Answer to Haltiwanger's 2005 Comp Question

Part A

In the Mortensen-Pissarides model, the steady state conditions are given by

$$p + \sigma\epsilon_d = b + \frac{\beta c}{1-\beta}\frac{v}{u} - \frac{\sigma\lambda}{r+\lambda}\int_{e_d}^{e_u}[1-F(x)]dx \tag{1}$$

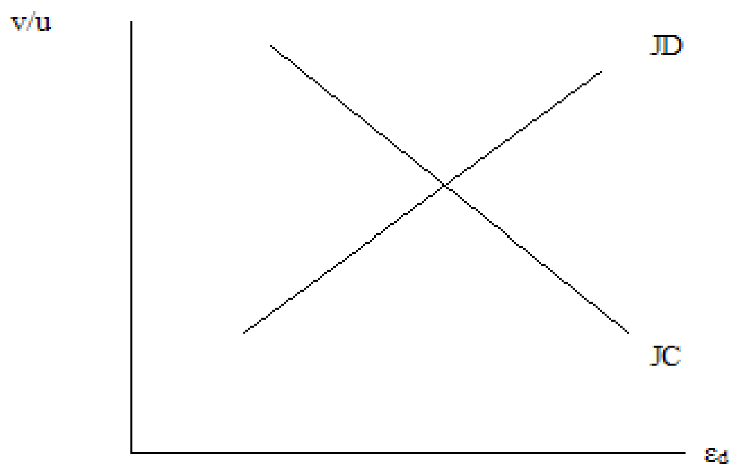
$$q = \frac{c}{1-\beta}\frac{r+\lambda}{\sigma(\epsilon_u - \epsilon_d)} \tag{2}$$

$$u = \frac{\lambda F(\epsilon_d)}{\lambda F(\epsilon_d) + m(v/u, 1)} \tag{3}$$

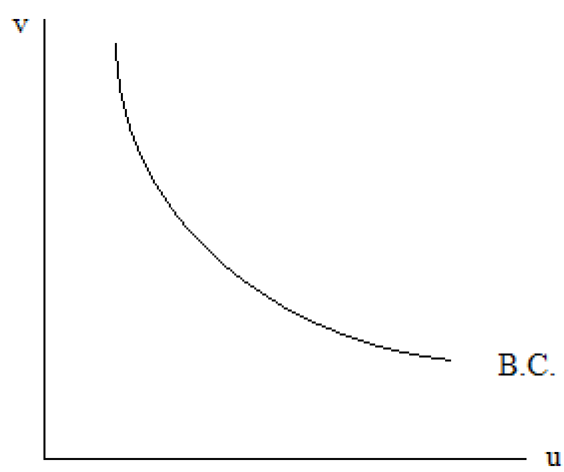
Equation (1) above is the job destruction equation."It gives the reservation productivity in terms of the ratio of vacancies to unemployment and the parameters of the model ... The left-hand-side of (1) is the lowest price acceptable to firms with a filled job. This is less than the opportunity cost of employment because of the existence of a hiring cost. The opportunity cost of employment to the workers is the value of leisure b plus the expected gain from search, which in equilibrium is equal to the second term on the right-hand-side of (1). The third term is a measure of the extent to which the employer is willing to incur an operational loss now in anticipation of a future improvement in the value of the match's product, i.e. it is option value of retaining an existing match." (Mortensen&Pissarides, 1994)

Equation (2) is the job creation equation. It is derived from the assumption that vacancies will be posted until the point where the value of posting a vacancy becomes zero. It can be noticed from equation (2) that the transition rate for a vacancy q is positively related to the interest rate, the cost of posting a vacancy and the arrival rate of idiosyncratic shocks and negatively related to the firm's share of the surplus and the dispersion parameter.

