

3. (Read the entire question before starting to write your answer.) Consider a growth model where the creation of new “ideas” is endogenous. The production of the final good  $Y$  takes place according to the technology

$$Y = L_y \sum_{j=1}^A \left( \frac{x_j}{L_y} \right)^\alpha$$

where  $L_y$  is labor used in the final goods sector,  $x_j$  is the amount of intermediate good of type  $j$  used in the production of final goods,  $A$  is the total number of types of intermediate goods, and  $0 < \alpha < 1$  is a fixed technological parameter. Intermediate goods of every type are produced according to a linear technology such that every unit of intermediate good  $x_j$  produced requires  $\eta$  units of final good and no labor. Finally, new ideas (new types of intermediate goods) are produced according to the technology

$$\dot{A} = \delta A L_a$$

where  $L_a$  is labor used in the “ideas” sector and  $\delta$  is a productivity parameter. Total labor in the economy is fixed and equal to  $\bar{L}$ .

(a) Describe the market structure of the economy and write down the optimization problem of each of the three sectors of the economy. Characterize the equilibrium factor demands and the competitive equilibrium wage.

(b) Find the total amount of labor  $L_a$  devoted to research. Find the growth rate of the economy. Explain why this model displays endogenous growth.

(c) Suppose that the government decides to subsidize the purchase of intermediate goods by final good firms. That is, instead of paying  $p_j$  per unit of  $x_j$  they pay  $(1 - \theta_x) p_j$  where  $0 < \theta_x < 1$  is the subsidy rate. (Do not worry about where the revenues to finance such a policy come from). Find the new equilibrium values of: i) the quantity of intermediate goods  $x$ ; ii)  $L_a$ ; iii)  $L_Y$ ; and iv) the economy’s growth rate. Explain what is happening.

(d) Now, suppose that alternatively the government decides to subsidize the production of final goods. That is, the government gives producers  $\theta_y$  per unit (above revenues from sales) for every unit of final output sold. (Again, do not worry about the government budget constraint). Find the new equilibrium values of: i) the quantity of intermediate goods  $x$ ; ii)  $L_a$ ; iii)  $L_Y$ ; and iv) the economy’s growth rate. Explain what is happening.

(e) Suppose instead that the government subsidizes labor hiring in the “ideas” sector. That is, instead of  $w_a$ , the unit labor cost to firms in the ideas sector is  $w_a (1 - \theta_a)$ , where the subsidy rate is  $0 < \theta_a < 1$ . Find the new equilibrium  $L_a$ ,  $L_Y$ , and growth rate for the economy. Explain what is happening.

(f) Explain the difference between the result in sections (c), (d), and (e).

(g) Briefly sketch how your answer to (a) and (b) would differ if final goods firms faced a cost in adjusting their labor supply similar to adjustment costs in the theory of investment.

Note:  $L_y = L_Y$