

ECON 602 - Macroeconomic Analysis II

Solutions to Comprehensive Exam

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Prof. Borağan Aruoba

Note that the timing convention of these questions are different from the one adopted in Spring 2008 and later.

3. Here is the description of the economy. Use the common framework for all the models we discussed in the lectures with the following changes:

- There is no capital in the economy. Just like the models we worked with before, there are two assets in the economy: a one-period nominal bond that is issued by the government and held by the households with gross nominal return R_t and fiat money. The beginning-of-the-period stocks are denoted by (b_t, m_t) .
- A neoclassical firm has a constant-returns-to-scale production function.
- When an agent consumes c_t units of consumption goods when he has m_{t+1}/p_t units of real money balances (end-of-period, as usual) he incurs a **time** loss of $\phi_t \equiv \phi(c_t, m_{t+1}/p_t)$. All we know about the function ϕ is that $\phi(c, z) > 0$, $\phi(0, z) = 0$, $\phi_1 > 0$ and $\phi_2 < 0$. This is the **key** difference in this question.
- The agent gets utility from consumption and leisure, given by $u(c, \ell)$.
- There is also a government in this economy with a standard budget constraint using income from a labor income tax (τ_t^h), printing money and issuing bonds and spending G_t every period, where $\{G_t\}_{t=1}^\infty$ is a deterministic and known sequence. To be clear, there are no lump-sum taxes.

1. (a) (25 points) (**Households**)

- (5 points) Write down the problem of the representative household in this economy, being careful about what it chooses and what it takes as given. Do not solve this problem yet, just state the problem.
- (15 points) Write down the conditions that characterize the solution to this problem. (No multipliers should appear here.)
- (5 points) Write down a condition (derived from the conditions above) that summarize the money demand decision on the margin and briefly interpret it. (MB=MC)

(b) (3 points) (**Firm**) Write down the problem of the firm and solve it.

(c) (2 points) (**Government**) Write down the budget constraint of the government.

- (d) (15 points) (**Equilibrium**) Using all the information so far, carefully define the equilibrium. Your definition must be self-contained. [At the expense of being redundant, write down all the equations that need to be in this definition. Don't worry about simplifying the equations further.]
- (e) (20 points) (**Ramsey Planner's Problem**) Write down the Ramsey planner's problem with the objective function, things that he chooses, constraints and multipliers carefully specified. Do not proceed further, just write down the problem. [Hint: In order to get the implementability constraint, think about the analogy between this model and a MIU model. This is an important part of this question.]
- (f) (35 points) (**Optimal Policy**) Assume the following functional forms

$$u(c, \ell) = c\ell$$

$$\phi(c, z) = \gamma\left(\frac{z}{c}\right)c$$

where $\gamma(\cdot)$ is a function such that $\gamma \geq 0$, $\gamma' < 0$ and $\gamma'' > 0$ (note the strict inequalities).

- i. (5 points) What property does the $\phi(c, z)$ function has with the given restriction?
- ii. (15 points) Solve the Ramsey problem in part (e). You need to write down all the equations and inequalities that characterize the solution. Do not proceed any further in this part.
- iii. (15 points) Is the Friedman Rule optimal? If so, prove. If not, find additional restrictions on the functions that would make it optimal.
- iv. (BONUS) Link the result in (iii) to the results you know from the MIU model. [This is optional. Attempt to answer this only if you have time.]