SOLUTION Question 3 Comps 8/2005 – Allan Drazen

(a) The key point in this problem is that growth is determined by the allocation of labor L_a to the ideas sector, but unlike the simple example in class, this will be determined by equalizing wages across the two competitive sectors, ideas and final goods, as well as the monopolistic behavior of the intermediate goods sector. (There are a few "tip-offs" that the intermediate sector is not competitive: modeling it as competitive makes it impossible to determine the size of the final and intermediate goods sectors, hence L_Y and L_a; also, it should remind one of Dixit-Stiglitz or of specialized goods.)

When the ideas sector produces a "blueprint" for a new intermediate good, this is sold (or rented) to a new intermediate goods producer, who has a monopoly position in that good and hence can set the price for it that final goods producers must pay. Let final good be numeraire. The final goods is competitive. Let profits be

$$\Pi_y = L_y^{1-\alpha} \sum_{j=1}^{A} x_j^{\alpha} - w_y L_y - \sum_{j=1}^{A} p_j x_j$$

where w_y is the wage in the Y sector and p_j is price of intermediate good j, which is set by the monopolistic producer. Maximizing with respect to inputs yields FOC and factor demands

$$\frac{\partial \Pi_y}{\partial L_y} = 0 \implies L_y = \frac{(1-\alpha)\, Y}{w_y}$$

and

$$\frac{\partial \Pi_y}{\partial x_j} = 0 \implies p_j\left(x_j\right) = \alpha x_j^{\alpha-1} L_y^{1-\alpha} \qquad (1)$$

where the latter is represented as an inverse demand function.

Intermediate goods producer j chooses x_j to maximize profits:

$$\Pi_{x_j} = p_j\left(x_j\right) x_j - \eta x_j$$

(this formulation differs from Romer's original formulation). Using (1), this yields a price $p = \eta/\alpha$ for each intermediate and associated quantity $x = \left(\frac{\alpha^2}{\eta}\right)^{\frac{1}{1-\alpha}} L_y$ and monopoly profit $\pi = (\frac{1}{\alpha} - 1)px$.

The "ideas" sector is competitive with free entry, where labor input produces new idea according to $\dot{A} = \delta A L_a$. Profits are

$$\Pi_a = P_a \delta A L_a - w_a L_a$$

where P_a is the price of a new blueprint. Profit-maximizing choice of L_a implies

$$w_a = P_a \delta A$$

P_a is set to extract profits from the buyer of the blueprint, which for simplicity one can assume is infinitely lived, that is, $P_a = \frac{(\frac{1}{\alpha}-1)px}{r}$.

(b) Labor allocation is given by equating w_a and w_y. We may write P_a as

$$P_a = \frac{(\frac{1}{\alpha} - 1)px}{r} = \frac{1-\alpha}{r} x^{\alpha} L_y^{1-\alpha}$$

Using the equations for w_a and w_y, one obtains for $w_a = w_y$:

$$A\delta\frac{1-\alpha}{r}x^a L_y^{1-\alpha} = (1-\alpha)\, L_y^{-\alpha} A x^{\alpha} \tag{2}$$

$$L_y = \frac{r}{\delta} \text{ so that } L_a = \bar{L} - \frac{r}{\delta}$$

(Note that CRTS in final output would not tie down scale of the firm. Something else must determine the division of labor between the final output and the "ideas" sector.)

We may write the production function for final goods as

$$\begin{aligned} Y &= L_y^{1-\alpha}\sum_{j=1}^{A}\left(\left(\frac{\eta}{\alpha^2}\right)L_y\right)^{\alpha} \\ &= A\left(\frac{\alpha^2}{\eta}\right)^{\frac{\alpha}{1-\alpha}} L_Y \end{aligned}$$

so that when the effect of endogenous technical progress is taken into account, we no longer have decreasing returns. "Endogenous growth" could refer to either endogenous technical progress or to the fact that growth can continue even though factors show decreasing returns.

The rate of growth of the economy is simply:

$$\gamma = \frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} = \delta L_a$$

(c) Intermediate good seller faces same price, so he still sets $p = \eta/\alpha$. Monopoly profits are higher as are P_a and w_a. However, so is w_y, and from (2), L_y and L_a are unchanged, so that the growth rate is unchanged. Demand for intermediates by final goods producers is now $x = \left(\frac{\alpha^2}{(1-\theta_x)\eta}\right)^{\frac{1}{1-\alpha}} L_y$ is higher. The subsidy raises demand for both inputs by the final goods sector and for labor by the ideas sector. Since w_y and w_y increase by the same proportion, labor allocation and growth are unchanged.

(d) $p = \eta/\alpha$, monopoly profits, P_a, w_a, and w_y are all unchanged. No effect at all. The key is that the intermediate goods sector, the monopolistic sector, determines what is happening.

(e) Now, given the fall in effective w_a in the ideas sectors, their demand for labor is higher. Using (2), one finds that now $L_y = (1-\theta_a)\delta/r < \delta/r$. L_a rises, as does the growth rate. and growth are unchanged.

(f) The key point is that to increase the rate of growth the subsidy must work directly on the demand for labor by the ideas sector. The government has an incentive to do so because the market solution delivers growth below the social optimum.

(g) Go back to the discussion of investment in class. With CRTS in final output, the scale of the firm is not tied down. Internal adjustment by themselves do so. One could solve for the demand for inputs by this kind of firm and go from there.