

ECON 602 - Macroeconomic Analysis II

Comprehensive Exam

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Prof. Borağan Aruoba

Note that the timing convention of these questions are different from the one adopted in Spring 2008 and later.

1. (a) (15 points) (**Households**) Consider the following problem of a household who takes as given prices $\{p_t, w_t, R_t\}_{t=0}^{\infty}$ and taxes $\{\tau_t^h\}_{t=0}^{\infty}$

$$\max_{\{c_t, h_t, m_t, b_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - h_t, z_t) \quad (1)$$

$$z_t \equiv \frac{m_{t+1}}{p_t} \quad (2)$$

$$p_t c_t + b_{t+1} + m_{t+1} \leq (1 - \tau_t^h) p_t w_t h_t + R_{t-1} b_t + m_t \quad (3)$$

$$\lim_{z \rightarrow \bar{z}} u_3(c, 1 - h, z) = 0 \text{ for } \bar{z} < \infty \quad (4)$$

$$u_1(c, 1 - h, z) > 0 \text{ for all } z \quad (5)$$

where b_t is the stock of a one-period nominal bond that issued by the government and held by the households and R_t is its gross nominal interest which is paid in $t + 1$. Use subscripts 1,2 and 3 to refer to marginal utilities with respect to consumption, leisure and real money balances.

- i. (10 points) Write down the conditions that characterize the solution to the household's problem.
 - ii. (5 points) What is the implication of (4) for the solution to the household's problem and therefore the equilibrium?
- (b) (5 points) (**Firms**) A Neoclassical firm has access to production function

$$Y_t = H_t \quad (6)$$

Write down the problem of the firm and solve it.

- (c) (20 points) (**Equilibrium**) There is also a government in this economy with a budget constraint

$$p_t G_t + R_{t-1} B_t + M_t = \tau_t^h p_t w_t H_t + B_{t+1} + M_{t+1} \quad (7)$$

where $\{G_t\}_{t=0}^{\infty}$ is a **deterministic** and known sequence of government expenditures that needs to be financed. Using all the information so far, carefully define the equilibrium. Your definition must be self-contained. At the expense of being redundant, write down all the equations that need to be in this definition. (No multipliers should appear in these definitions.)

- (d) (15 points) (**Ramsey Planner's Problem**) Write down the Ramsey planner's problem with the objective function, things that he chooses, constraints and multipliers carefully specified. (It is very important that you remember your answer in question 1b above.) Do not proceed further, just write down the problem. [You do not need to derive the implementability constraint, unless you do not remember how it should look like.]
- (e) (45 points) (**Optimal Policy**) Assume the utility function takes the form

$$u(c, 1 - h, z) = cz + (1 - h) \tag{8}$$

- i. (15 points) Solve the Ramsey problem in question 4. You need to write down all the equations and inequalities that characterize the solution. Do not proceed any further in this part.
- ii. (10 points) What is the optimal interest rate that comes out of the Ramsey problem?
- iii. (10 points) What is the optimal labor income tax that comes out of the Ramsey problem?
- iv. (10 points) State the "uniform taxation" result as formally and generally as you can and relate it to your findings above.