (1 + \delta\alpha(\gamma + 1))x_{n_{|S(\rho^{*})|}} - (1 - \delta)\alpha(M - (\gamma + 1)x_{n_{|S(\rho^{*})|}})$$

$$(1 + \alpha(\gamma + 1))x_{n_{1}} - (1 - \delta)\alpha M = (1 + \alpha(\gamma + 1))x_{n_{|S(\rho^{*})|}} - (1 - \delta)\alpha M$$

$$x_{n_{1}} = x_{n_{|S(\rho^{*})|}} \text{ (because } 1 + \alpha(\gamma + 1) > 0 \text{ by assumption)},$$

which contradicts with $x_{n_1} < x_{n_{|S(\rho^*)|}}$. Hence, the initial assumption is wrong and we have $x_{n_{|S(\rho^*)|}} > M/(\gamma + 1)$.

Now, suppose ρ^* is optimal but $x_{n_1} \geq M/(\gamma+1)$. It follows that $x_{n_j} > M/(\gamma+1)$ for all j > 1 and $\sum_k \rho_k^* x_k > M/(\gamma+1)$. Hence, ρ^* must be a maximizer of $V^h(\rho)$. Following a similar idea as above by writing down the Lagrangian function and then taking the first-order conditions, we get

$$(1 - \delta\beta(\gamma + 1))x_k - (1 - \delta)T_k = (1 - \delta\beta(\gamma + 1))x_{k'} - (1 - \delta)T_{k'}$$
 for all $k, k' \in S(\rho^*)$,

where $T_k = \beta((\gamma + 1)x_k - M)$ for all $k \in S(\rho^*)$. This means

$$(1 - \beta(\gamma + 1))x_k + (1 - \delta)\beta M = (1 - \beta(\gamma + 1))x_{k'} + (1 - \delta)\beta M$$
 for all $k, k' \in S(\rho^*)$, (7)

As $\beta \neq 1/(\gamma + 1)$, the equation above implies that $x_k = x_{k'}$ for all $k, k' \in S(\rho^*)$. This contradicts with the assumption that the optimal choice is stochastic. This completes our proof of part (i) of Lemma B.1.

For part (ii), proof is again by contradiction. Suppose ρ is optimal and $n_{|S(\rho)|} - n_1 \ge 2$ (i.e., the highest and lowest amounts kept by the dictator are not consecutive). We will show that such ρ cannot be optimal. The idea is constructing another strategy ρ' from ρ by moving

weights from allocations (x_{n_1}, y_{n_1}) and $(x_{n_{|S(\rho)|}}, y_{n_{|S(\rho)|}})$ to allocation (x_i, y_i) , for some i such that $x_{n_1} < x_i < x_{n_{|S(\rho)|}}$, to increase $V(\rho)$. The goal is to reduce the guilt of the dictator (i.e., $\sum_{k:x_k < M/(\gamma+1)} \rho_k(M-(\gamma+1)x_k)$) while keeping the dictator's expected earnings (i.e., $\sum_{k=1}^n \rho_k x_k$) unchanged.

First, as ρ is optimal, it follows from part (i) of the Lemma that $x_{n_1} < M/(\gamma + 1) < x_{n_{|S(\rho)|}}$. As $n_{|S(\rho)|} - n_1 \ge 2$, there must exist i such that $n_{|S(\rho)|} > i > n_1$. Equivalently, $x_{n_{|S(\rho)|}} > x_i > x_{n_1}$. Define ρ' as follows

$$\rho'_{k} = \begin{cases} \rho_{k} & \text{if } k \neq n_{1}, i, n_{|S(\rho)|} \\ \rho_{k} - \varepsilon \frac{x_{n_{|S(\rho)|}} - x_{i}}{x_{n_{|S(\rho)|}} - x_{n_{1}}} & \text{if } k = n_{1} \\ \rho_{k} + \varepsilon & \text{if } k = i \\ \rho_{k} - \varepsilon \frac{x_{i} - x_{n_{1}}}{x_{n_{|S(\rho)|}} - x_{n_{1}}} & \text{if } k = n_{|S(\rho)|} \end{cases}$$

where $\varepsilon>0$ is sufficiently small. It is routine to verify that $\rho'_{n_1}+\rho'_i+\rho'_{n_{|S(\rho)|}}=\rho_{n_1}+\rho_i+\rho_{n_{|S(\rho)|}}$ and $x_{n_1}\rho'_{n_1}+x_i\rho'_i+x_{n_{|S(\rho)|}}\rho'_{n_{|S(\rho)|}}=x_{n_1}\rho_{n_1}+x_i\rho_i+x_{n_{|S(\rho)|}}\rho_{n_{|S(\rho)|}}$. This implies $\sum_{k=1}^n\rho'_k=1$ and $\sum_{k=1}^n\rho'_kx_k=\sum_{k=1}^n\rho_kx_k$. Hence, when ε is small enough, $\rho'\in\Delta(\mathcal{A})$ and is thus a feasible choice. We will show that ρ' dominates ρ for the dictator (i.e., $V(\rho')>V(\rho)$). Note that both ρ' and ρ are either from $\Delta^l(\mathcal{A})$ or $\Delta^h(\mathcal{A})$. Also note that the first two terms of $V^l(\rho)$ and $V^h(\rho)$ functions are identical for ρ and ρ' given that the dictator's expected payoffs under ρ and ρ' are the same; hence, these terms will be canceled out in $V(\rho)-V(\rho')$. Furthermore, our assumption of $|S(\rho)|\geq 2$ implies that $(1-\delta)(\alpha+\beta)>0$; we will prove this at the end of the proof.

Suppose $x_i \geq M/(\gamma + 1)$. Using the properties of ρ and definition of ρ' , we have

$$\frac{V(\rho) - V(\rho')}{(1 - \delta)(\alpha + \beta)} = \sum_{k: x_k < M/(\gamma + 1)} \rho'_k(M - (\gamma + 1)x_k) - \sum_{k: x_k < M/(\gamma + 1)} \rho_k(M - (\gamma + 1)x_k) =$$

$$= \rho'_{n_1}(M - (\gamma + 1)x_{n_1}) - \rho_{n_1}(M - (\gamma + 1)x_{n_1}) \quad \text{(using } x_i \ge M/(\gamma + 1))$$

$$= (\rho'_{n_1} - \rho_{n_1})(M - (\gamma + 1)x_{n_1}) < 0 \quad \text{(because } \rho'_{n_1} < \rho_{n_1} \text{ and } x_{n_1} < M/(\gamma + 1)),$$

which contradicts with the optimality of ρ .

Now, suppose $x_i < M/(\gamma + 1)$. Then

$$\frac{V(\rho) - V(\rho')}{(1 - \delta)(\alpha + \beta)} = \sum_{k: x_k < M/(\gamma + 1)} \rho_k'(M - (\gamma + 1)x_k) - \sum_{k: x_k < M/(\gamma + 1)} \rho_k(M - (\gamma + 1)x_k) =$$

$$= \sum_{k: k \in \{n_1, i\}} \rho_k'(M - (\gamma + 1)x_k) - \sum_{k: k \in \{n_1, i\}} \rho_k(M - (\gamma + 1)x_k) \quad \text{(using } x_i < M/(\gamma + 1))$$

$$= M \sum_{k: k \in \{n_1, i\}} (\rho_k' - \rho_k) - (\gamma + 1) \sum_{k: k \in \{n_1, i\}} (\rho_k' x_k - \rho_k x_k) \quad \text{(rearranging terms)}$$

$$= M(\rho_{n_{|S(\rho)|}} - \rho_{n_{|S(\rho)|}}') - (\gamma + 1)(\rho_{n_{|S(\rho)|}} x_{n_{|S(\rho)|}} - \rho_{n_{|S(\rho)|}}' x_{n_{|S(\rho)|}})$$

$$= (\rho_{n_{|S(\rho)|}} - \rho_{n_{|S(\rho)|}}')(M - (\gamma + 1)x_{n_{|S(\rho)|}}) < 0 \quad \text{(because } \rho_{n_{|S(\rho)|}}' < \rho_{n_{|S(\rho)|}} \text{ and } M < (\gamma + 1)x_{n_{|S(\rho)|}})$$

where the equation on the fourth line uses the facts that $\rho'_{n_1} + \rho'_i + \rho'_{n_{|S(\rho)|}} = \rho_{n_1} + \rho_i + \rho_{n_{|S(\rho)|}}$ and $x_{n_1}\rho'_{n_1} + x_i\rho'_i + x_{n_{|S(\rho)|}}\rho'_{n_{|S(\rho)|}} = x_{n_1}\rho_{n_1} + x_i\rho_i + x_{n_{|S(\rho)|}}\rho_{n_{|S(\rho)|}}$. As $V(\rho) < V(\rho')$, ρ' dominates ρ for the dictator and that contradicts with the optimality of ρ .