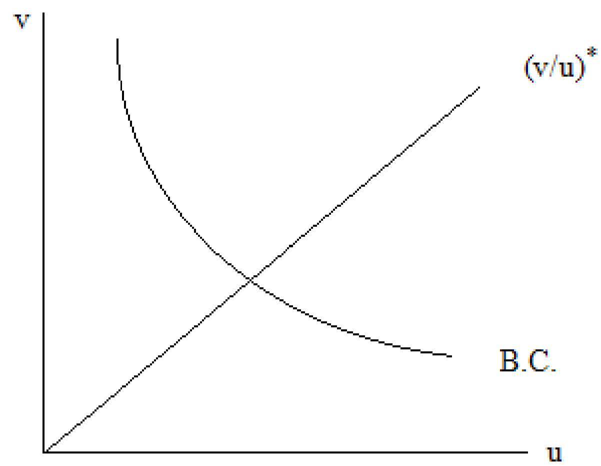
It is important to notice that equation (1) and (2) relate the vacancy-unemployment ratio to the reservation productivity $\epsilon_d$. It can be shown that equation (1) can be represented as a downward sloping loci and equation (2) as a positively sloped loci. Graphically




Equation (3) is a steady-state condition for unemployment and it is known as the Beveridge curve. In the Mortensen&Pissarides model, the inflow to unemployment is given by $(1-u)\lambda F(\ \epsilon_d)$. The flow out of unemployment is given by $u * m(\frac{v}{u}, 1)$. Equation (3) above is obtained by equating inflows and outflows. In the M&P model, the slope of the curve cannot be determined analytically but the data seem to imply a downward sloping Beveridge curve. Graphically,




To solve the model, note that equation (1) and (2) determined the steady-state vacancy-unemployment ratio. Plugging that ratio in equation (3) one can get the steady-state unemployment level and, for the vacancy-unemployment ratio one get the steady-state level of vacancies. Graphically,




Part B

Change in p
Differentiating (1) w.r.t p we get

$1+\sigma\frac{\partial\epsilon_d}{\partial p}=\frac{\beta c}{1-\beta}\frac{\partial\theta}{\partial p}+\frac{\sigma\lambda}{r+\lambda}\frac{\partial\epsilon_d}{\partial p}-\frac{\sigma\lambda}{r+\lambda}F(\epsilon_d)\frac{\partial\epsilon_d}{\partial p}$ where $\theta=\frac{v}{u}$

$$1+\frac{r+\lambda F(\epsilon_d)}{r+\lambda}\sigma\frac{\partial\epsilon_d}{\partial p}=\frac{\beta c}{1-\beta}\frac{\partial\theta}{\partial p} \tag{4}$$

Take logs of (2)

$$\ln q=\ln(\frac{c}{1-\beta})+\ln(r+\lambda)-\ln\sigma-\ln(\epsilon_u-\epsilon_d) \tag{5}$$

Now differentiate (5) w.r.t $p$

$\frac{1}{q}q'(\theta)\frac{\partial\theta}{\partial p}=\frac{1}{(\epsilon_u-\epsilon_d)}\frac{\partial\epsilon_d}{\partial p}$

Divide and multiple the left-hand-side by $\theta$

$$\frac{\eta}{\theta}\frac{\partial\theta}{\partial p}=-\frac{1}{(\epsilon_u-\epsilon_d)}\frac{\partial\epsilon_d}{\partial p} \tag{6}$$

where $\eta\equiv-\frac{q'(\theta)}{q}\theta$ is the elasticity of the transition rate for a vacancy. Note that by homogeneity of the matching function $0<\eta<1$.

$$\frac{\partial \theta}{\partial p} = -\frac{\theta}{\eta(\epsilon_u - \epsilon_d)} \frac{\partial \epsilon_d}{\partial p} \tag{7}$$

