<u>Solution to Haltiwanger's question in August 2004 Comp</u>

<u>PART i</u>

$$V(Z_{t-1}, L_t, p_t) = \max_{u_t, w_t, S_t}\{(\text{argmax} p_t)S_t - w_t u_t(\text{argmax} L_t) - H(z_t) - \tau(1-u_t)(\text{argmax} L_t) + ...$$

$$... + E_t \beta V(Z_t, L_{t+1}, p_{t+1})\} \quad (1)$$

subject to $u_t \leq 1$ (2)

$$S_t \leq m(p_t) + \varepsilon_t \quad (3)$$

$$S_t \leq Z_{t-1} + g(u_t L_t) \quad (4)$$

$$Z_t = Z_{t-1} + g(u_t L_t) - S_t \quad (5)$$

$$\int w_t u_t f(\varepsilon) d\varepsilon + (1 - \int u_t f(\varepsilon) d\varepsilon) Y \geq V \quad (6)$$

where argmax p_t and argmax L_t are the values of p_t and L_t that solve the following problem

$$V(Z_{t-1}) = \max_{p_t, L_t} E_t\{p_t S_t(p_t, L_t, \varepsilon_t) - w_t(p_t, L_t, \varepsilon_t) u_t(p_t, L_t, \varepsilon_t) L_t - H(z_t(p_t, L_t, \varepsilon_t)) - \tau(1 - u_t(p_t, L_t, \varepsilon_t)) L_t + \beta V(Z_t)\}$$

(7)

subject to the same restrictions above.

<u>PART ii</u>

Note that (6) will always hold as equality.

Now multiply (6) by L_t

$$L_t \int w_t u_t f(\varepsilon) d\varepsilon + (1 - \int u_t f(\varepsilon) d\varepsilon) L_t Y = V L_t$$

rearranging we get

$$E_t(w_t u_t L_t) = V L_t - E_t(1 - u_t) L_t Y \quad (8)$$

<u>Solving for p_t and L_t</u> (remember S_t and u_t are a function of p_t, L_t and ε_t. Below we assume that functions $S_t = S(p_t, u_t, \varepsilon_t)$ and $u_t = u(p_t, u_t, \varepsilon_t)$ are differentiable)

$$V(Z_{t-1}) = \max_{p_t, L_t} E_t\{p_t S_t - V L_t + (1 - u_t) L_t Y - H(Z_{t-1} + g(u_t L_t) - S_t) - \tau(1 - u_t) L_t + ...$$

$$... + \lambda_t(1 - u_t) + \mu_t[m(p_t) + \varepsilon_t - S_t] + \Omega_t(Z_{t-1} + g(u_t L_t) - S_t) + \beta V(Z_{t-1} + g(u_t L_t) - S_t)\} \quad (9)$$

First Order Conditions

$$E_t\left[S_t + p_t\frac{\partial S_t}{\partial p_t} - \frac{\partial u_t}{\partial p_t}L_tY + \left(\Omega_t - H'(Z_t) + \beta\frac{\partial V}{\partial p_t}\right)\left(g'(u_tL_t)\frac{\partial u_t}{\partial p_t}L_t - \frac{\partial S_t}{\partial p_t}\right) + (\tau L_t - \lambda_t)\frac{\partial u_t}{\partial p_t} + \mu_t\left(m'(p_t) - \frac{\partial S_t}{\partial p_t}\right)\right] = 0 \quad (10)$$

$$E_t\left\{(p_t - \mu_t)\frac{\partial S_t}{\partial L_t} - V + (1-u_t)(Y-\tau) + (\tau L_t - L_tY - \lambda_t)\frac{\partial u_t}{\partial L_t} + \left(\Omega_t - H'(Z_t) + \beta V'(z_t)\right)\left[g'(u_tL_t)\left(u_t + \frac{\partial u_t}{\partial L_t}L_t\right) - \frac{\partial S_t}{\partial L_t}\right]\right\} = 0 \quad (11)$$

Solving for u_t, w_t and S_t (for notational convenience define $p_t^* = \arg\max p_t$ and $L_t^* = \arg\max L_t$)

$$V(Z_{t-1}, L_t, p_t) = \max_{u_t, S_t}\{p_t^*S_t - VL_t^* + (1-u_t)L_t^*Y - H(Z_{t-1} + g(u_tL_t^*) - S_t) - \tau(1-u_t)L_t^* + \ldots$$

$$\ldots + \lambda_t(1-u_t) + \mu_t[m(p_t^*) + \varepsilon_t - S_t] + \Omega_t(Z_{t-1} + g(u_tL_t^*) - S_t) + \beta E_tV(Z_{t-1} + g(u_tL_t^*) - S_t, L_{t+1}, p_{t+1})\} \quad (12)$$

First Order Conditions

$$-L_t^*Y - H'(Z_t)g'(u_tL_t^*)L_t^* + \tau L_t^* - \lambda_t + \Omega_t g'(u_tL_t^*)L_t^* + \beta E_t\{V_{Zt}(Z_t, L_{t+1}, p_{t+1})g'(u_tL_t^*)L_t^*\} \leq 0$$

with equality if $u_t \geq 0$ (13)

$$p_t^* + H'(Z_t) - \mu_t - \Omega_t - \beta E_{Zt}\{V(Z_t, L_{t+1}, p_{t+1})\} \leq 0$$ with equality if $S_t \geq 0$ (14)

Remember that w_t can be recovered from (7)

PART iii

The effect of an adverse demand shock on inventories and on the number of workers laid off is going to depend on the size of the shock and the level of inventories at the moment of the shock. This is due to the fact that the cost of holding inventories is a strictly convex function while the cost of laying worker off is a linear function. Laying off workers also reduces production and therefore, at a given level of sales, the inventories carried over the next period are reduced. Because the production function is concave, laying off more workers increases the size of the reduction in production at an increasing rate.

If the cost of holding inventories is small for small inventories, one would expect to observe firm only increasing inventories after an adverse demand shock if the shock is not big enough and the original level of stock is small. If the original inventories are not all that big, one would expect to observe an increase in inventories and workers being laid off after a shock. If the original level of stock is high, it may be the case that the firm only lays workers off.

PART iv

The main difference of the model presented here with the standard inventory model and the standard implicit contract model is that the firm here has two margins in which it can adjust behavior in the presence of a negative demand shock. In the cases where the firm adjusts the two margins, for the same negative demand shock, the increase in inventories will be smaller than in the standard inventories case and the number of workers being laid off is going to be smaller than in the standard implicit contract model.