Above we claimed that $(1 - \delta)(\alpha + \beta) > 0$. Next, we will prove it by contradiction. Suppose $(1 - \delta)(\alpha + \beta) = 0$. As $\delta < 1$ in part (i) of Proposition B.1, this implies $\beta = -\alpha$. In this case, $V(\rho)$ simplifies to $V(\rho) = (1 + \alpha(\gamma + 1)) \sum_{k=1}^{n} \rho_k x_k - \alpha M$. Since $1 + \alpha(\gamma + 1) > 0$ by assumption, such a dictator will choose $\rho = (0, \dots, \rho_n = 1)$. Hence, $|S(\rho)| = 1$ at the optimum, contradicting the assumption that $|S(\rho)| \geq 2$. This completes our proof of Lemma B.1.

Back to the main proof. We prove cases (i)a and (i)b of Proposition B.1 separately.

Case (i)a of Proposition B.1: Suppose $a_e = (M/(\gamma+1), M\gamma/(\gamma+1)) \in \mathcal{A}$. We show that the dictator's optimal behavior must be deterministic by a contradiction. Suppose the optimal ρ^* is stochastic (i.e., $|S(\rho^*)| \geq 2$). Lemma B.1 then implies that $S(\rho^*) = \{n_1, n_2\}$ and $x_{n_1} < M/(\gamma+1) < x_{n_2}$, which implies $n_2 - n_1 > 1$ as $a_e \in \mathcal{A}$. This contradicts with Lemma B.1. Hence, the dictator's optimal behavior must be deterministic. In such a case, there exists $i \in \mathbb{N}$ such that $\rho_i^* = 1$. Plug this strategy into (6) and get

$$V(\rho^*) = \begin{cases} V^l(\rho^*) & \text{if } \rho^* \in \Delta^l(\mathcal{A}) \\ V^h(\rho^*) & \text{if } \rho^* \in \Delta^h(\mathcal{A}) \end{cases} = \begin{cases} x_i - \alpha(M - (\gamma + 1)x_i) & \text{if } x_i \leq M/(\gamma + 1) \\ x_i - \beta((\gamma + 1)x_i - M) & \text{if } x_i \geq M/(\gamma + 1) \end{cases}$$

Note that this piecewise linear function of x_i will achieve its maximum at the boundaries. Hence, there are three possible candidates for ρ^* : $\rho^* = (\rho_1^* = 0, \dots, 0)$, $\rho^* = (0, \dots, \rho_{a_e}^* = 1, \dots, 0)$, or $\rho^* = (0, \dots, \rho_n^* = 1)$. In the first case, the dictator's utility is $-\alpha M$. In the second case, the dictator's utility is $M/(\gamma + 1)$. In the last case, the dictator's utility is $M(1 - \beta \gamma)$. As $1 + \alpha(\gamma + 1) > 0$ and $\beta \neq 1/(\gamma + 1)$ by assumptions, the dictator's optimal choice is unique and given by $\rho^* = (0, \dots, \rho_{a_e}^* = 1, \dots, 0)$ if $\beta > 1/(\gamma + 1)$ and $\rho^* = (0, \dots, \rho_n^* = 1)$ if $\beta < 1/(\gamma + 1)$. This completes our proof of part (i)a of Proposition B.1.

Case (i)b of Proposition B.1: Suppose $a_e \notin \mathcal{A}$. Let $[e]^- = (x_{e^-}, y_{e^-})$ and $[e]^+ = (x_{e^+}, y_{e^+})$ be two feasible allocations that are left-closest and right-closest to a_e , respectively. Following the same logic as in the proof of Case (i)a and the results in Lemma B.1, there are four candidates for the optimal strategy. Among them, three are deterministic and given by

- $\rho = (0, ..., \rho_{[e]^-} = 1, ..., 0)$, which corresponds to an utility of $x_{e^-} \alpha (M (\gamma + 1)x_{e^-})$;
- $\rho = (0, ..., \rho_{[e]^+} = 1, ..., 0)$, which corresponds to an utility of $x_{e^+} \beta((\gamma + 1)x_{e^+} M)$;
- $\rho = (0, ..., \rho_n = 1)$, which corresponds to an utility of $M(1 \beta \gamma)$.

The last one is stochastic and given by $\rho=(0,...,\rho_{[e]^-}=\lambda,\rho_{[e]^+}=1-\lambda,...,0)$ for some $\lambda\in(0,1)$. Note that the stochastic candidate takes this form because Lemma B.1 implies that any probabilistic play involving randomization between allocations different from $[e]^-$ and $[e]^+$ is not optimal. Notice that the first two deterministic candidates, $\rho=(0,...,\rho_{[e]^-}=1,...,0)$ and $\rho=(0,...,\rho_{[e]^+}=1,...,0)$, also share the form $\rho=(0,...,\rho_{[e]^-}=\lambda,\rho_{[e]^+}=\lambda,...,0)$ for some $\lambda\in[0,1]$. Hence, the possible candidates for the optimal strategies are either $\rho=(0,...,\rho_n=1)$ or $\rho=(0,...,\rho_{[e]^-}=\lambda,\rho_{[e]^+}=\lambda,...,0)$ for some $\lambda\in[0,1]$.

We first characterize the optimal λ when $\rho=(0,...,\rho_{[e]^-}=\lambda,\rho_{[e]^+}=\lambda,...,0)$. In this case, $\sum_{k=1}^n \rho_k x_k = \lambda x_{e^-} + (1-\lambda) x_{e^+}$. Let $\bar{\lambda}$ be the solution of $\lambda x_{e^-} + (1-\lambda) x_{e^+} = M/(\gamma+1)$, i.e., $\bar{\lambda}=(x_{e^+}-M/(\gamma+1))/(x_{e^+}-x_{e^-})$. The dictator's optimization problem of choosing

 $\rho = (0, ..., \rho_{[e]^-} = \lambda, \rho_{[e]^+} = \lambda, ..., 0)$ can be rewritten as an optimization problem of choosing λ . Specifically, the dictator solves

$$\max_{\rho} V(\rho) = \begin{cases} V^l(\rho) & \text{if } \rho \in \Delta^l(\mathcal{A}) \\ V^h(\rho) & \text{if } \rho \in \Delta^h(\mathcal{A}) \end{cases} = \max_{\lambda \in [0,1]} \begin{cases} \Gamma_1(\lambda) & \text{if } \lambda \in [\bar{\lambda},1] \\ \Gamma_2(\lambda) & \text{if } \lambda \in [0,\bar{\lambda}], \end{cases}$$

where $\Gamma_1(\lambda)$ and $\Gamma_2(\lambda)$ are linear and continuous functions of λ . Note that the optimal λ depends on the exact values of $x_{e^-}, x_{e^+}, \alpha, \beta, \gamma, \delta$, and M. Lemma B.2 below provides a complete characterization of the optimal strategy of the dictator when $\delta \neq 1$ and $a_e \notin \mathcal{A}$.

Lemma B.2. Suppose $\delta \neq 1$.

- (i) If $\beta < 1/(\gamma + 1)$ then $\rho^* = (0, ..., \rho_n^* = 1)$.
- (ii) If $\beta > 1/(\gamma + 1)$ then the dictator randomizes between $[e]^-$ and $[e]^+$, with the optimal weight on $[e]^-$, λ^* , given by the following table

| | $\Gamma_2'(\lambda) < 0$ | $\Gamma_2'(\lambda) = 0$ | $\Gamma_2'(\lambda) > 0$ |
|--------------------------|--------------------------|------------------------------------|------------------------------------|
| $\Gamma'_1(\lambda) < 0$ | $\lambda^* = 0$ | $\lambda^* \in [0, \bar{\lambda}]$ | $\lambda^* = \bar{\lambda}$ |
| $\Gamma_1'(\lambda) = 0$ | _ | $\lambda^* \in [0,1]$ | $\lambda^* \in [\bar{\lambda}, 1]$ |
| $\Gamma'_1(\lambda) > 0$ | _ | _ | $\lambda^* = 1$ |

Notes: The symbol " – " in the table above implies that the corresponding case cannot happen since $\Gamma_1'(\lambda) < \Gamma_2'(\lambda)$. Here,

$$\Gamma'_{1}(\lambda) = \delta(x_{e^{-}} - x_{e^{+}})(1 + \alpha(\gamma + 1)) + (1 - \delta)[x_{e^{-}} - \alpha(M - (\gamma + 1)x_{e^{-}}) - x_{e^{+}} + \beta((\gamma + 1)x_{e^{+}} - M)]$$

$$\Gamma'_{2}(\lambda) = \delta(x_{e^{-}} - x_{e^{+}})(1 - \beta(\gamma + 1)) + (1 - \delta)[x_{e^{-}} - \alpha(M - (\gamma + 1)x_{e^{-}}) - x_{e^{+}} + \beta((\gamma + 1)x_{e^{+}} - M)]$$

