2. (Read the entire question before starting to write your answer.) Consider a i/iree-period OLG model. For an individual born at time *t*. let c_t^y, c_{t+1}^m and c_{t+2}^o be consumption in the first period of life ("youth'[1]), the second period of life ("middle age"), and the third period of life ("old"), respectively. The utility function of a young person born at time *t* is

$$U(c_t^y, c_t^m, c_t^o) = \ln c_t^y + \beta \ln c_{t+1}^m + \beta^2 \ln c_{t+1}^o$$

where 1 > β > 0. Individuals supply inelastically *i* units of labor when young, one unit of labor when middle aged, and do not work when old. There is no human capital accumulation and no bequests. The production technology in the model satisfies the same assumptions as in the two-period Diamond model, and capital to be used in period *t + 1* must be set aside in period *t.* Individuals have perfect foresight, and all markets are competitive. The rate of population growth is a constant n > —1.

a) From now, assume 0 < *l* < *1*. Do you think this assumption is natural? Why or why not?

b) Set up the optimization problem of the young, being careful with the specification of factors returns as seen by the individual. (For notational simplicity, you may disregard for a moment the time indices *t, t + 1* and *t* + 2.)

c) Derive the intertemporal budget constraint and find the optimal c^y, c^m. and $c°$.

d) Find the financial wealth a^y held by a young person at the end of the first period of his life. (Since there are no bequests, a^y = the saving s^y of the young) and the financial wealth a^m of a middle-aged individual at the end of the second period of his life as functions of the relevant parameters. Do a^y and a^m depend on the rate of interest between the first and the second period of life? Comment!

e) Give conditions that could make $a^y{}_t < 0$. Comment!

f) Derive the equation for capital accumulation in the model. Show that the fundamental difference equation of the model is a second order non-linear difference equation.

g) How do you think the dynamics might differ from the two-period model with $y_t = k_t^\alpha$?