1 Overview

The programs made available on this web page are sample programs for the computation of estimators introduced in Kelejian and Prucha (2004). In particular we provide sample programs for the estimation of the following system of spatially interrelated cross sectional equations corresponding to n cross sectional units:

$$Y_n = Y_n B + X_n C + \overline{Y}_n \Lambda + U_n, \tag{1}$$

with

$$\begin{aligned} Y_n &= (y_{1,n}, ..., y_{m,n}), \\ X_n &= (x_{1,n}, ..., x_{k,n}), \\ U_n &= (u_{1,n}, ..., u_{m,n}), \\ \overline{Y}_n &= (\overline{y}_{1,n}, ..., \overline{y}_{m,n}), \\ \overline{y}_{j,n} &= W_n \; y_{j,n}, \quad j = 1, ..., m, \end{aligned}$$

where $y_{j,n}$ is the $n \times 1$ vector of cross sectional observations on the dependent variable in the j-th equation, $x_{l,n}$ is the $n \times 1$ vector of cross sectional observations on the l-th exogenous variable, $u_{j,n}$ is the $n \times 1$ disturbance vector in the j-th equation, W_n are $n \times n$ spatial weighting matrices of known constants, and B, C and Λ are correspondingly defined parameter matrices of dimension $m \times m$, $k \times m$ and $m \times m$, respectively. The vector $\overline{y}_{j,n}$ is typically referred to as a spatial lag of $y_{j,n}$. We note that although not explicitly shown, some of the exogenous variables can be spatial lags of other exogenous variables.

In addition to allowing for general spatial lags in the endogenous and exogenous variables we also allow for spatial autocorrelation in the disturbances. In particular, the disturbances are assumed to be determined by the following first-order autoregressive process:

$$U_n = \overline{U}_n R + E_n, \tag{2}$$

with

$$\begin{aligned} E_n &= (\varepsilon_{1,n}, ..., \varepsilon_{m,n}), \\ R &= diag_{j=1}^{m}(\rho_j), \\ \overline{U}_n &= (\overline{u}_{1,n}, ..., \overline{u}_{m,n}), \\ \overline{u}_{j,n} &= W_n u_{j,n}, \quad j = 1, ..., m, \end{aligned}$$

where $\varepsilon_{j,n}$ denotes the $n \times 1$ vector of innovations and ρ_j the spatial autoregressive parameter in the j-th equation. The vector $\overline{u}_{j,n}$ is typically referred to as the spatial lag of $u_{j,n}$. A set of explicit assumptions is given in the paper. Given those assumptions

$$E\varepsilon_{j,n} = 0, \qquad E\varepsilon_{j,n}\varepsilon'_{k,n} = \sigma_{jk} I_n.$$

Using obvious notation, let $Z_{j,n} = (Y_{j,n}, X_{j,n}, \overline{Y}_{j,n})$ denote the matrix of observations of right hand side variables that appear in the j-th equation, and let $\delta_j = (\beta'_j, \gamma'_j, \lambda'_j)'$ denote the corresponding parameter vector. We can then rewrite the j-th equation in (1) and (2) as

$$\begin{aligned} y_{j,n} &= Z_{j,n}\delta_j + u_{j,n}, \\ u_{j,n} &= \rho_j W_n u_{j,n} + \varepsilon_{j,n}. \end{aligned} \tag{3}$$

Applying a Cochrane-Orcutt transformation to this model yields furthermore

$$y^*_{j,n}(\rho_j) = Z^*_{j,n}(\rho_j)\delta_j + \varepsilon_{j,n}, \tag{4}$$

where

$$\begin{aligned} y^*_{j,n}(\rho_j) &= y_{j,n} - \rho_j W_n y_{j,n}, \\ Z^*_{j,n}(\rho_j) &= Z_{j,n} - \rho_j W_n Z_{j,n}. \end{aligned}$$

Stacking the equations yields

$$y^*_n(\rho) = Z^*_n(\rho)\delta + \varepsilon_n \tag{5}$$

where

$$\begin{aligned} y^*_n(\rho) &= (y^*_{1,n}(\rho_1)', ..., y^*_{m,n}(\rho_m)')', \\ Z^*_n(\rho) &= diag^m_{j=1}(Z^*_{j,n}(\rho_j)), \\ \varepsilon_n &= (\varepsilon'_{1,n}, ..., \varepsilon'_{m,n})', \end{aligned}$$

and $\rho = (\rho_1, \ldots, \rho_m)'$ and $\delta = (\delta'_1, \ldots, \delta'_m)'$. Clearly $E\varepsilon_n\varepsilon'_n = \Sigma \otimes I_n$ where $\Sigma = (\sigma_{jk})$.