Proof. We begin with some calculations. When $\rho = (0, ..., \rho_{[e]^-} = \lambda, \rho_{[e]^+} = 1 - \lambda, ..., 0)$, the dictator's utility is given by $\Gamma_1(\lambda)$ if $\lambda \in [\bar{\lambda}, 1]$ and $\Gamma_2(\lambda)$ if $\lambda \in [0, \bar{\lambda}]$. Note that both $\Gamma_1(\lambda)$ and $\Gamma_2(\lambda)$ are linear functions of λ . Therefore, they achieve their maximum values at boundary values of λ . This means

$$\max \left\{ \max_{\lambda \in [\bar{\lambda},1]} \Gamma_1(\lambda), \max_{\lambda \in [0,\bar{\lambda}]} \Gamma_2(\lambda) \right\} = \max \{ \Gamma_1(1), \Gamma_1(\bar{\lambda}), \Gamma_2(\bar{\lambda}), \Gamma_2(0) \}.$$

It is straightforward that $\Gamma_1(\bar{\lambda}) = \Gamma_2(\bar{\lambda})$ and

$$\begin{split} &\Gamma_{1}(1) &= u(x_{e^{-}},y_{e^{-}}) = x_{e^{-}} - \alpha(y_{e^{-}} - \gamma x_{e^{-}}) = x_{e^{-}} - \alpha(M - (\gamma + 1)x_{e^{-}}) = (1 + \alpha(\gamma + 1))x_{e^{-}} - \alpha M \\ &\Gamma_{2}(0) &= u(x_{e^{+}},y_{e^{+}}) = x_{e^{+}} - \beta(\gamma x_{e^{+}} - y_{e^{+}}) = x_{e^{+}} - \beta(\gamma M - (\gamma + 1)y_{e^{+}}) = x_{e^{+}} + \beta(\gamma + 1)y_{e^{+}} - \beta\gamma M \\ &\Gamma_{1}(\bar{\lambda}) &= \delta u(\bar{\lambda}x_{e^{-}} + (1 - \bar{\lambda})x_{e^{+}}, \bar{\lambda}y_{e^{-}} + (1 - \bar{\lambda})y_{e^{+}}) + (1 - \delta)[\bar{\lambda}u(x_{e^{-}},y_{e^{-}}) + (1 - \bar{\lambda})u(x_{e^{+}},y_{e^{+}})] \\ &= \delta u(M/(\gamma + 1), M\gamma/(\gamma + 1)) + (1 - \delta)[\bar{\lambda}((1 + \alpha(\gamma + 1))x_{e^{-}} - \alpha M) + (1 - \bar{\lambda})(x_{e^{+}} - \beta((\gamma + 1)x_{e^{+}} - M))] \\ &= \delta M/(\gamma + 1) + (1 - \delta)[\bar{\lambda}x_{e^{-}} + (1 - \bar{\lambda})x_{e^{+}} + \alpha\bar{\lambda}((\gamma + 1)x_{e^{-}} - M) - \beta(1 - \bar{\lambda})((\gamma + 1)x_{e^{+}} - M)] \\ &= \delta M/(\gamma + 1) + (1 - \delta)[M/(\gamma + 1) + \alpha\bar{\lambda}((\gamma + 1)x_{e^{-}} - M) - \beta(1 - \bar{\lambda})((\gamma + 1)x_{e^{+}} - M)] \\ &= M/(\gamma + 1) - (1 - \delta)(\gamma + 1)[\alpha\bar{\lambda}(M/(\gamma + 1) - x_{e^{-}}) + \beta(1 - \bar{\lambda})(x_{e^{+}} - M/(\gamma + 1))]. \end{split}$$

<u>Proof of Part (i) of Lemma B.2</u>: Suppose $\beta < 1/(\gamma+1)$. The dictator's utility when choosing $\rho = (0, ..., \rho_n = 1)$ is given by $M - \beta \gamma M$. To show that $\rho = (0, ..., \rho_n = 1)$ is optimal, it is

sufficient to prove $M - \beta \gamma M \ge \max\{\Gamma_1(1), \Gamma_1(\bar{\lambda}), \Gamma_2(\bar{\lambda}), \Gamma_2(0)\}$. We show that $M - \beta \gamma M$ is greater than or equal to each term separately.

• $M - \beta \gamma M > \Gamma_1(1)$ is equivalent to

$$M - \beta \gamma M > (1 + \alpha(\gamma + 1))x_{e^-} - \alpha M \Leftrightarrow M(1 + \alpha - \beta \gamma) \ge (1 + \alpha(\gamma + 1))x_{e^-}.$$

The inequality above holds because $M(1 + \alpha - \beta \gamma) > M(1 + \alpha - \gamma/(\gamma + 1)) > (1 + \alpha(\gamma + 1))x_{e^-}$. The first inequality uses the assumption that $\beta < 1/(\gamma + 1)$. The second inequality comes from the fact that $x_{e^-} < M/(\gamma + 1)$ (because (x_{e^-}, y_{e^-}) is the allocation closest to the left of allocation $a_e = (M/(\gamma + 1), M\gamma/(\gamma + 1))$) and $1 + \alpha(\gamma + 1) > 0$.

- $M \beta \gamma M > \Gamma_1(\bar{\lambda})$ holds because $\Gamma_1(\bar{\lambda}) \leq M/(\gamma + 1)$ (see calculation above) and $M \beta \gamma M > M/(\gamma + 1)$ as $\beta < 1/(\gamma + 1)$.
- $M \beta \gamma M \ge \Gamma_2(0)$ is equivalent to

$$M - \beta \gamma M \ge x_{e^+} + \beta (\gamma + 1) y_{e^+} - \beta \gamma M \Leftrightarrow M - x_{e^+} \ge \beta (\gamma + 1) y_{e^+} \Leftrightarrow y_{e^+} \ge \beta (\gamma + 1) y_{e^+}.$$

The last inequality holds as $\beta < 1/(\gamma + 1)$ and $y_{e^+} \ge 0$. Note that the inequality $M - \beta \gamma M \ge \Gamma_2(0)$ is strict when $y_{e^+} > 0$. When $y_{e^+} = 0$, $\rho^* = (0, ..., \rho_n^* = 1)$ and $\rho^* = (0, ..., \rho_{[e]^+}^* = 1, ..., 0)$ coincide.

This completes our proof of part (i) of Lemma B.2.

Proof of Part (ii) of Lemma B.2: Suppose $\beta > 1/(\gamma + 1)$. Using the proof of Part (i) of Lemma B.2, we know that $M - \beta \gamma M \leq \Gamma_2(0)$ when $\beta > 1/(\gamma + 1)$. Additionally, the inequality $M - \beta \gamma M \leq \Gamma_2(0)$ is strict when $y_{e^+} > 0$. Hence, it is not optimal to choose $\rho = (0, ..., \rho_n = 1)$ when $y_{e^+} > 0$. When $y_{e^+} = 0$, as argued above, $\rho = (0, ..., \rho_n = 1)$ and $\rho = (0, ..., \rho_{[e]^+} = 1, ..., 0)$ are identical. Therefore, we can safely ignore $\rho = (0, ..., \rho_n = 1)$ in characterizing the dictator's optimal choice. First, we show $\Gamma'_1(\lambda) < \Gamma'_2(\lambda)$. This inequality is equivalent to

$$\delta(x_{e^-} - x_{e^+})(1 + \alpha(\gamma + 1)) < \delta(x_{e^-} - x_{e^+})(1 - \beta(\gamma + 1)),$$

which holds because $x_{e^-} - x_{e^+} < 0$ and $1 + \alpha(\gamma + 1) > 0 > 1 - \beta(\gamma + 1)$. As both $\Gamma_1(\lambda)$ and $\Gamma_2(\lambda)$ are linear functions of λ , the maximizers of $\Gamma_1(\lambda)$ and $\Gamma_2(\lambda)$ are determined based on the signs of $\Gamma'_1(\lambda)$ and $\Gamma'_2(\lambda)$. Additionally, note that $\Gamma_1(\bar{\lambda}) = \Gamma_2(\bar{\lambda})$. The optimal λ is then given in the table in part (ii) of Lemma B.2. \square

Back to the main proof. Note that part (i) of Lemma B.2 proves Proposition B.1 part (i)b for the case of $\beta < 1/(\gamma + 1)$. Similarly, part (ii) of Lemma B.2 proves Proposition B.1 part (i)b for the case of $\beta > 1/(\gamma + 1)$. This completes our proof of Proposition B.1.