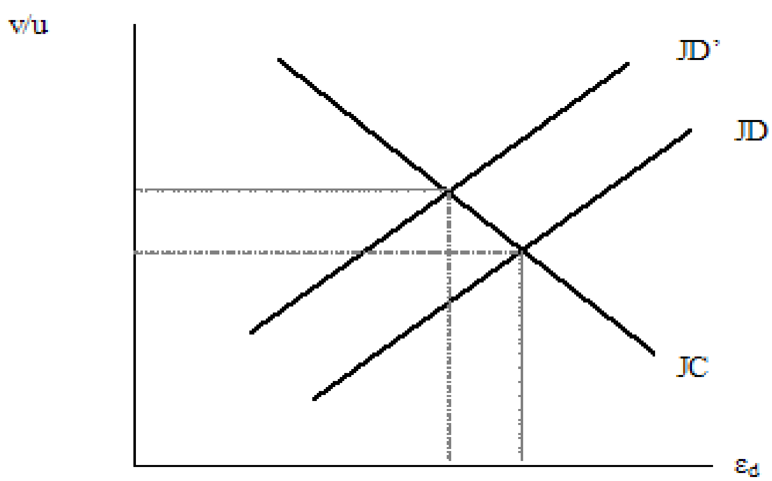
Now, replace (7) into (4)

$$1 + \frac{r+\lambda F(\epsilon_d)}{r+\lambda} \sigma \frac{\partial \epsilon_d}{\partial p} = -\frac{\beta c}{1-\beta} \frac{\theta}{\eta(\epsilon_u - \epsilon_d)} \frac{\partial \epsilon_d}{\partial p}$$

$$\left[\frac{\beta c}{1-\beta} \frac{\theta}{\eta(\epsilon_u - \epsilon_d)} + \frac{r+\lambda F(\epsilon_d)}{r+\lambda} \sigma\right] \frac{\partial \epsilon_d}{\partial p} = -1$$

The term in brackets is positive, so for the equation above to hold $\frac{\partial \epsilon_d}{\partial p} < 0$. Equation (7) says that $\frac{\partial \epsilon_d}{\partial p} < 0 \Longrightarrow \frac{\partial \theta}{\partial p} > 0$.

To see this graphically, note that p does not enter in the job creation equation, so the a change in p is only going to affect the location of the job destruction curve




The intuition behind this result is that an increase in the aggregate component of productivity p makes a filled job more valuable to the firm so they are willing to accept lower idiosyncratic components of productivity. Given the a filled job is more valuable more vacancies are opened and because $\epsilon_d$ went down less jobs are destroyed. More vacancies and lower destruction make the vacancy-unemployment ratio to go down.

Change in b

Differentiating (1) w.r.t b we get
$\sigma \frac{\partial \epsilon_d}{\partial b} = 1 + \frac{\beta c}{1-\beta}\frac{\partial \theta}{\partial b} + \frac{\sigma\lambda}{r+\lambda}\frac{\partial \epsilon_d}{\partial b} - \frac{\sigma\lambda}{r+\lambda}F(\epsilon_d)\frac{\partial \epsilon_d}{\partial b}$

$$-1 + \frac{r+\lambda F(\epsilon_d)}{r+\lambda}\sigma\frac{\partial \epsilon_d}{\partial p} = \frac{\beta c}{1-\beta}\frac{\partial \theta}{\partial p} \tag{8}$$

Differentiating (5) w.r.t b we get

$\frac{1}{q}q'(\theta)\frac{\partial \theta}{\partial b} = \frac{1}{(\epsilon_u - \epsilon_d)}\frac{\partial \epsilon_d}{\partial b}$

$$\frac{\partial \theta}{\partial b} = -\frac{\theta}{\eta(\epsilon_u - \epsilon_d)}\frac{\partial \epsilon_d}{\partial b} \tag{9}$$