Note: We provide two sets of sample programs, one for TSP and one for Stata. In the following we only describe the use of the TSP programs. The use of the Stata files is analogous.

2 Data Files

The sample program involves two exogenous variables per equation and two simultaneous equations and an idealized spatial weighting matrix W_n. This matrix corresponds to the case where each unit has "one neighbor ahead and one neighbor behind" in a wrap around world, and the row sums of the weighting matrix are normalized to one; for a more detailed description of this idealized matrix see the Monte Carlo section of Kelejian and Prucha (1999). The sample size is taken to be 100. The actual estimation programs assume that the data for the exogenous variables and spatial weighting matrix are stored in files named VAR4.DAT and MMAT.DAT, respectively.

3 Estimation Programs

The main estimation program is contained in the file PROGRAM4.TSP. This program calls three "subroutines" contained in the files GMPROC1.TSP, TSLSPROC4.TSP and FGS3SLSPROC.TSP. Those subroutines compute the GM estimator for ρ, the Generalized Spatial 2SLS (GS2SLS) estimator for β and λ, and the Generalized Spatial 3SLS (GS3SLS) estimator for β and λ, respectively.

The program PROGRAM4.TSP first reads in the data for the exogenous variables and spatial weighting matrix from the files VAR4.DAT and MMAT.DAT. The actual estimation of the parameters of the model (1)-(2) is performed in three steps for the GS2SLS estimator and four steps for the GS3SLS estimator.

Step 1: In the first step we estimate the model parameters δ_j from (3) by applying 2SLS applied to each equation:

$$\widetilde{\delta}_{j,n} = (\widetilde{Z}'_{j,n}Z_{j,n})^{-1}\widetilde{Z}'_{j,n}y_{j,n}, \tag{6}$$

where $\widetilde{Z}_{j,n} = P_H Z_{j,n} = (\widetilde{Y}_{j,n}, X_{j,n}, \widetilde{\overline{Y}}_{j,n})$, and $\widetilde{Y}_{j,n} = P_H Y_{j,n}$, $\widetilde{\overline{Y}}_{j,n} = P_H \overline{Y}_{j,n}$, $P_H = H_n(H_n' H_n)^{-1} H_n'$, and where H_n is the matrix of instruments which is formed as a subset of linearly independent columns of $(X_n, W_n X_n, W_n^2 X_n, ...)$; see the paper for more details. Based on $\widetilde{\delta}_{j,n}$ we can compute 2SLS residuals:

$$\widetilde{u}_{j,n} = y_{j,n} - Z_{j,n}\widetilde{\delta}_{j,n}. \tag{7}$$

Step 2: In the second step the spatial autoregressive parameters ρ_j are estimated in terms of the residuals obtained via the first step and the generalized moments procedure suggested in Kelejian and Prucha (1999). More specifically the estimators of ρ_j and σ_{jj}, $\widetilde{\rho}_{j,n}$ and $\widetilde{\sigma}_{jj,n}$ respectively are defined as the nonlinear least squares estimators that minimize

$$\left[g_{j,n} - G_{j,n} \begin{bmatrix} \rho_j \\ \rho_j^2 \\ \sigma_{jj} \end{bmatrix} \right]' \left[g_{j,n} - G_{j,n} \begin{bmatrix} \rho_j \\ \rho_j^2 \\ \sigma_{jj} \end{bmatrix} \right], \tag{8}$$

