2. Consider first a single two-period lived household with a utility function $u(c_t^y) + \beta E_t\left[u\left(c_{t+1}^o\right)\right]$, where $1 > \beta > 0$ and $u(c) = ac - \frac{b}{2}c^2$. The individual receives an exogenous endowment x_t in her first period of life, and can save s_t to finance second period consumption. The budget constraints in the first and second period are, respectively:

$$
\begin{aligned}
c_t^y &= x_t - T_t^y - s_t \\
c_{t+1}^o &= Rs_t - T_{t+1}^o
\end{aligned}
$$

where R is the interest factor and T_t^y and T_{t+1}^o are lump-sum taxes on the individual in the two periods. Government spending is exogenous, equal to $\bar{g} > 0$ in the first period and equal to the random value $\tilde{g}$ in the second period with a CDF $F(\tilde{g})$. Assume that $\beta R = 1$.

(a) Derive the household's intertemporal budget constraint.

(b) Assume the government runs a balanced budget: $T_t^y = \bar{g}$ and $T_{t+1}^o = \tilde{g}$. Derive the consumption and saving functions.

(c) Continue to assume that the government runs a balanced budget. What is the effect of a mean-preserving spread of $\tilde{g}$ on savings s_t, consumption, and utility as seen from period t? (A mean-preserving spread of a random variable z is a new random variable $\hat{z} = z + \varepsilon$ where ε is mean zero white noise.)

(d) Now assume that the government collects taxes $\bar{g} - m$ rather than $\bar{g}$ in the first period, and increases the random tax in period $t+1$ from $\tilde{g}$ to $\tilde{g} + Rm$. Simultaneously, the government issues (safe) debt in period t, repaying it in $t+1$. How much debt does the government have to issue? How does this policy change affect consumption, household savings, government savings, and national (household plus government) savings? Explain. What is the effect on utility as seen from period t? Explain.

Now suppose that there are overlapping generations of such individuals, where population grows at rate n. Government spending is $g_0 = \bar{g}$ at time 0 and follows a random path $\{\tilde{g}_t\}_{t=1}^{\infty}$ thereafter, where $\tilde{g}_t$ is an i.i.d. random variable with the same CDF $F(\cdot)$ as above. The word starts at time 0 with the $T_0^y = \bar{g}$ and $T_0^o = 0$. Thereafter, taxes are given by $T_t^y = \bar{g}$ and $T_t^o = \tilde{g}_t - \bar{g}$. The government issues debt each period with a constant real value per young person of b.

(e) Characterize the time path of this economy for the case where $R = 1/\beta$.

(f) Characterize the time path of this economy when the interest factor R_t is random with a CDF $H(R)$. Discuss the difference between this case and your results from part (e). [redacted]

(g) If you can, discuss how your results in part (f) would be affected if the random variables $\tilde{g}_t$ and R_t were correlated.