# SOLUTION 601 Comp 1/2008

a)

$$s_t = \frac{\beta}{1+\beta}\left(1-\mu_t\right) w_t$$

Unaffected by $\gamma$.

b)

$$f\left(k_t\right) = c_t^y + \frac{c_t^o}{1+n} + \left(1+n\right) k_{t+1} + g_t$$

where $g_t \equiv \frac{G_t}{L_t} = \frac{G_0}{L_0} = g$, as it is constant over time.

$$g_t = \mu_t w_t$$

c)

$$\mu_t = \frac{g}{f\left(k_t\right) - k_t f'\left(k_t\right)}$$

If $k_t < k^{SS}$, assume that $G_0$ is "small" enough in relation to $K_0$ and $L_0$ so that $g/w_0\left(=\mu_0\right) < 1$. With $k_t$ rising over time, this will ensure $\mu_t < 1\ \forall t$. If $k_t \geq k^{SS}$, assume $g/w^{SS} < 1$.

d) $\frac{\beta}{1+\beta}\left(1-\mu_t\right) w_t = \left(1+n\right) k_{t+1}$ which implies

$$\begin{aligned} k_{t+1} &= \frac{\beta}{\left(1+\beta\right)\left(1+n\right)}\big(f\left(k_t\right) - k_t f'\left(k_t\right) - g\big) \\ &= B\left[\left(1-\alpha\right) k_t - g\right] \text{ where } B = \frac{\beta}{\left(1+\beta\right)\left(1+n\right)} < 1 \end{aligned}$$

e) Set $k_{t+1} = k_t = k^{SS}$ in above and differentiate. i) $k^{SS}$ is independent of $K_0$

ii) $G_0$ moves as $g$. Shift down:

$$\frac{\partial k^{SS}}{\partial g} = -\frac{B}{1-\left(1-\alpha\right) B} < 0$$

iii)$\alpha$. Shift down

$$\frac{\partial k^{SS}}{\partial g} = -\frac{B k^{SS}}{1-\left(1-\alpha\right) B} < 0$$

f) Gov't budget constraint becomes $g = \tau_t\left(w_t + R_t k_t\right)$, so that $\tau_t = g/f\left(k_t\right)$ for $t, t+1, t+2, \ldots$ implying a lower rate of tax than in previous case.

g) Saving function is $s_t = \frac{\beta}{1+\beta}\left(1-\tau_t\right) w_t$, so that saving is higher with the capital tax. Given the same dynamic function, $k^{SS}$ is higher. This seemingly counterintuitive result reflects the fact that saving is independent of the interest rate and only depends on after-tax labor income. Taxing capital reduces the tax burden on labor income, hence raising $s_t$ and $k^{SS}$.