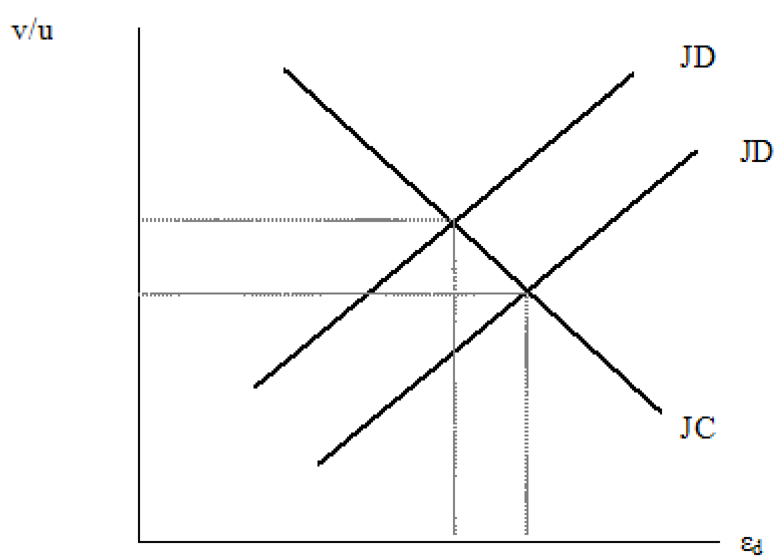
Now, replace (9) into (8)

$-1 + \frac{r+\lambda F(\epsilon_d)}{r+\lambda}\sigma\frac{\partial \epsilon_d}{\partial b} = -\frac{\beta c}{1-\beta}\frac{\theta}{\eta(\epsilon_u - \epsilon_d)}\frac{\partial \epsilon_d}{\partial b}$

$\left[\frac{\beta c}{1-\beta}\frac{\theta}{\eta(\epsilon_u - \epsilon_d)} + \frac{r+\lambda F(\epsilon_d)}{r+\lambda}\sigma\right]\frac{\partial \epsilon_d}{\partial b} = 1$

The term in brackets is positive, so for the equation above to hold $\frac{\partial \epsilon_d}{\partial b} > 0$. Equation (9) says that $\frac{\partial \epsilon_d}{\partial b} > 0 \Longrightarrow \frac{\partial \theta}{\partial p} < 0$.

To see this graphically,



An increase in the worker's taste for leisure will force firm to pay workers higher wages. A filled job is now less valuable to the firm so they require higher idiosyncratic components of productivity. Given the a filled job is less valuable, fewer vacancies are opened and because $\epsilon_d$ went up more jobs are destroyed. Fewer vacancies and more job destruction make the vacancy-unemployment ratio to go up.

Change in $\sigma$

Differentiating (1) w.r.t $\sigma$ we get

$$\epsilon_d + \sigma \frac{\partial \epsilon_d}{\partial \sigma} = \frac{\beta c}{1-\beta}\frac{\partial \theta}{\partial \sigma} - \frac{\lambda}{r+\lambda}\int_{e_d}^{e_u}[1-F(x)]dx + \frac{\sigma\lambda}{r+\lambda}\frac{\partial \epsilon_d}{\partial \sigma} - \frac{\sigma\lambda}{r+\lambda}F(\epsilon_d)\frac{\partial \epsilon_d}{\partial \sigma}$$

$$\epsilon_d + \frac{r+\lambda F(\epsilon_d)}{r+\lambda}\sigma\frac{\partial \epsilon_d}{\partial \sigma} = \frac{\beta c}{1-\beta}\frac{\partial \theta}{\partial \sigma} - \frac{\lambda}{r+\lambda}\int_{e_d}^{e_u}[1-F(x)]dx \qquad (10)$$

Take logs of (2)

$$\ln q = \ln(\frac{c}{1-\beta}) + \ln(r+\lambda) - \ln\sigma - \ln(\epsilon_u - \epsilon_d)$$