Proof of Proposition 3: Let $\alpha = \frac{\eta(\theta-1)}{2}$ and $\beta = \frac{\eta(\theta+1)}{2}$. Note that when $\eta, \theta \in [0,1]$ and $(\eta, \theta) \neq (1, 0)$, we have $\alpha > -1/2$ and $\alpha + \beta \geq 0$. Hence, the assumptions in Propositions 1 and 2 are satisfied. We show $u^{CR}(x, y) = u^{FS}(x, y) - \alpha M$ for all allocations (x, y) such that x+y=M. As $\mathcal{V}(\Phi_{\text{ex-a}}, \Phi_{\text{ex-p}})$ is assumed to be linear following (1), the constant term $(-\alpha M)$

does not impact the dictator's optimal behavior. Consequently, the two models generate the same set of optimal strategies.

First, suppose $x \geq y$. Then

$$u^{FS}(x,y) - \alpha M = x - \beta(x-y) - \alpha M \quad \text{(using definition of } u^{FS}(x,y))$$

$$= x - \beta(x-y) - \alpha(x+y) \quad \text{(using } x+y=M)$$

$$= x - \frac{\eta(\theta+1)}{2}(x-y) - \frac{\eta(\theta-1)}{2}(x+y) \quad \text{(using definition of } \alpha,\beta)$$

$$= (1 - \eta\theta)x + \eta y$$

$$= (1 - \eta)x + \eta\theta y + \eta(1 - \theta)(x+y)$$

$$= (1 - \eta)x + \eta[\theta \min\{x,y\} + (1 - \theta)(x+y)] = u^{CR}(x,y)$$

Now, suppose $y \geq x$. Then

$$u^{FS}(x,y) - \alpha M = x - \alpha(y-x) - \alpha M \quad \text{(using definition of } u^{FS}(x,y))$$

$$= x - 2\alpha y \quad \text{(using } x + y = M)$$

$$= x - 2\frac{\eta(\theta - 1)}{2}y \quad \text{(using definition of } \alpha)$$

$$= x - \eta(\theta - 1)y$$

$$= (1 - \eta)x + \eta\theta x + \eta(1 - \theta)(x + y)$$

$$= (1 - \eta)x + \eta[\theta \min\{x, y\} + (1 - \theta)(x + y)] = u^{CR}(x, y).$$

This completes our proof of Proposition 3. \square

Proof of Proposition 4 and Remark 1: We first show that there are at most two allocations in the support of the dictator's optimal choice when u(x, M - x) is a strictly convex or concave function of $x \in (0, M)$. This result is a corollary of the following Lemma:

Lemma B.3. Let u''(x, M-x) be the second-order derivative of u(x, M-x) with respect to x. Suppose there are $q \geq 3$ allocations in the support of ρ^* . Then equation u''(x, M-x) = 0 must have at least one real root in (0, M).

Proof. Suppose ρ^* is optimal and there exist $i_1, i_2, \ldots, i_q \in \mathbb{N}$ with $i_1 < i_2 < \cdots < i_q$ such that $\rho_{i_k}^* \in (0,1)$ for all $k = 1, 2, \ldots, q$, where $q \geq 3$. The dictator chooses ρ^* by maximizing the following Lagrangian:

$$\mathcal{L}(\rho) = \mathcal{V}(\Phi_{\text{ex-a}}(\rho), \Phi_{\text{ex-p}}(\rho)) + \sum_{k=1}^{n} \omega_k \rho_k + \sum_{k=1}^{n} \psi_k (1 - \rho_k) + \nu \left(1 - \sum_{k=1}^{n} \rho_k\right),$$

where $\omega_k \geq 0$, $\psi_k \geq 0$, and ν are the Lagrange multipliers associated with the conditions $\rho_k \geq 0$, $1 - \rho_k \geq 0$, and $1 - \sum_{k=1}^n \rho_k = 0$, respectively. The FOCs are

$$\frac{\partial \mathcal{L}(\rho)}{\partial \rho_k} = \mathcal{V}_1(\Phi_{\text{ex-a}}(\rho), \Phi_{\text{ex-p}}(\rho)) \cdot \frac{\partial \Phi_{\text{ex-a}}(\rho)}{\partial \rho_k} + \mathcal{V}_2(\Phi_{\text{ex-a}}(\rho), \Phi_{\text{ex-p}}(\rho)) \cdot \frac{\partial \Phi_{\text{ex-p}}(\rho)}{\partial \rho_k} + \omega_k - \psi_k - \psi_k$$

We have $\frac{\partial \mathcal{L}(\rho)}{\partial \rho_k}|_{\rho=\rho^*}=0$ for $k=i_1,i_2,\ldots,i_q$ as ρ^* is optimal and $\rho_{i_k}^*\in(0,1)$. Note that the complementary slackness requires $\omega_k\rho_k^*=0$ and $\psi_k(1-\rho_k^*)=0$ for all k and $\nu(1-\sum_{k=1}^n\rho_k^*)=0$

0. As $\rho_k^* \in (0,1)$ for all $k = i_1, i_2, \dots, i_q$, this implies that $\omega_k = \psi_k = 0$ for all $k = i_1, i_2, \dots, i_q$. Additionally, we have

$$\frac{\partial \Phi_{\text{ex-a}}(\rho)}{\partial \rho_k} = x_k u_1(\sum_k \rho_k x_k, M - \sum_k \rho_k x_k) - x_k u_2(\sum_k \rho_k x_k, M - \sum_k \rho_k x_k)
\frac{\partial \Phi_{\text{ex-p}}(\rho)}{\partial \rho_k} = u(x_k, M - x_k),$$

where $u_1(a,b) = \frac{\partial u(a,b)}{\partial a}$ and $u_2(a,b) = \frac{\partial u(a,b)}{\partial b}$. Then $\frac{\partial \mathcal{L}(\rho)}{\partial \rho_k}|_{\rho=\rho^*} = 0$ is equivalent to

$$\mathcal{V}_{1} \quad (\Phi_{\text{ex-a}}(\rho^{*}), \Phi_{\text{ex-p}}(\rho^{*})) \cdot \left[x_{k} u_{1} \left(\sum_{k} \rho_{k}^{*} x_{k}, M - \sum_{k} \rho_{k}^{*} x_{k} \right) - x_{k} u_{2} \left(\sum_{k} \rho_{k}^{*} x_{k}, M - \sum_{k} \rho_{k}^{*} x_{k} \right) \right] + \\
+ \quad \mathcal{V}_{2} \left(\Phi_{\text{ex-a}}(\rho^{*}), \Phi_{\text{ex-p}}(\rho^{*}) \right) \cdot u(x_{k}, M - x_{k}) + \omega_{k} - \psi_{k} - \nu = 0.$$

Define a function $\mathcal{H} \colon \mathbb{R}_+ \to \mathbb{R}$ as follows:

$$\mathcal{H}(x) = \mathcal{V}_{1}(\Phi_{\text{ex-a}}(\rho^{*}), \Phi_{\text{ex-p}}(\rho^{*})) \cdot x \left[u_{1} \left(\sum_{k} \rho_{k}^{*} x_{k}, M - \sum_{k} \rho_{k}^{*} x_{k} \right) - u_{2} \left(\sum_{k} \rho_{k}^{*} x_{k}, M - \sum_{k} \rho_{k}^{*} x_{k} \right) \right] + \mathcal{V}_{2}(\Phi_{\text{ex-a}}(\rho^{*}), \Phi_{\text{ex-p}}(\rho^{*})) \cdot u(x, M - x) - \nu$$

The analysis above suggests that $\mathcal{H}(x_k) = 0$ for all $k = i_1, i_2, \dots, i_q$; this is because $\omega_k = \psi_k = 0$ for all $k = i_1, i_2, \dots, i_q$. As $q \geq 3$, $\mathcal{H}(x_{i_1}) = \mathcal{H}(x_{i_2}) = \mathcal{H}(x_{i_3}) = 0$. The Rolle's theorem then implies that H''(x) = 0 must have at least one real root in $(x_1, x_3) \subseteq (0, M)$. As $\mathcal{H}''(x) = \mathcal{V}_2(\Phi_{\text{ex-a}}(\rho^*), \Phi_{\text{ex-p}}(\rho^*))u''(x, M - x)$ and \mathcal{V} is strictly increasing in the second argument (Assumption 1 in Section 2.1.2), we have $\mathcal{H}''(x) = 0$ if and only if u''(x, M - x) = 0. Hence, equation u''(x, M - x) = 0 must have at least one real root in (0, M).

Back to the main proof of Proposition 4 and Remark 1. Note that when u(x, M - x) is strictly convex or concave in (0, M), equation u''(x, M - x) = 0 has no real root $x \in (0, M)$. Hence, it follows from Lemma B.3 that there are at most two allocations in the support of ρ^* , which is the result stated in Remark 1 and in the first part of Proposition 4.

