

Question No. 4

Consider a two-sector optimal growth problem in which the social planner maximizes the utility of the representative agent given by:

$$W = \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

where c_t = consumption and l_t is leisure in period t . Sector 1 produces consumption goods using capital k_{1t} and labor n_{1t} , according to the production function:

$$c_t = f_1(k_{1t}, n_{1t})$$

Sector 2 produces capital goods using capital k_{2t} and labor n_{2t} according to:

$$k_{t+1} = f_2(k_{2t}, n_{2t})$$

Total employment is $n_t = n_{1t} + n_{2t}$, leisure l_t is constrained by the time endowment \bar{l} so that is $n_t + l_t \leq \bar{l}$. Also, $k_{1t} + k_{2t} \leq k_t$, where k_0 is given.

a. Formulate this problem as a dynamic programming problem. Write down the functional equation (or equations) that the value function satisfies and clearly specify the state and control variables.

b. Find the equations describing optimal paths

c. Consider another economy that is similar, except that capital is sector specific. The economy starts period t with given amounts of capital k_{1t} and k_{2t} that may be used only in their respective sectors. In the period the capital-goods sector produces capital that is specific to each sector according to the transformation curve:

$$g(k_{1t+1}, k_{2t+1}) \leq f_2(k_{2t}, n_{2t})$$

Write down the Bellman equation associated with this problem, clearly specifying state and control variables and characterize the optimal path. How does the growth path of this economy differ from the growth path of the previous economy?