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Anticipated loser regret in third price auctions

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ABSTRACT

Losing the auction at an affordable price generates loser regret. In third price auctions if bidders anticipate loser regret, then in line with the experimental findings, in a symmetric equilibrium the bids are more than the risk neutral Nash equilibrium.

1. Introduction

In independent-private value auctions, overbidding comparing to risk neutral Nash equilibrium (RNNE) bidding strategy is one of the robust findings of the experimental economics (see e.g. Cox et al., 1982; 1988; and Kagel, 1995 for a detailed survey). It has been proposed that risk aversion can be an explanation for overbidding behavior (see e.g. Cox et al., 1988). However, the literature has not been fully convinced by this explanation (see e.g. Kagel and Levin, 2007, see also Armantiera and Treich, 2009). The most well-known objection to risk aversion explanation is by Kagel and Levin (1993). They showed that in third price auctions although risk aversion implies underbidding, in the experiment the bids were significantly higher than the RNNE.

Following Filiz-Ozbay and Ozbay (2007), we show that anticipation of loser regret[1] —anticipation of a disutility affecting a loser of an auction when he learns that he lost at an affordable price — derives the observed deviations from RNNE predictions in both first and third price auctions. We take an equilibrium approach. Utility function includes a disutility term in addition to monetary utility when a bidder loses the object at an affordable price and we characterize the symmetric equilibrium bidding strategy. This disutility is a function of the difference between his valuation and the third highest bid (price of the object). We show that in the third price auction, overbidding is the equilibrium bidding strategy.

It has been observed in the auction literature that feedback regarding the bids change the bidding behavior (see e.g. Isaac and Walker, 1985; Ockenfels and Selten, 2005). These feedback should not have a direct effect on the risk attitudes and hence the risk aversion explanation is silent on differences between setups with and without feedback. Filiz-Ozbay and Ozbay (2007) showed theoretically that anticipated loser regret leads to overbidding in first price auctions. Furthermore they showed experimentally that bidders indeed anticipated loser regret and reflected this in their bids via overbidding. In Section 2, we analyze third price auctions with regretful bidders and make an equilibrium analysis. Section 3 is the conclusion.

2. Model

There is a single object for sale, and there are n potential bidders, indexed by $i=1,\dots,n$. Bidder i assigns a private value of v_i to the object. Each v_i is independently and identically drawn from $[0,\overline{v}\,]$ according to an increasing distribution function F, and f is the density function corresponding to F. Let the reservation price of the seller be equal to 0.

The object is sold in a third price auction and suppose at the end of the auction, the bidders learn their winning/losing position and also the third highest bid. If bidder i with valuation v_i bids b_i, then his utility with loser regret is

$$u_i(v_i,b_i,b_{-i}) = \begin{cases} v_i-b^2 & \text{if } b_i > b^1 \,(i \text{ wins}) \\ -g\left(v_i-b^2\right) & \text{if } b_i<b^1 \text{ and } b^2<v_i \,(i \text{ loses at an affordable price}) \\ 0 & \text{if } b_i<b^1 \text{ and } b^2\geq v_i \,(i \text{ loses at an unaffordable price}) \end{cases}$$

where b^1 denotes the highest bid among all the bids except bidder i's and b^2 denotes the second highest bid among all the *bids* except

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[1] Regret in auction setting is introduced by Engelbrecht-Wiggans (1989).

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bidder i's. Loser regret is a negative emotion and the bigger the difference between a bidder's value and the third highest bid is, the more loser regret he may feel. So, in line with the loser regret function in Filiz-Ozbay and Ozbay (2007), $g(\cdot): R \rightarrow R_+$ is assumed to be a non-negative, non-decreasing, differentiable real valued function. Moreover assume $g(x)=0$ for all $x \leq 0$, in other words, if a bidder loses and learns that he could not afford the price, i.e. $v \leq b^2$, then there is no loser regret.

Theorem 1. In a third price auction with loser regret, the increasing, symmetric equilibrium bidding strategy satisfies the following condition:

$$b(v) = v + \frac{F(v)}{(n-2)f(v)} + \int_0^{b^{-1}(v)} \frac{g'(v-b(y))f(y)F(y)^{n-3}dy}{f(v)F(v)^{n-3}}$$

for all $v \in (0, \overline{v}]$ and $b(0)=0$.

Proof. Consider any representative bidder motivated by loser regret and participating in a third price auction. Let $b(\cdot)$ be his optimum increasing bidding strategy[2]. The expected utility is

$$EU(v, b(s)) = \int_0^s \int_y^s (v-b(y)) f_{v^1|v^2}(x|y) f_{v^2}(y) dx dy - \int_0^{b^{-1}(v)} \int_s^{\infty} g(v-b(y)) f_{v^1|v^2}(x|y) f_{v^2}(y) dx dy$$

where $f_{v^1}(y)$ is the density of highest valuation among all the valuations except bidder i's, $f_{v^2}(y)$ is the density of second highest valuation among all the valuations except bidder i's, and $f_{v^1|v^2}(x|y)$ is the conditional density. The expected utility above is equal to

$$EU(v, b(s)) = \int_0^s (v-b(y)) \left[F_{v^1|v^2}(s|y) - F_{v^1|v^2}(y|y)\right] f_{v^2}(y) dy - \int_0^{b^{-1}(v)} g(v-b(y)) \left[1 - F_{v^1|v^2}(s|y)\right] f_{v^2}(y) dy$$

The first order condition should hold at $s=v$:

$$\int_0^s (v-b(y)) f_{v^1|v^2}(s|y) f_{v^2}(y) dy + \int_0^{b^{-1}(v)} g(v-b(y)) f_{v^1|v^2}(s|y) f_{v^2}(y) dy|_{s=v} = 0$$

