Solution to Drazen's question in January 2005 Comp

NOTE: Given that this is a "report" question, there are several ways of answering this question. Below is a draft of a possible answer.

PART I

Overlapping generations models are based on the fact that agents live for a given number of periods, after which they die. New agents are also entering the economy. To simplify the exposition, I will assume that agents live two periods and they only work in the first period of their lives. The objective of the agents is to maximize a utility function of the form $U(c_t^{young}, c_{t+1}^{old})$ subject to $c_t^{young} = w_t l_t - s_t$

$$c_{t+1}^{old} = (1+r_{t+1}) s_t$$

Given that the agent only work in the first period, he has to decide how much to save in order to be able to consume in the second period. How much people are going to save depend on wages and interest rates. Then, $s_t = s(r_{t+1}, w_t)$.

If we assume that the production function $Y= F(K,L)$ is homogenous of degree one, then we have that $y=Y/L= F(K/L,1)=f(k)$. It can be shown that for a competitive economy $r_{t+1} = f'(k_t)$ and $w_t = f(k_t) - f'(k_t) k_t$. Therefore, the saving function can be written as

$$s_t = s(f'(k_t), f(k_t) - f'(k_t) k_t).$$

In order to have an equilibrium, we need that net savings equal net investment, that is $K_t – K_{t-1} = L_t s_t - L_{t-1} s_{t-1}$, where capital letters represent aggregate levels. Then, we need

$$K_{t+1} = L_t s_t = s(f'(k_t), f(k_t) - f'(k_t) k_t).$$

Given the constant growth assumption, $L_{t+1}= (1+n) L_t$. Then the equilibrium equation is given by

$$(1+n)k_{t+1} = s(f'(k_t), f(k_t) - f'(k_t) k_t)$$

Notice that this last equation is giving that the evolution of capital through time. Depending on the properties of $F(K,L)$ and $U(c_t^{young}, c_{t+1}^{old})$ we may have either a unique steady state equilibrium or several steady state.

For a more detailed explanation of the model refer to professor Drazen's handout on this topic and to Blanchard and Fischer book.

PART II

Part a

One of the features of the overlapping generation model that is not present in neither the Solow model nor in Cass-Koopmans model is that the equilibrium reached by the economy can be dynamically inefficient. An equilibrium is dynamically inefficient is the economy ends up with "too much" capital in equilibrium, where by "too much" we mean that the equilibrium level of capital is greater than that implied by the Golden Rule. In an inefficient equilibrium, it is possible to consume some of the capital in any given period and still be able to raise consumption from then on.

A criterion for the existence of dynamic efficiency is that net capital income must exceed net investment. To obtain this criterion remember that $I_t = Y_t - C_t$. Also, in the steady state equilibrium k_t is fixed and therefore K_t must be growing at a rate of n. Therefore net investment in the steady state equilibrium is nK_t. Net capital income is given by rK_t. If $rK_t < nK_t$ then $r < n$ which is the theoretical criterion to identify an inefficient equilibrium in the OLG model.

For more on this criterion see Abel, Mankiw, Summers and Zeckhauser (1989) "Assessing Dynamic Inefficiency: Theory and Evidence" in the Review of Economic Studies.

Part b

In the overlapping generation model, dynamic inefficiency may appear because the only way an agent can ensure himself having a positive consumption level in the future is by accumulating capital. If the agents want to have a high level of consumption in the future, they will end up accumulating too much capital.

In the presence of a pay-as-you-go social security system, individuals have now a new way of allocating current resources to future consumption. By paying taxes today, the agent is entitled to receiving benefits in the future. Then, if the agents are paying the social security taxes, they will reduce their savings and therefore the level of capital in the economy will go down.

In a pay-as-you-go system, the rate of return is given by the population growth n. This is because the proceeds from taxation are TL_t and those proceeds have to be divided between L_{t-1} people. Therefore each of them will be (1+n)T. If they originally pay T on taxes, the return is then n. If the economy is originally dynamic inefficient, we know that $r_{t+1} < n$. Because the return of the system is higher that the return on savings, it is possible for agents to increase their consumption level, which mean that everyone can be made better off. Therefore, if the economy is originally dynamically inefficient, implementing a pay-as-you-go social security system is a Pareto improvement.

Part c

In a model where agents are forward looking and have perfect information and where there is no social security in the economy, the introduction of a fully funded social security may have either no effect on individual's welfare or reduce it. The return that the government can obtain by taxing individual and investing the proceeds in the market is the same that the individual can obtain on his own by saving and investing in the market. If social security taxes are lower than the amount the individual was saving on his own, once social security is introduced, the individual will reduce savings in the same amount of the taxes paid in order to achieve his preferred allocation. If this is the case, the individual is as well off in both situations. If taxes are higher than what the individual was previously saving, the individual is forced to save more than what he prefers (he is forced to consume too little in the present). Because he can't achieve his preferred allocation any longer he is made worse off.

In order to compare the fully funded scheme and the pay-as-you-go scheme remember that the rate of return of the pay-as-you-system is related to the rate of growth of the population while the rate of return of the fully funded scheme is related to the marginal productivity of capital. If the marginal productivity of capital is higher than the rate of growth of the population, the amount of saving required to finance the same level of future consumption would be lower in the fully funded system. This means taht, given a certain consumption levels in future periods, the agents can have a higher consumption level in the present. This implies that replacing the pay-as-you-go system with a fully funded system represents a Pareto improvement.

Even if the previous argument in favor of a fully funded system is strong, some people's concerns are not related to the rate of returns of different systems. I can think of two concerns. First is that of redistribution. In a **pure** fully funded system the money obtained from social security taxes goes to personal accounts, therefore there is no place for income redistribution policies. The second concern is that of the transition from one system to the other. If the government decides to switch schemes all of the sudden it will have to pay the old without being able to get resources from the young (all the money it get from them is now going to the personal accounts). This switch will generate big budget deficits.

Part d

For a small open economy, the interest rate is not determined endogenously. It is that determined in the international market. For the economy to be in a dynamic inefficient equilibrium, the international interest rate must be lower than economy's population growth.

If the government decides to implement a pay-as-you-go system, the agents will reduce savings. This reduction in savings is going to be cancelled out by an increase in the inflow of capital from the rest of the world. This must happen in order to satisfy the condition that the local interest rate equals the world interest rate. Therefore, after the creation of the social security system, the economy will end up with the same amount of capital. Again the implementation of pay-as-you-go social security system will be Pareto improving only if the economy was previous in dynamic inefficient equilibrium.