2. Consider an economy consisting of overlapping generations of two-period lived individuals. Let N denote the population of young individuals and assume that there is no population growth. A representative young person born at time t has preferences defined over consumption in the two periods of his life denoted by (c_t^y, c_{t+1}^o) . His utility function is given by

$$U(c_t^y, c_{t+1}^o) = u(c_t^y) + \beta c_{t+1}^o$$

where $u(c_t^y)$ has the usual "nice" properties and $1 > \beta > 0$. There is an initial old generation at t = 1 who care only about consumption when old, denoted c_0^o.

Each young person is endowed with income of e > 0 units of output when young and nothing when old. There is an individual storage technology with the following properties: s, units of output invested in "shmoos" at t yields $zf(s_t)$ units of output at t + 1, where $f(\cdot)$ is strictly increasing and concave, and z > 0 is a technology parameter. Shmoos fully depreciate fully after use in production. Each of the initial old is endowed with s_0 shmoos.

a) Write down the individual's choice problem and find the optimal individual holding of shmoos. Find the equilibrium level of shmoos per capita in each period and the growth path of this economy. How does the steady state level of shmoos per capita s* depend on the parameters z and P? Explain.

Now suppose that the above economy is in steady state when suddenly, and unexpectedly, a government emerges at some arbitrary date. Relabel time to call this date t = 0 and call it the "initial" period. (Note that an "initial" old person has s* shmoos, as you derived in part (a)). Assume that the government has an exogenous expenditure requirement: it needs to acquire Ng units of output in each and every period starting at t = 0 for the foreseeable future, where 0 < g < e. Consider two types of taxes to finance g: lump-sum taxes, where (x_t^y, x_{t+1}^o) denotes the tax imposed on generation t and proportional income taxes, where (τ_t^y, τ_{t+1}^o) denotes the tax rate imposed on generation t to finance its expenditures. (When only lump-sum taxes are used, the "initial" old at relabeled t = 0 face a tax x_0^o; when only proportional taxes are used, the second-period tax rate on the "initial" old is given by (τ_{0t}^o=g/zf(s*) where we assume that zf (s*) > g.) Assume the government runs a balanced budget.

b) Write down the period-by-period budget constraints of a representative individual, of the initial old, and of the government under each tax regime.

c) Compare the steady-state effects on s of the government financing its purchases via: i) only x_t^y ; ii) only x_t^o ; iii) only τ_t^y ; and iv) only τ_t^o. Explain the differences.

d) Compare the steady-state effects on welfare of the first two options in part (c) (that is, using solely x_t^y versus solely x_t^o for the case where (x_t^y, x_t^o) = (x^y, x^o) (that is, same tax across generations as a function of age, except possibly for initial old). Explain.

e) In part (d) suppose that $zf(s^*) < 1$. Show that there is a balanced budget fiscal policy with $x^y > 0$ that makes every generation better off relative to the s^* allocation. Explain why the no-

government equilibrium not Pareto optimal.

Suppose now that the government decides to finance at least a part of its expenditures by issuing one-period debt, as in the Diamond model, where debt issued at t pays interest r_{t+1} in period $t + 1$. Consider a policy of a constant level of debt Nb, where all taxation is lump-sum.

f) Write down the representative individual's optimization problem and first-order conditions. Write down the government budget constraints in the "initial" period and in each subsequent period. Discuss the effect of different levels of b for a given level of g on the real interest rate, holding of shmoos, the welfare of the initial old, and welfare of subsequent generations under a policy of using only x^y versus using only x°.

g) If possible, design a fiscal policy that yields Ricardian equivalence and explain why Ricardian equivalence holds under this policy (or why no such policy is possible).