Now differentiate w.r.t $\sigma$

$$\frac{1}{q}q'(\theta)\frac{\partial \theta}{\partial \sigma} = -\frac{1}{\sigma} + \frac{1}{(\epsilon_u-\epsilon_d)}\frac{\partial \epsilon_d}{\partial \sigma}$$

$$-\frac{\sigma(\epsilon_u-\epsilon_d)}{q}q'(\theta)\frac{\partial \theta}{\partial \sigma} = \epsilon_u - \epsilon_d - \sigma\frac{\partial \epsilon_d}{\partial \sigma}$$

$$\sigma(\epsilon_u-\epsilon_d)\frac{\eta}{\theta}\frac{\partial \theta}{\partial \sigma} = \epsilon_u - \epsilon_d - \sigma\frac{\partial \epsilon_d}{\partial \sigma} \qquad (11)$$

Now, from equation (10) we get

$$\sigma\frac{\partial \epsilon_d}{\partial \sigma} = \frac{r+\lambda}{r+\lambda F(\epsilon_d)}\left[\frac{\beta c}{1-\beta}\frac{\partial \theta}{\partial \sigma} - \epsilon_d - \frac{\lambda}{r+\lambda}\int_{e_d}^{e_u}[1-F(x)]dx\right] \qquad (12)$$

Replacing (12) into (11)

$$\sigma(\epsilon_u-\epsilon_d)\frac{\eta}{\theta}\frac{\partial \theta}{\partial \sigma} = \epsilon_u - \epsilon_d - \frac{r+\lambda}{r+\lambda F(\epsilon_d)}\left[\frac{\beta c}{1-\beta}\frac{\partial \theta}{\partial \sigma} - \epsilon_d - \frac{\lambda}{r+\lambda}\int_{e_d}^{e_u}[1-F(x)]dx\right]$$

$$\left[\ \sigma(\epsilon_u-\epsilon_d)\frac{\eta}{\theta} + \frac{r+\lambda}{r+\lambda F(\epsilon_d)}\frac{\beta c}{1-\beta}\right]\frac{\partial \theta}{\partial \sigma} = \epsilon_u - \epsilon_d + \frac{r+\lambda}{r+\lambda F(\epsilon_d)}\left[\epsilon_d + \frac{\lambda}{r+\lambda}\int_{e_d}^{e_u}[1-F(x)]dx\right]$$

$$\left[\ \sigma(\epsilon_u-\epsilon_d)\frac{\eta}{\theta} + \frac{r+\lambda}{r+\lambda F(\epsilon_d)}\frac{\beta c}{1-\beta}\right]\frac{\partial \theta}{\partial \sigma} = \epsilon_u + \frac{r+\lambda}{r+\lambda F(\epsilon_d)}\left[\epsilon_d - \frac{r+\lambda F(\epsilon_d)}{r+\lambda}\epsilon_d + \frac{\lambda}{r+\lambda}\int_{e_d}^{e_u}[1-F(x)]dx\right]$$

$$\left[\ \sigma(\epsilon_u-\epsilon_d)\frac{\eta}{\theta}+\frac{r+\lambda}{r+\lambda F(\epsilon_d)}\frac{\beta c}{1-\beta}\right]\frac{\partial\theta}{\partial\sigma}=\epsilon_u+\frac{r+\lambda}{r+\lambda F(\epsilon_d)}\left[\frac{\lambda-\lambda F(\epsilon_d)}{r+\lambda}\epsilon_d+\frac{\lambda}{r+\lambda}\frac{1-F(\epsilon_d)}{1-F(\epsilon_d)}\int_{e_d}^{e_u}[1-F(x)]dx\right]$$

$$\left[\ \sigma(\epsilon_u-\epsilon_d)\frac{\eta}{\theta}+\frac{r+\lambda}{r+\lambda F(\epsilon_d)}\frac{\beta c}{1-\beta}\right]\frac{\partial\theta}{\partial\sigma}=\epsilon_u+\frac{\lambda[1-F(\epsilon_d)]}{r+\lambda F(\epsilon_d)}\left[\epsilon_d+\int_{e_d}^{e_u}[\frac{1-F(x)}{1-F(\epsilon_d)}]dx\right]$$