The second part of Proposition 4 is proved by a contradiction. Suppose the dictator randomizes between allocations (x_i, y_i) and (x_j, y_j) , with i < j, and there is another feasible allocation (x_k, y_k) located between them. In other words, $\rho_i^* > 0$ and $\rho_j^* = 1 - \rho_i^* > 0$ and i < k < j for some k. Note that by assumption $x_i < x_k < x_j$. Hence, $\rho_i^* x_i + \rho_j^* x_j \in [x_i, x_j] = [x_i, x_k] \cup [x_k, x_j]$. Consider two following cases:

Case 1: $\rho_i^* x_i + \rho_j^* x_j \in [x_i, x_k]$. In this case, there exists $\lambda \in [0, 1]$ such that $\rho_i^* x_i + \rho_j^* x_j = \lambda x_i + (1 - \lambda) x_k$. Construct another choice $\rho' \in \Delta(\mathcal{A})$ as follows:

$$\rho'_t = \begin{cases} 0 & \text{if } t \neq i, k \\ \lambda & \text{if } t = i \\ 1 - \lambda & \text{if } t = k \end{cases}$$

As $\rho_i^* \in (0,1)$, it is the case that $\lambda \in (0,1)$ and hence $\rho_k' > 0$. Additionally, by definition of ρ'

$$\rho_i^* x_i + \rho_j^* x_j = \rho_i' x_i + \rho_k' x_k. \tag{8}$$

Note that (8) implies $\rho'_i < \rho^*_i$ because $x_i < x_k < x_j$ and $\rho^*_i = 1 - \rho^*_i$ and $\rho'_k = 1 - \rho'_i$.

We prove $\Phi_{\text{ex-a}}(\rho^*) = \Phi_{\text{ex-a}}(\rho')$ and $\Phi_{\text{ex-p}}(\rho^*) < \Phi_{\text{ex-p}}(\rho')$ so it follows that $\mathcal{V}(\Phi_{\text{ex-a}}(\rho^*), \Phi_{\text{ex-p}}(\rho^*)) < \mathcal{V}(\Phi_{\text{ex-a}}(\rho'), \Phi_{\text{ex-p}}(\rho'))$ because \mathcal{V} is strictly increasing in the second argument (Assumption 1 in Section 2.1.2). This will lead to a contradiction to the optimality of ρ^* .

• First, we show $\Phi_{\text{ex-a}}(\rho^*) = \Phi_{\text{ex-a}}(\rho')$. Note that

$$\sum_{t} \rho_{t}^{*} x_{t} = \rho_{i}^{*} x_{i} + \rho_{j}^{*} x_{j} = \rho_{i}' x_{i} + \rho_{k}' x_{k} = \sum_{t} \rho_{t}' x_{t}.$$

The first equation comes from the properties of ρ^* . The second equation is (8). The last equation results from the definition of ρ' . Hence, $\Phi_{\text{ex-a}}(\rho^*) = u(\sum_t \rho_t^* x_t, M - \sum_t \rho_t^* x_t) = u(\sum_t \rho_t' x_t, M - \sum_t \rho_t' x_t) = \Phi_{\text{ex-a}}(\rho')$.

• Second, we show $\Phi_{\text{ex-p}}(\rho^*) < \Phi_{\text{ex-p}}(\rho')$. By properties of ρ^* and definition of ρ'

$$\Phi_{\text{ex-p}}(\rho^*) = \sum_{t} \rho_t^* u(x_t, M - x_t) = \rho_i^* u(x_i, M - x_i) + \rho_j^* u(x_j, M - x_j)$$

$$\Phi_{\text{ex-p}}(\rho') = \sum_{t} \rho_t' u(x_t, M - x_t) = \rho_i' u(x_i, M - x_i) + \rho_k' u(x_k, M - x_k)$$

Hence, $\Phi_{\text{ex-p}}(\rho^*) < \Phi_{\text{ex-p}}(\rho')$ is equivalent to

$$\begin{array}{lcl} \rho_{i}^{*}u(x_{i},M-x_{i})+\rho_{j}^{*}u(x_{j},M-x_{j}) & < & \rho_{i}^{\prime}u(x_{i},M-x_{i})+\rho_{k}^{\prime}u(x_{k},M-x_{k}) \\ (\rho_{i}^{*}-\rho_{i}^{\prime})u(x_{i},M-x_{i})+\rho_{j}^{*}u(x_{j},M-x_{j}) & < & \rho_{k}^{\prime}u(x_{k},M-x_{k}) \\ \frac{\rho_{i}^{*}-\rho_{i}^{\prime}}{\rho_{k}^{\prime}}u(x_{i},M-x_{i})+\frac{\rho_{j}^{*}}{\rho_{k}^{\prime}}u(x_{j},M-x_{j}) & < & u\Big(\frac{\rho_{i}^{*}-\rho_{i}^{\prime}}{\rho_{k}^{\prime}}x_{i}+\frac{\rho_{j}^{*}}{\rho_{k}^{\prime}}x_{j},M-\frac{\rho_{i}^{*}-\rho_{i}^{\prime}}{\rho_{k}^{\prime}}x_{i}-\frac{\rho_{j}^{*}}{\rho_{k}^{\prime}}x_{j}\Big), \end{array}$$

where the last inequality uses $x_k = \frac{\rho_i^* - \rho_i'}{\rho_k'} x_i + \frac{\rho_j^*}{\rho_k'} x_j$, which follows from (8). Note that the last inequality follows from the strict concavity of u(x, M - x) if $\frac{\rho_i^* - \rho_i'}{\rho_k'} + \frac{\rho_j^*}{\rho_k'} = 1$ and $\frac{\rho_i^* - \rho_i'}{\rho_k'} > 0$. We already showed that $\rho_i^* > \rho_i'$, so the latter is satisfied. The former is also satisfied as

$$\frac{\rho_i^* - \rho_i'}{\rho_{l_*}'} + \frac{\rho_j^*}{\rho_{l_*}'} = \frac{\rho_i^* + \rho_j^* - \rho_i'}{\rho_{l_*}'} = \frac{1 - \rho_i'}{\rho_{l_*}'} = \frac{\rho_k'}{\rho_{l_*}'} = 1.$$

Case 2: $\rho_i^* x_i + \rho_j^* x_j \in [x_k, x_j]$. This case is similar to Case 1 so we omit a formal proof. This completes our proof of the consecutiveness of the two allocations in the support of the optimal strategy (the moreover part in Proposition 4). \square

Proof of Proposition 5: We first show that an optimal strategy assigns positive weights to at most two allocations. The proof is by contradiction. Suppose ρ^* is optimal but there are at least three non-zero components in ρ^* . Let $S(\rho^*) = \{i : \rho_i^* > 0\}$ be the support of ρ^* . By assumption, $|S(\rho^*)| \geq 3$. The dictator chooses $\rho \in \Delta(\mathcal{A})$ to maximize

$$c_1U_D^2(\rho) + c_2U_R^2(\rho) + c_3U_D(\rho)U_R(\rho) + c_4U_D(\rho) + c_5U_R(\rho)$$

The Lagrangian function of the optimization problem is

$$\mathcal{L} = c_1 U_D^2(\rho) + c_2 U_R^2(\rho) + c_3 U_D(\rho) U_R(\rho) + c_4 U_D(\rho) + c_5 U_R(\rho) + \sum_{j=1}^n \eta_j \rho_j + \sum_{j=1}^n \gamma_j (1 - \rho_j) + \nu \Big(1 - \sum_{j=1}^n \rho_j\Big),$$

where $\eta_j \geq 0$, $\gamma_j \geq 0$, and ν are Lagrangian multipliers associated with inequalities $\rho_j \geq 0$, $1 - \rho_j \geq 0$, and equation $1 - \sum_{j=1}^n \rho_j = 0$, respectively. As both $U_D(\rho)$ and $U_R(\rho)$ admit expected utility representations, we have

$$\frac{\partial U_D(\rho)}{\partial \rho_j} = u_D(x_j)$$
 and $\frac{\partial U_R(\rho)}{\partial \rho_j} = u_R(M - x_j).$

Therefore, for all j

$$\frac{\partial \mathcal{L}}{\partial \rho_j} = 2c_1 U_D(\rho) u_D(x_j) + 2c_2 U_R(\rho) u_R(M - x_j) + c_3 [U_R(\rho) u_D(x_j) + U_D(\rho) u_R(M - x_j)] + c_4 u_D(x_j) + c_5 u_R(M - x_j) + \eta_j - \gamma_j - \nu.$$

