Sketch of Solution to My Macro Comps Question, August 2008

Allan Drazen

(a) Budget constraints:

$$\begin{aligned} c_t^y + s_t &= (1-\tau)\, w_t h_t \\ c_{t+1}^o &= (1 + (1-\tau)\, r_{t+1})\, s_t \end{aligned}$$

NOTE that labor and capital *income* are taxed

Individual choice problem:

$$\max_{s_t} \ln\left((1-\tau)\, w_t h_t - s_t\right) + \beta \ln\left((1 + (1-\tau)\, r_{t+1})\, s_t\right)$$

Firm choice problem:

$$\max_{k_t, n_t} A k_t^{\alpha} (n_t h_t)^{1-\alpha} - w_t n_t h_t - r_t k_t$$

(b) Definition of competitive equilibrium

Given initial k_o and h_0, a competitive equilibrium for this economy is a sequence of allocations $\left\{n_t, k_t, h_t, y_t, c_t^y, c_{t+1}^o\right\}_{t=0}^{t=\infty}$ and factor prices $\{w_t, r_{t+1}\}_{t=0}^{t=\infty}$ such that

(i) given w_t and r_{t+1}, the allocation $\left(c_t^y, c_{t+1}^o\right)$ solves the generation t young agent's problem;

(ii) given w_t and r_t, the allocation (n_t, k_t, y_t) maximizes the representative firm's profits subject to the production technology;

(iii) $k_{t+1} = s_t$ and $n_t = 1$;

(iv) h_t evolves according to $h_{t+1} = B \ldots$

(v)$E_t = \tau\,(w_t h_t + r k_t)$.

(c) Evolution of k_t and h_t:

Individual FOC: $s_t = \frac{\beta}{1+\beta}(1-\tau)\, w_t h_t$. Since $w_t h_t = (1-\alpha)\, A k_t^{\alpha} h_t^{1-\alpha}$, we have:

$$k_{t+1} = \frac{\beta}{1+\beta}(1-\tau)(1-\alpha)\, A k_t^{\alpha} h_t^{1-\alpha} \tag{1}$$

Also, since $w_t h_t + r k_t = A k_t^{\alpha} h_t^{1-\alpha}$, we have $E_t = \tau A k_t^{\alpha} h_t^{1-\alpha}$, so that

$$h_{t+1} = B \tau^{1-\mu} k_t^{\alpha(1-\mu)} h_t^{1-\alpha+\alpha\mu} \tag{2}$$

(1) and (2) are dynamic equations. They imply

$$x_{t+1} \equiv \frac{h_{t+1}}{k_{t+1}} = \frac{(1+\beta)\, B \tau^{1-\mu}}{\beta (1-\tau)(1-\alpha)\, A^{\mu}} \left(\frac{h_t}{k_t}\right)^{\alpha\mu}$$

Clearly, this ratio converges monotonically to a unique steady state, call it x^*, which is given by

$$\ln x^* = \frac{1}{1-\alpha\mu} \ln\left(\frac{(1+\beta)\, B}{\beta(1-\alpha)\, A^{\mu}}\right) + \frac{1-\mu}{1-\alpha\mu} \ln \tau - \frac{1}{1-\alpha\mu} \ln(1-\tau) \tag{3}$$

(d) Using (1) to find k_{t+1}/k_t, one may write the growth rate as

$$\gamma = \ln \frac{\beta (1-\alpha) A}{(1+\beta)} + \ln (1-\tau) + (1-\alpha) \ln x^* \tag{4}$$

Substituting (3) into (4) and differentiating with respect to τ, the tax rate that maximizes the steady state growth rate is $\tau = 1 - \alpha$, the elasticity of output with respect to human capital.

(e) Conceptually, the answer is yes if the government can commit to a sequence of tax rates. But, if the tax rate is chosen every year, there is a time inconsistency problem. Choosing at the beginning of each generation is like committing.

(f) Human capital accumulation is the "engine of growth" here. Accumulation of h_t causes the production function in terms of k_t to shift up over time, unlike the Diamond model. Hence, this is an endogenous growth model. If h_t did not cause this to occur, for whatever reason, the dynamics in this economy would look similar to the Diamond model.