$$\left[\ \sigma(\epsilon_u-\epsilon_d)\frac{\eta}{\theta}+\frac{r+\lambda}{r+\lambda F(\epsilon_d)}\frac{\beta c}{1-\beta}\right]\frac{\partial\theta}{\partial\sigma}=\epsilon_u+\frac{\lambda[1-F(\epsilon_d)]}{r+\lambda F(\epsilon_d)}E(\epsilon\mid\epsilon\geq\epsilon_d)$$

Given that in the model $E(\epsilon)=0$ , then $E(\epsilon\mid\epsilon\geq\epsilon_d)>0$. The right hand side of the equation above is then positive and so is the term in the square brackets on the left-hand side. Therefore, $\frac{\partial\theta}{\partial\sigma}>0$

From (11) we get that

$$\frac{\partial\theta}{\partial\sigma}=\frac{\theta}{\sigma(\epsilon_u-\epsilon_d)\eta}\left[\epsilon_u-\epsilon_d-\sigma\frac{\partial\epsilon_d}{\partial\sigma}\right]$$

$$\frac{\partial\theta}{\partial\sigma}=\frac{\theta}{\sigma\eta}-\frac{\theta}{(\epsilon_u-\epsilon_d)\eta}\frac{\partial\epsilon_d}{\partial\sigma}\tag{13}$$

Replacing (13) into (10)

$$\epsilon_d+\frac{r+\lambda F(\epsilon_d)}{r+\lambda}\sigma\frac{\partial\epsilon_d}{\partial\sigma}=\frac{\beta c}{1-\beta}\left[\frac{\theta}{\sigma\eta}-\frac{\theta}{(\epsilon_u-\epsilon_d)\eta}\frac{\partial\epsilon_d}{\partial\sigma}\right]-\frac{\lambda}{r+\lambda}\int_{e_d}^{e_u}[1-F(x)]dx$$

$$\left[\frac{r+\lambda F(\epsilon_d)}{r+\lambda}\sigma+\frac{\beta c}{1-\beta}\frac{\theta}{(\epsilon_u-\epsilon_d)\eta}\right]\frac{\partial\epsilon_d}{\partial\sigma}=\frac{\beta c}{1-\beta}\frac{\theta}{\sigma\eta}-\epsilon_d-\frac{\lambda}{r+\lambda}\int_{e_d}^{e_u}[1-F(x)]dx$$

Multiply everything by $\sigma$

$$\sigma\left[\frac{r+\lambda F(\epsilon_d)}{r+\lambda}\sigma+\frac{\beta c}{1-\beta}\frac{\theta}{(\epsilon_u-\epsilon_d)\eta}\right]\frac{\partial\epsilon_d}{\partial\sigma}=\frac{\beta c}{1-\beta}\frac{\theta}{\eta}-\sigma\epsilon_d-\frac{\lambda\sigma}{r+\lambda}\int_{e_d}^{e_u}[1-F(x)]dx\tag{14}$$

Equation (1) implies

$$p-b-\frac{\beta c}{1-\beta}\frac{v}{u}=-\sigma\epsilon_d-\frac{\sigma\lambda}{r+\lambda}\int_{e_d}^{e_u}[1-F(x)]dx\tag{15}$$

Replacing (15) into (14)

$$\sigma\left[\frac{r+\lambda F(\epsilon_d)}{r+\lambda}\sigma+\frac{\beta c}{1-\beta}\frac{\theta}{(\epsilon_u-\epsilon_d)\eta}\right]\frac{\partial\epsilon_d}{\partial\sigma}=\frac{\beta c}{1-\beta}\frac{\theta}{\eta}+p-b-\frac{\beta c}{1-\beta}\theta$$

$$\sigma\left[\frac{r+\lambda F(\epsilon_d)}{r+\lambda}\sigma+\frac{\beta c}{1-\beta}\frac{\theta}{(\epsilon_u-\epsilon_d)\eta}\right]\frac{\partial\epsilon_d}{\partial\sigma}=p-b+\frac{\beta c\theta}{1-\beta}\left(\frac{1}{\eta}-1\right)\tag{16}$$