As ρ^* is optimal, complementary slackness implies $\eta_j \rho_j^* = 0$ and $\gamma_j (1 - \rho_j^*) = 0$ for all j. By definition of $S(\rho^*)$, $\rho_j^* \in (0,1)$ for all $j \in S(\rho^*)$. Complementary slackness then implies $\eta_j = \gamma_j = 0$ for all $j \in S(\rho^*)$. Hence, for all $j \in S(\rho^*)$

$$\frac{\partial \mathcal{L}}{\partial \rho_j}\bigg|_{\rho=\rho^*} = 2c_1 U_D(\rho^*) u_D(x_j) + 2c_2 U_R(\rho^*) u_R(M-x_j) + c_3 [U_R(\rho^*) u_D(x_j) + U_D(\rho^*) u_R(M-x_j)] + c_4 u_D(x_j) + c_5 u_R(M-x_j) - \nu$$

$$= 0.$$

Group all terms with $u_D(x_j)$ and $u_R(M-x_j)$ together, for all $j \in S(\rho^*)$, we have

$$u_D(x_i)[2c_1U_D(\rho^*) + c_3U_R(\rho^*) + c_4] + u_R(M - x_i)[2c_2U_R(\rho^*) + c_3U_D(\rho^*) + c_5] = \nu.$$
 (9)

Let i_1 , i_2 , i_3 be pairwise distinct elements in $S(\rho^*)$ such that $i_1 < i_2 < i_3$. Then equation (9) holds for $j = i_1, i_2, i_3$. Subtract equation (9) for $j = i_1, i_2, i_3$ side by side, we have

$$[u_D(x_{i_1}) - u_D(x_{i_2})][2c_1U_D(\rho^*) + c_3U_R(\rho^*) + c_4] +$$

$$+[u_R(M - x_{i_1}) - u_R(M - x_{i_2})][2c_2U_R(\rho^*) + c_3U_D(\rho^*) + c_5] = 0$$

$$[u_D(x_{i_2}) - u_D(x_{i_3})][2c_1U_D(\rho^*) + c_3U_R(\rho^*) + c_4] +$$

$$+[u_R(M - x_{i_2}) - u_R(M - x_{i_3})][2c_2U_R(\rho^*) + c_3U_D(\rho^*) + c_5] = 0$$
(11)

Note that $i_1 < i_2 < i_3$ implies $x_{i_1} < x_{i_2} < x_{i_3}$ because we order \mathcal{A} in increasing magnitude of the dictator's payoff. By the monotonicity of $u_D(x)$ and $u_R(x)$ (Assumption 4), we have

$$u_D(x_{i_1}) < u_D(x_{i_2}) < u_D(x_{i_3})$$
 and $u_R(M - x_{i_1}) > u_R(M - x_{i_2}) > u_R(M - x_{i_3})$.

Note that $2c_1U_D(\rho^*) + c_3U_R(\rho^*) + c_4 = \mathbf{V}_1(U_D(\rho^*), U_R(\rho^*)) > 0$ and $2c_2U_R(\rho^*) + c_3U_D(\rho^*) + c_5 = \mathbf{V}_2(U_D(\rho^*), U_R(\rho^*)) > 0$ by the assumption that \mathbf{V} is strictly increasing in both argu-

ments. Therefore, equations (10) and (11) imply

$$\frac{u_D(x_{i_1}) - u_D(x_{i_2})}{u_R(M - x_{i_1}) - u_R(M - x_{i_2})} = \frac{u_D(x_{i_2}) - u_D(x_{i_3})}{u_R(M - x_{i_2}) - u_R(M - x_{i_3})}.$$
(12)

By Assumption 4, $u_D(x)$ and $u_R(x)$ are twice differentiable on [0, M], and $u'_R(x) > 0$ for all $x \in (0, M)$. By Cauchy mean value theorem (also known as extended mean value theorem), there exist $c^* \in (x_{i_1}, x_{i_2})$ and $c^{**} \in (x_{i_2}, x_{i_3})$ such that

$$-\frac{u_D'(c^*)}{u_R'(M-c^*)} = \frac{u_D(x_{i_1}) - u_D(x_{i_2})}{u_R(M-x_{i_1}) - u_R(M-x_{i_2})} \quad \text{and} \quad -\frac{u_D'(c^{**})}{u_R'(M-c^{**})} = \frac{u_D(x_{i_2}) - u_D(x_{i_3})}{u_R(M-x_{i_2}) - u_R(M-x_{i_3})}.$$

Hence, equation (12) implies that

$$\frac{u_D'(c^*)}{u_R'(M-c^*)} = \frac{u_D'(c^{**})}{u_R'(M-c^{**})}.$$
(13)

Note that $c^* < c^{**}$ because $x_{i_1} < x_{i_2} < x_{i_3}$. We have

$$\frac{\partial}{\partial x} \frac{u_D'(x)}{u_R'(M-x)} = \frac{u_D''(x)u_R'(M-x) + u_R''(M-x)u_D'(x)}{[u_R'(M-x)]^2}.$$

By Assumption 4, we have $u'_D(x) > 0 > u''_D(x)$ and $u'_R(M-x) > 0 > u''_R(M-x)$ for all $x \in (0,M)$. Hence, $u''_D(x)u'_R(M-x) + u''_R(M-x)u'_D(x) < 0$ for all $x \in (0,M)$. This implies that $\frac{u'_D(x)}{u'_R(M-x)}$ is a decreasing function of x in (0,M). Therefore, (13) cannot happen with $c^* < c^{**}$ (contradiction). Hence, the initial assumption is wrong and there are at most two distinct elements in the support of ρ^* .

To show that the two allocations in the support of ρ^* are consecutive, proof is by contradiction. Suppose the dictator randomizes between allocations (x_i, y_i) and (x_j, y_j) , with i < j, but there is another feasible allocation (x_k, y_k) located between them. In other words, $\rho_i^* > 0$ and $\rho_j^* = 1 - \rho_i^* > 0$ and i < k < j for some k. Note that by assumption $x_i < x_k < x_j$. Hence, $\rho_i^* x_i + \rho_j^* x_j \in [x_i, x_j] = [x_i, x_k] \cup [x_k, x_j]$. Consider two following cases:

Case 1: $\rho_i^* x_i + \rho_j^* x_j \in [x_i, x_k]$. In this case, there exists $\lambda \in [0, 1]$ such that $\rho_i^* x_i + \rho_j^* x_j = \lambda x_i + (1 - \lambda) x_k$. Construct another choice $\rho' \in \Delta(\mathcal{A})$ as follows:

$$\rho'_t = \begin{cases} 0 & \text{if } t \neq i, k \\ \lambda & \text{if } t = i \\ 1 - \lambda & \text{if } t = k \end{cases}$$

As $\rho_i^* \in (0,1)$, it is the case that $\rho_k' > 0$. Additionally, by definition of ρ'

$$\rho_i^* x_i + \rho_j^* x_j = \rho_i' x_i + \rho_k' x_k. \tag{14}$$

Note that (14) implies $\rho'_i < \rho^*_i$ because $x_i < x_k < x_j$ and $\rho^*_j = 1 - \rho^*_i$ and $\rho'_k = 1 - \rho'_i$.

We prove $U_D(\rho') > U_D(\rho^*)$ and $U_R(\rho') > U_R(\rho^*)$ so it follows that $\mathbf{V}(U_D(\rho'), U_R(\rho')) > \mathbf{V}(U_D(\rho^*), U_R(\rho^*))$ because \mathbf{V} is assumed to be strictly increasing in the both arguments. This will lead to a contradiction to the optimality of ρ^* . We show $U_D(\rho') > U_D(\rho^*)$; showing

 $U_R(\rho') > U_R(\rho^*)$ is similar. By properties of ρ^* and definition of ρ'

$$U_D(\rho^*) = \sum_{t=1}^n \rho_t^* u_D(x_t, M - x_t) = \rho_i^* u_D(x_i, M - x_i) + \rho_j^* u_D(x_j, M - x_j)$$

$$U_D(\rho') = \sum_{t=1}^n \rho_t' u_D(x_t, M - x_t) = \rho_i' u_D(x_i, M - x_i) + \rho_k' u_D(x_k, M - x_k)$$

Hence, $U_D(\rho^*) < U_D(\rho')$ is equivalent to

$$\rho_{i}^{*}u_{D}(x_{i}, M - x_{i}) + \rho_{j}^{*}u_{D}(x_{j}, M - x_{j}) < \rho_{i}^{'}u_{D}(x_{i}, M - x_{i}) + \rho_{k}^{'}u_{D}(x_{k}, M - x_{k})
(\rho_{i}^{*} - \rho_{i}^{'})u_{D}(x_{i}, M - x_{i}) + \rho_{j}^{*}u_{D}(x_{j}, M - x_{j}) < \rho_{k}^{'}u_{D}(x_{k}, M - x_{k})
\frac{\rho_{i}^{*} - \rho_{i}^{'}}{\rho_{k}^{'}}u_{D}(x_{i}, M - x_{i}) + \frac{\rho_{j}^{*}}{\rho_{k}^{'}}u_{D}(x_{j}, M - x_{j}) < u_{D}\left(\frac{\rho_{i}^{*} - \rho_{i}^{'}}{\rho_{k}^{'}}x_{i} + \frac{\rho_{j}^{*}}{\rho_{k}^{'}}x_{j}, M - \frac{\rho_{i}^{*} - \rho_{i}^{'}}{\rho_{k}^{'}}x_{j} - \frac{\rho_{j}^{*}}{\rho_{k}^{'}}x_{j}\right),$$

where the last inequality uses $x_k = \frac{\rho_i^* - \rho_i'}{\rho_k'} x_i + \frac{\rho_j^*}{\rho_k'} x_j$, which follows from (14). Note that the last inequality follows from the strict concavity of $u_D(x, M-x)$ if $\frac{\rho_i^* - \rho_i'}{\rho_k'} + \frac{\rho_j^*}{\rho_k'} = 1$ and $\frac{\rho_i^* - \rho_i'}{\rho_k'} > 0$. We already showed that $\rho_i^* > \rho_i'$, so the latter is satisfied. The former is also satisfied as

$$\frac{\rho_i^* - \rho_i'}{\rho_k'} + \frac{\rho_j^*}{\rho_k'} = \frac{\rho_i^* + \rho_j^* - \rho_i'}{\rho_k'} = \frac{1 - \rho_i'}{\rho_k'} = \frac{\rho_k'}{\rho_k'} = 1.$$

