Macroeconomic Comprehensive Theory Examination (August 2001)

Sketch of the Solution – Haltiwanger's Question (#2)

Constraints:

$$\underbrace{H_{t+1}}_{(a)} = \underbrace{(1-\sigma\ )H_t}_{(b)} + \underbrace{(1-H_t+\sigma_t H_t)\theta_t}_{(c)} \qquad (1)$$

(a) # of *H* job sites at *t+1*
(b) # of *H* job sites at *t* that were not hit by the adverse shock in that period
(c) # of *L* job sites that were shut down in *t* to become *H* job sites in *t+1*

$$\underbrace{C_t}_{(d)} = \underbrace{(1-\sigma_t)H_t Y^H}_{(e)} + \underbrace{(1-\theta_t)(1-H_t+\sigma_t H_t)Y^L}_{(f)} \qquad (2)$$

(d) Aggregate consumption in *t*
(e) Total production coming from *H* job sites (only those not hit by adverse shock)
(f) Total production coming from *L* job sites (only those not shut down)

Note: Equation (2) implies costly labor reallocation

Optimization Problem:

$$Max \quad E_0 \sum_{t=0}^{\infty} \beta^t A_t U(C_t) \qquad (3)$$

Subject to (1), (2), $A_{t+1} = \Omega(A_t)$, and $\sigma_{t+1} = \Gamma(\sigma_t)$,

Note: *A*: Aggregate demand shock
σ: Idiosyncratic allocative shock

Part (i)

Bellman Equation:

$$V(H,A,\sigma) = \underset{\theta}{Max} \quad \{AU(C) + \beta EV(H',A',\sigma')\} \qquad (4)$$

Subject to $H' = (1-\sigma)H + (1-H+\sigma H)\theta \qquad (1)$

$$C = (1-\sigma)HY^H + (1-\theta)(1-H+\sigma H)Y^L \quad (2)$$

$$A' = \Omega(A)\,,\ \sigma' = \Gamma(\sigma)$$

The Bellman Equation can be rewritten as:

$$V(H,A,\sigma) = \underset{\theta}{Max} \quad \begin{Bmatrix} AU((1-\sigma)HY^H + (1-\theta)(1-H+\sigma H)Y^L) \\ +\beta EV((1-\sigma)H + (1-H+\sigma H)\theta, A', \sigma') \end{Bmatrix}$$

Subject to $A' = \Omega(A)\,,\ \sigma' = \Gamma(\sigma)$

First-order condition (after simple algebra):

$$\theta: \quad AU'(C)Y^L = \beta EV_1(H',A',\sigma') \qquad (5)$$

Envelope condition (after simple algebra):

$$V_1(H,A,\sigma) = AU'(C)(1-\sigma)(Y^H - (1-\theta)Y^L) + \beta EV_1(H',A',\sigma')(1-\theta)(1-\sigma) \quad (6)$$

Using (5) to eliminate V_1 from the right-hand side of equation (6), we have:

$$V_1(H,A,\sigma) = AU'(C)(1-\sigma)Y^H \qquad (7)$$

The Euler equation is finally obtained forwarding (7) one period, and plugging it into (5):

$$\underbrace{AU'(C)Y^L}_{(g)} = \underbrace{\beta E(A'U'(C')(1-\sigma')Y^H)}_{(h)} \qquad (8)$$

(g) Utility cost of foregoing one unit of current output to move one additional worker from a low productivity to a high productivity site
(h) Discounted expected gains that result from an improved allocation of employment at the beginning of the next period

Part (ii)

A key result of the model is that job reallocation is counter-cyclical. To see this, remember that (5) can be written as (using (1) and (2)):

$$AU'((1-\sigma)HY^H + (1-H+\sigma H)(1-\theta)Y^L)Y^L = \beta EV_1((1-\sigma)H + (1-H+\sigma H)\theta, A', \sigma')$$

Using the expression above, define:

$$\Omega = AU'((1-\sigma)HY^H + (1-H+\sigma H)(1-\theta)Y^L)Y^L - EV_1((1-\sigma)H + (1-H+\sigma H)\theta, A', \sigma')$$

Now, calculate:

$$\frac{\partial\theta^*}{\partial A} = -\frac{\partial\Omega/\partial A}{\partial\Omega/\partial\theta} = -\frac{U'(C)Y^L}{-AU''(C)(1-H+\sigma H)(Y^L)^2 - \beta EV_{11}(H', A', \sigma')(1-H+\sigma H)} < 0$$

Where we assume iid A shocks. Thus, the pace of reallocation (job flows) increases in response to a decline in aggregate demand. The intuition for this result is that, during recessions, the one period of foregone production (necessary in the reallocation process to make high productivity job sites operational) has a low opportunity cost.

Serial correlation in the allocative shocks affects the pace of reallocation in two distinct and offsetting ways. Consider the case of highly persistent allocative shocks. A positive innovation in σ today implies that another positive innovation tomorrow is likely to occur, and thus current reallocation to high productivity sites is unattractive because those sites have a considerable chance of being reverted in the near future. This first effect (substitution effect) works in favor of reducing the reallocation pace. On the other hand, a positive innovation in the current σ when allocative shocks are highly persistent also reduces the expected wealth of the consumer. In response to it, agents will consume less and engage in more reallocation (income effect). So, the final effect is ambiguous.

Finally, the dynamics of the allocative shocks can certainly affect the counter-cyclical nature of job reallocation. Indeed, highly persistent allocative shocks may even overcome the incentives placed by the feature of foregone production. As an example, consider a situation of low aggregate demand (low A_t), where a highly persistent positive innovation in σ occurs. If the substitution effect is sufficiently powerful, it may be the case that despite of the "cheap times" no reallocation is going to take place.

Part (iii)

Barriers to job destruction could be model as:

$$C_t = (1-\sigma_t)H_tY^H + (1-\theta_t(1+\tau))(1-H_t+\sigma_tH_t)Y^L \qquad (9)$$

The formulation above creates an additional disincentive for job destruction as it makes the mobility cost higher than the one required by the reallocation process. In this case, a lower proportion of low-productivity sites are destroyed, but less high-productivity sites are created as well. This will reduce unemployment at the expenses of a worse long-term productivity level of the economy. In the long-term, consumption will also be lower than the no-tax environment.

Part (iv)

The assumption that workers can perfectly pool idiosyncratic risks implies that per capita consumption is the same across all agents, even tough there is heterogeneity with respect to their employment conditions. This assumption is also necessary to guarantee the equivalence between the competitive equilibrium outcome and the solution to the social planner's problem.

If workers could not pool idiosyncratic risks, barriers to job destruction could actually improve the utility of some agents with recurrent unemployment histories.