

ECON 602 - Macroeconomic Analysis II

Comprehensive Exam

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3. Consider the following modification to the real model used in Chari and Kehoe (1999). In addition to labor and capital, there is a third factor of production, labeled L_t , which also enters utility. We assume this factor is supplied elastically and we normalize the total available stock to unity. To be specific, we modify the utility function of the representative household as

$$U(c, 1 - h, 1 - L) \equiv \log(c) + \log(1 - h) + \log(1 - L)$$

and the production function is $F(K, H, L)$, where the technology shock is subsumed in $F(\cdot)$. The third factor of production brings in a factor payment denoted by s_t and we denote the factor payments for capital and labor by r_t and w_t . Throughout the question, we assume that the factor income from L cannot be taxed, while the income from the other two factors can. The setup is otherwise entirely standard. (Two shocks, state-contingent bond issued by the government etc.) Use U_i and F_i to denote partial derivatives. [This question does not require a lot of complicated derivations. If you find yourself doing that, re-evaluate your strategy.]

- a. **(10 points)** Solve the household's problem, being very clear about things that it takes as given and things that it chooses.
- b. **(5 points)** Solve the firm's problem.
- c. **(15 points)** Carefully define a competitive equilibrium. In defining the equilibrium, you can refer to equations you derived above. After you define the equilibrium, on one page list and number all the equations you need to characterize a competitive equilibrium, minimizing the number of equations (and variables).

For the next two questions, you need to write Propositions which you will prove. The statements should read as "Under assumptions X, Y will happen." Since you don't know what results you will get, leave some empty space for the statement of the Proposition and work on the proof. Once you're done with the proof, the statement of the Proposition will hopefully be obvious. While you need a correctly stated proposal and proof for full credit, having the correct derivations will earn you substantial credit.

The PVIC you will need for this problem is

$$E_0 \sum_{t=0} [U_{1,t}C_t - U_{2,t}H_t - U_{3,t}L_t] = A_0$$

- d. **(35 points)** Think about solving a “naive” (or standard) Ramsey problem. Show **how/if** one can implement the Ramsey allocations **using the set of policy tools available to the government**. [Hint: This is the problem one would solve without regard to the completeness of the tax system.]
- e. **(35 points)** Determine the sign (positive/negative/zero) of the long-run capital income tax rate under the Ramsey policy, **implemented using available tools**.
- f. To think about (answer only if you have time): What would happen to the optimal capital income tax rate if we did not allow labor income to be taxed in the standard model? [Hint: You don't need any derivations.]