Since $f_{v^1|v^2}(s|y) f_{v^2}(y) = f_{v^2|v^1}(y|s) f_{v^1}(s)$, the first order condition becomes

$$\int_0^s (v-b(y)) f_{v^2|v^1}(y|s) f_{v^1}(s) dy + \int_0^{b^{-1}(v)} g(v-b(y)) f_{v^2|v^1}(y|s) f_{v^1}(s) dy|_{s=v} = 0$$

By substituting $f_{v^2}|_{v^1}(y|s) = \frac{(n-2)f(y)F(y)^{n-3}}{F(s)^{n-2}}$ (see e.g. David, 1980) and dividing both sides by $f_{v^1}(s)$:

$$\int_0^v (v-b(y)) f(y) F(y)^{n-3} dy + \int_0^{b^{-1}(v)} g(v-b(y)) f(y) F(y)^{n-3} dy = 0$$

take the derivative with respect to v

$$(v-b(v)) f(v) F(v)^{n-3} + \int_0^v f(y) F(y)^{n-3} dy + \int_0^{b^{-1}(v)} g'(v-b(y)) f(y) F(y)^{n-3} dy = 0$$

since $g(0)=0$. Then

$$(v-b(v)) f(v) F(v)^{n-3} + \frac{F(v)^{n-2}}{n-2} + \int_0^{b^{-1}(v)} g'(v-b(y)) f(y) F(y)^{n-3} dy = 0$$

$$b(v) = v + \frac{F(v)}{(n-2)f(v)} + \frac{1}{f(v)F(v)^{n-3}} \int_0^{b^{-1}(v)} g'(v-b(y)) f(y) F(y)^{n-3} dy$$

Also $b(0)=0$ otherwise the bidder's expected utility would be less than 0 (since $g(\cdot)$ is non-negative) which is less than bidding 0. □

The symmetric risk neutral Nash equilibrium bidding strategy of third price auction is $b^{RN}(v) = v + \frac{F(v)}{(n-2)f(v)}$ (see Monderer and Tennenholtz, 2000). Therefore, the equilibrium bidding behavior with anticipated loser regret can be written as:

$$b(v) = b^{RN}(v) + \int_0^{b^{-1}(v)} \frac{g'(v-b(y)) f(y) F(y)^{n-3}}{f(v) F(v)^{n-3}} dy$$

Since g is assumed to be non-decreasing, i.e. $g' \geq 0$, in a third price auction, loser regret leads to overbidding:

Corollary 1. The symmetric equilibrium bidding strategy in a third price auction with loser regret is higher than the symmetric risk neutral Nash equilibrium bidding strategy. i.e. $b(v) \geq b^{RN}(v)$ for all $v \in [0, \overline{v}]$.

3. Conclusion

In auctions, learning the winning/losing position and price of the object leads to anticipation of loser regret (see the experiment of Filiz-Ozbay and Ozbay, 2007). Indeed, in the third price auction experiment of Kagel and Levin (1993), subjects knew that at the end of the auction all the bids were going to be announced. Therefore, it is plausible to think that overbidding in their experiment is driven by anticipated loser regret.

In this paper we theoretically demonstrate that overbidding in third price auctions can be explained by anticipated loser regret. Together with the results of Filiz-Ozbay and Ozbay (2007) on first price auctions, we conclude that bidders do not want to leave the auction with empty hands if they could have done better. To avoid that feeling, they overbid in first and third price auctions.

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References

Armantiera, O., Treich, N., 2009. Star-shaped probability weighting functions and overbidding in first-price auctions. Economics Letters 104, 83–85.

Cox, J.C., Roberson, B., Smith, V.L., 1982. Theory and behavior of single object auctions. In: Smith, V.L. (Ed.), Research in Experimental Economics, vol. 2. JAI Press, Greenwich, Connecticut, pp. 1–43.

Cox, J.C., Smith, V.L., Walker, J.M., 1988. Theory and individual behavior of first-price auctions. Journal of Risk and Uncertainty 1, 61–99.

David, H.A., 1980. A. Order Statistics. John Wiley and Sons, Inc.

[2] Since we consider the symmetric equilibrium, the identity index of bidder can be dropped. The proof technique is similar to finding RNNE for the third price auctions (see e.g. Krishna, 2002).

Engelbrecht-Wiggans, R., 1989. The effect of regret on optimal bidding in auctions. Management Science 35, 685–692.
Filiz-Ozbay, E., Ozbay, E.Y., 2007. Auctions with anticipated regret: theory and experiment. American Economic Review 97, 1407–1418.
Isaac, R.M., Walker, J.M., 1985. Information and conspiracy in sealed bid auctions. Journal of Economic Behavior and Organization 6, 139–159.
Kagel, J.H., 1995. Auctions: a survey of experimental research. In: Kagel, J.H., Roth, A.E. (Eds.), The Handbook of Experimental Economics. Princeton University Press, Princeton), pp. 501–585.
Kagel, J.H., Levin, D., 1993. Independent private value auctions: bidder behavior in first-, second- and third-price auctions with varying number of bidders. Economic Journal 103, 868–879.
Kagel, J.H., Levin, D., 2007. Auctions: Experiments (for New Palgrave Dictionary of Economics).
Krishna, V., 2002. Auction Theory. Academic Press, San Diego.
Monderer, D., Tennenholtz, M., 2000. k-Price auctions. Games and Economic Behavior 31, 220–244.
Ockenfels, A., Selten, R., 2005. Impulse balance equilibrium and feedback in first price auctions. Games and Economic Behavior 51, 155–170.