Case 2: $\rho_i^* x_i + \rho_j^* x_j \in [x_k, x_j]$. This case is similar to Case 1 so we omit a formal proof. This completes our proof of the consecutiveness of the two allocations in the support of the optimal strategy. This completes our proof of the Proposition. \square

Proof of Proposition 6: For part (i), suppose $\rho_{(5,5)}(\mathcal{A}_4) = a \in [0,1]$ and $\rho_{(0,10)}(\mathcal{A}_4) = 1-a$. We want to show that a=1 is optimal for the dictator. Note that ρ yields an expected allocation of (5a, 10-5a). In this expected allocation, the dictator's share is (weakly) smaller than that of the recipient: $5a \leq 10-5a$. Hence, the dictator's utility is given by

$$\delta u(5a, 10 - 5a) + (1 - \delta)[au(5, 5) + (1 - a)u(0, 10)]$$

$$= \delta[5a - \alpha(10 - 5a - 5a)] + (1 - \delta)[a(5 - \alpha \cdot 0) + (1 - a)(0 - \alpha \cdot 10)]$$

$$= \delta[5a - 10\alpha(1 - a)] + (1 - \delta)[5a - 10\alpha(1 - a)]$$

$$= 5a(1 + 2\alpha) - 10\alpha.$$

By assumption $1 + 2\alpha > 0$. Hence, the dictator's utility function is strictly increasing in a. Therefore, a = 1 is optimal for the dictator.

For part (ii), suppose $\rho_{(5,5)}(\mathcal{A}_2) = b \in [0,1]$ and $\rho_{(10,0)}(\mathcal{A}_2) = 1 - b$. We want to show that b = 1 is optimal for the dictator when $\beta > 1/2$ and b = 0 is optimal for the dictator when $\beta < 1/2$. Note that ρ yields an expected allocation of (10 - 5b, 5b). In this expected allocation, the dictator's share is (weakly) greater than that of the recipient: $10 - 5b \ge 5b$. Hence, the dictator's utility is given by

$$\delta u(10-5b,5b) + (1-\delta)[bu(5,5) + (1-b)u(10,0)]$$

```
 = \delta[10 - 5b - \beta(10 - 5b - 5b)] + (1 - \delta)[b(5 - \beta \cdot 0) + (1 - b)(10 - \beta \cdot 10)] 
 = \delta[10 - 5b - 10\beta(1 - b)] + (1 - \delta)[5b + 10(1 - \beta)(1 - b)] 
 = \delta[10 - 5b - 10\beta(1 - b)] + (1 - \delta)[10 - 5b - 10\beta(1 - b)] 
 = 10 - 10\beta + 5b(2\beta - 1).
```

When $\beta > 1/2$, the dictator's utility is strictly increasing in b. Hence, the optimal choice is b = 1. Conversely, when $\beta < 1/2$, the dictator's utility is strictly decreasing in b. Hence, the optimal choice is b = 0. This completes our proof of the Proposition. \square

Appendix C Screenshots of Experimental Interface

C.1 Experiment 1

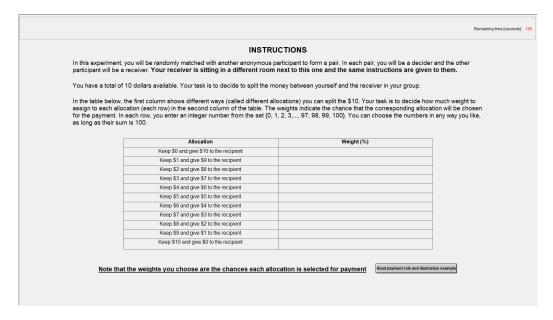


Figure 13: Instruction - Experiment 1

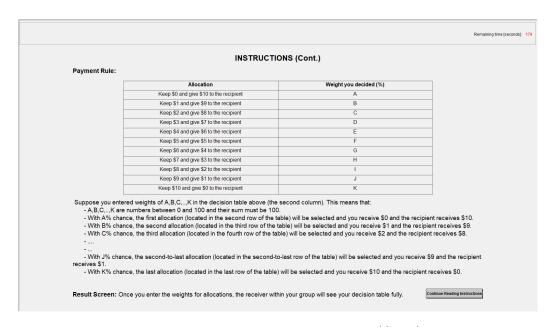


Figure 14: Instruction - Experiment 1 (Cont.)

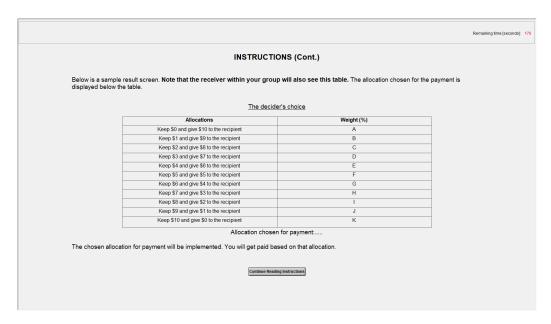


Figure 15: Instruction - Experiment 1 (Cont.)

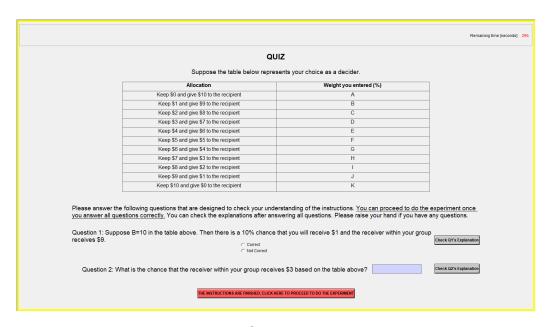


Figure 16: Quiz - Experiment 1

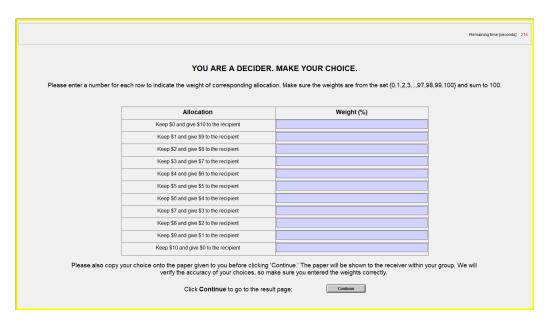


Figure 17: Main Question - Experiment 1

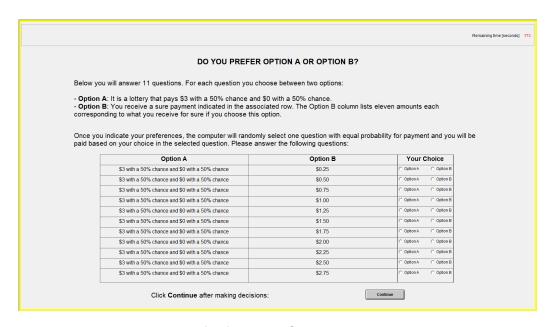


Figure 18: Risk Elicitation Question - Experiment 1

C.2 Experiment 2

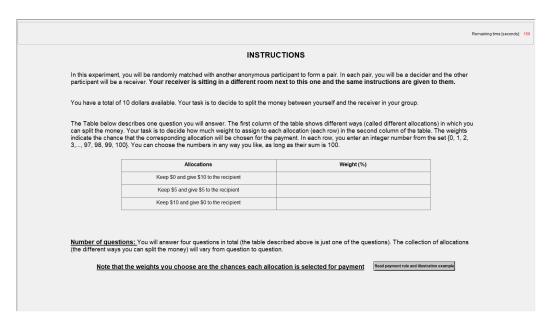


Figure 19: Introduction - Experiment 2

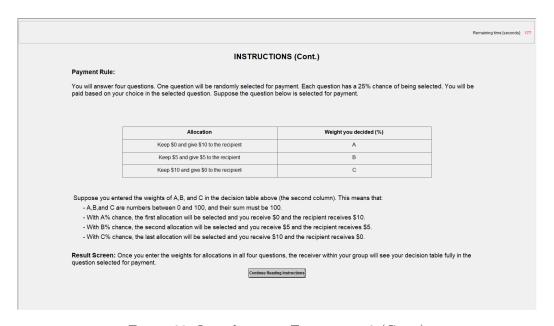


Figure 20: Introduction - Experiment 2 (Cont.)