where

$$G_{j,n} = \frac{1}{n} \begin{bmatrix} 2\widetilde{u}_{j,n}' \widetilde{\overline{u}}_{j,n} & -\widetilde{\overline{u}}_{j,n}' \widetilde{\overline{u}}_{j,n} & n \\ 2\widetilde{\overline{u}}_{j,n}' \widetilde{\overline{\overline{u}}}_{j,n} & -\widetilde{\overline{\overline{u}}}_{j,n}' \widetilde{\overline{\overline{u}}}_{j,n} & Tr(W_n' W_n) \\ \widetilde{u}_{j,n}' \widetilde{\overline{\overline{u}}}_{j,n} + \widetilde{\overline{u}}_{j,n}' \widetilde{\overline{u}}_{j,n} & -\widetilde{\overline{u}}_{j,n}' \widetilde{\overline{\overline{u}}}_{j,n} & 0 \end{bmatrix}, \qquad g_{j,n} = \frac{1}{n} \begin{bmatrix} \widetilde{u}_{j,n}' \widetilde{u}_{j,n} \\ \widetilde{\overline{u}}_{j,n}' \widetilde{\overline{u}}_{j,n} \\ \widetilde{u}_{j,n}' \widetilde{\overline{u}}_{j,n} \end{bmatrix},$$

and $\widetilde{\overline{u}}_{j,n} = W_n \widetilde{u}_{j,n}$, and $\widetilde{\overline{\overline{u}}}_{j,n} = W_n^2 \widetilde{u}_{j,n}$. The code for computing the GM estimators is contained in the file GMPROC1.TSP

Step 3: In the third step the GS2SLS estimator is computed from the transformed regression model in (4), with ρ_j replaced by $\widetilde{\rho}_{j,n}$, as follows:

$$\widehat{\delta}_{j,n}^F = \left[\widehat{Z}_{j,n}^{*\prime}(\widetilde{\rho}_{j,n}) Z_{j,n}^*(\widetilde{\rho}_{j,n}) \right]^{-1} \widehat{Z}_{j,n}^{*\prime}(\widetilde{\rho}_{j,n}) y_{j,n}^*(\widetilde{\rho}_{j,n}), \tag{9}$$

where $\widehat{Z}_{j,n}^*(\widetilde{\rho}_{j,n}) = P_H Z_{j,n}^*(\widetilde{\rho}_{j,n})$, $Z_{j,n}^*(\widetilde{\rho}_{j,n}) = Z_{j,n} - \widetilde{\rho}_{j,n} W_n Z_{j,n}$, $y_{j,n}^*(\widetilde{\rho}_{j,n}) = y_{j,n} - \widetilde{\rho}_{j,n} W_n y_{j,n}$. This procedure is executed in TSLSPROC4.TSP.

Step 4: In the fourth step the GS3SLS estimator is computed from the transformed regression model in (5), with ρ_j replaced by $\widetilde{\rho}_{j,n}$, as follows:

$$\check{\delta}_n^F = \left[\widehat{Z}_n^*(\widetilde{\rho}_n)' \left(\widehat{\Sigma}_n^{-1} \otimes I_n \right) Z_n^*(\widetilde{\rho}_n) \right] \widehat{Z}_n^*(\widetilde{\rho}_n)' \left(\widehat{\Sigma}_n^{-1} \otimes I_n \right) y_n^*(\widetilde{\rho}_n), \tag{10}$$

where $\widehat{\Sigma}_n$ is estimated as a $m \times m$ matrix whose (j,l)-th element is $\widehat{\sigma}_{jl,n} = n^{-1} \widetilde{\varepsilon}_{j,n}' \widetilde{\varepsilon}_{l,n}$ where $\widetilde{\varepsilon}_{j,n} = y_{j,n}^*(\widetilde{\rho}_{j,n}) - Z_{j,n}^*(\widetilde{\rho}_{j,n}) \widehat{\delta}_{j,n}^F$. This procedure is executed in FGS3SLSPROC.TSP

References

[1] Kelejian, H. H. and I. R. Prucha, (2004): "Estimation of Simultaneous system of spatially interrelated cross sectional equations," *Journal of Econometrics*, Vol 118, 27-50.

[2] Kelejian, H. H. and I. R. Prucha, (1998): "A Generalized Spatial Two-Stage Least Squares Procedure for Estimating a Spatial Autoregressive Model with Autoregressive Disturbances," *Journal of Real Estate Finance and Economics,* Vol. 17:1, 99-121.

[3] Kelejian, H. H. and I. R. Prucha, (1999): "A Generalized Moments of Estimator for the Autoregressive Parameter in a Spatial Model," *International Economic Review,* 40, 509-533.