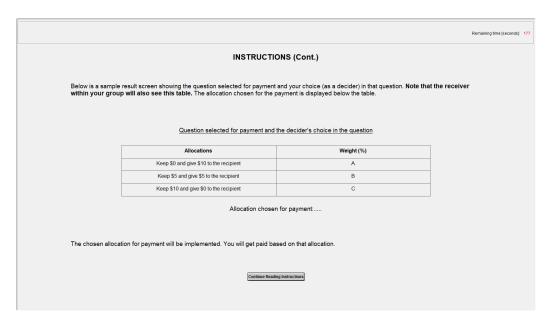


Figure 21: Introduction - Experiment 2 (Cont.)

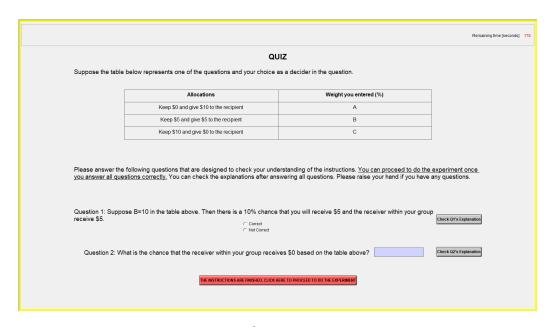


Figure 22: Quiz - Experiment 2

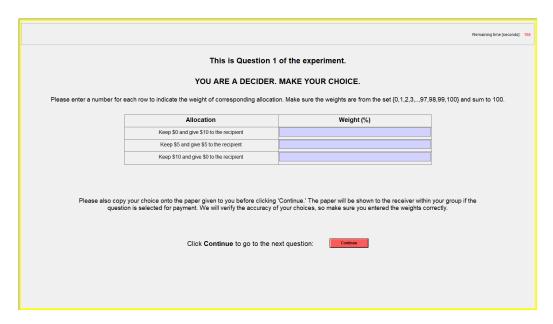


Figure 23: Main question 1 - Experiment 2 (using one specific order)

| | | | Remaining time [seconds]: 177 | | |
|--|---|--|-------------------------------|--|--|
| This is Question 2 of the experiment. | | | | | |
| | YOU ARE A DECIDER. | MAKE YOUR CHOICE. | | | |
| Please enter a number for | each row to indicate the weight of corresponding allocation | on. Make sure the weights are from the set {0,1,2,3,,97, | 98,99,100} and sum to 100. | | |
| | Allocation | Weight (%) | | | |
| | Keep \$5 and give \$5 to the recipient | | | | |
| | Keep \$10 and give \$0 to the recipient | | | | |
| | | | | | |
| | y your choice onto the paper given to you before clicking | | | | |
| quest | ion is selected for payment. We will verify the accuracy of | your choices, so make sure you entered the weights co | rrectly. | | |
| Click Continue to go to the next question: | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |

Figure 24: Main question 2 - Experiment 2 (using one specific order)

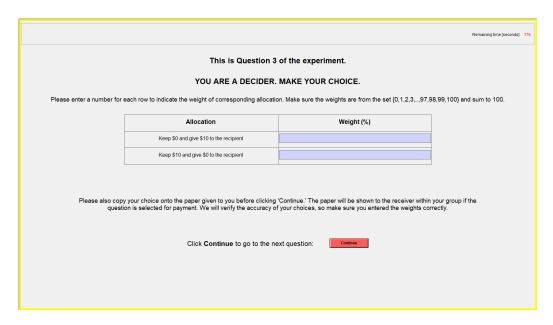


Figure 25: Main question 3 - Experiment 2 (using one specific order)

| | | | Remaining time [seconds]: 169 | | |
|--|---|--|-------------------------------|--|--|
| This is Question 4 of the experiment. | | | | | |
| | YOU ARE A DECIDER. | MAKE YOUR CHOICE. | | | |
| Please enter a number for | each row to indicate the weight of corresponding allocation | on. Make sure the weights are from the set {0,1,2,3,,97, | 98,99,100} and sum to 100. | | |
| | Allocation | Weight (%) | | | |
| | Keep \$0 and give \$10 to the recipient | | | | |
| | Keep \$5 and give \$5 to the recipient | | | | |
| | | | | | |
| | y your choice onto the paper given to you before clicking | | | | |
| quest | ion is selected for payment. We will verify the accuracy of | f your choices, so make sure you entered the weights co | rrectly. | | |
| Click Continue to go to the next question: | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |

Figure 26: Main question 4 - Experiment 2 (using one specific order)

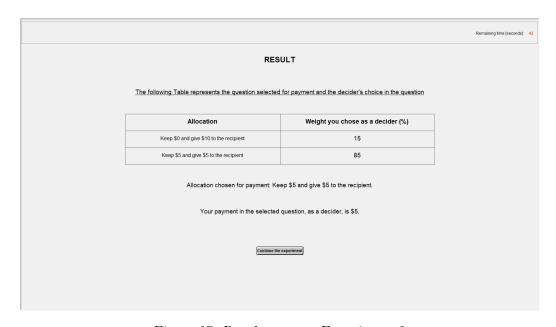


Figure 27: Result screen - Experiment 2

Appendix D Additional Details on Experimental Design

Order of questions in Experiment 2. In each experimental session in Experiment 2, the number of dictators was either 8 (one session) or 16 (six sessions). Each dictator answered four decision problems: $A_1 = \{(0,10), (5,5), (10,0)\}, A_2 = \{(5,5), (10,0)\}, A_3 = \{(0,10), (10,0)\},$ and $A_4 = \{(0,10), (5,5)\}$. The order of questions was randomized at the individual level. We used eight orders of questions as follows:

$$(1) \mathcal{A}_1 - \mathcal{A}_2 - \mathcal{A}_3 - \mathcal{A}_4;$$

$$(2) \mathcal{A}_1 - \mathcal{A}_3 - \mathcal{A}_4 - \mathcal{A}_2;$$

(3)
$$A_2 - A_4 - A_1 - A_3$$
;

(4)
$$A_2 - A_4 - A_3 - A_1$$
;

(5)
$$A_3 - A_2 - A_1 - A_4$$
;

(6)
$$A_3 - A_1 - A_4 - A_2$$
;

(7)
$$A_4 - A_1 - A_2 - A_3$$
;

(8)
$$A_4 - A_3 - A_2 - A_1$$
.

In these eight orders, for all i = 1, 2, 3, 4, decision problem A_i appears as the first, second, third, and fourth questions exactly twice. Additionally, for any i, j with $i \neq j$, decision problem A_i appears immediately before decision problem A_j exactly twice.

Questionnaires. We use the following set of questionnaires for both Experiments 1 and 2. Note that questions 9 and 10 are specific to Experiment 1.

- 1. How old are you?
- 2. What is your gender? Male/Female/Non-binary/Preferred not to say.
- 3. What is your race/ethnicity (Please enter 0 if you prefer not to disclose your race/ethnicity)?
- 4. What year are you in college? Freshman/Sophomore/Junior/Senior.
- 5. What is your current college major? Arts and communication (journalism, fine arts, film and media studies, etc.)/Business and economics (economics, marketing, finance, accounting, etc.)/Health and medical (nursing, pharmacy, public health, physical therapy, etc.)/Science, technology, engineering, and math (biology, computer science, physics, etc.)/Social sciences and humanities (psychology, sociology, political science, english literature, history, etc.)/Undeclared/Others.
- 6. Are you involved in a student or social organization? Please enter 0 if the answer is no. If the answer is yes, please specify what kind of organization.
- 7. When was the last time you donated to or volunteered for a nonprofit or charity organization (501(c) organizations)? Less than 1 week ago/Less than 1 month ago/Between 1 and 3 months ago/Between 3 and 6 months ago/Between 6 and 12 months ago/More than 12 months ago/Never.

- 8. When was the last time that you gave something to someone on the street? Less than 1 week ago/Less than 1 month ago/Between 1 and 3 months ago/Between 3 and 6 months ago/Between 6 and 12 months ago/More than 12 months ago/Never.
- 9. Did you choose more than one allocation with positive weights when you are the decider? If so, how many allocations did you select, and why did you choose many allocations instead of a single allocation? Please enter 0 if you chose one allocation.
- 10. Did you choose more than two allocations with positive weights when you are the decider? If so, why did you choose more than two allocations instead of a single allocation or just two allocations? Please enter 0 if this question does not apply to you.