Note that the term in the square brackets on the left-hand side is positive and, if $p \geq b$, the term in the right hand side will be positive. Then $\frac{\partial \epsilon_d}{\partial \sigma} > 0$

To analyze this case graphically, is not evident from (1) whether the job destruction curve will shift up or down as a consequence of a change in $\sigma$. To find that out notice that differentiating (1) w.r.t. $\sigma$ keeping $\theta$ constant we get

$\epsilon_d + \sigma \frac{\partial \epsilon_d}{\partial \sigma} = -\frac{\lambda}{r+\lambda} \int_{e_d}^{e_u} [1 - F(x)]dx + \frac{\sigma\lambda}{r+\lambda} \frac{\partial \epsilon_d}{\partial \sigma} - \frac{\sigma\lambda}{r+\lambda} F(\epsilon_d) \frac{\partial \epsilon_d}{\partial \sigma}$

$\left[\frac{r}{r+\lambda} + \frac{\lambda}{r+\lambda} F(\epsilon_d)\right] \sigma \frac{\partial \epsilon_d}{\partial \sigma} = -\epsilon_d - \frac{\lambda}{r+\lambda} \int_{e_d}^{e_u} [1 - F(x)]dx$

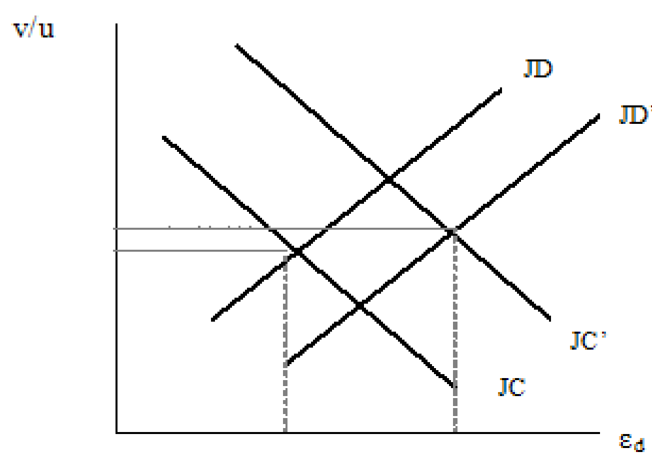
Multiplying both sides by $\sigma$

$$\left[\frac{\sigma r}{r+\lambda} + \frac{\sigma\lambda}{r+\lambda} F(\epsilon_d)\right] \sigma \frac{\partial \epsilon_d}{\partial \sigma} = -\sigma\epsilon_d - \frac{\lambda\sigma}{r+\lambda} \int_{e_d}^{e_u} [1 - F(x)]dx \qquad (17)$$

Replacing (15) into (17)

$\left[\frac{\sigma r}{r+\lambda} + \frac{\sigma\lambda}{r+\lambda} F(\epsilon_d)\right] \sigma \frac{\partial \epsilon_d}{\partial \sigma} = p - b - \frac{\beta c}{1-\beta} \frac{v}{u}$

The term in brackets on the left hand side is positive and the term in the right hand side is positive is the aggregate productivity component exceeds the opportunity of employment $(b + \frac{\beta c}{1-\beta} \frac{v}{u})$. Assuming that this last condition holds $\frac{\partial \epsilon_d}{\partial \sigma} > 0$ and so the job destruction curve shifts to the right.



If $p > b + \frac{\beta c}{1-\beta}\frac{v}{u}$, equation (1) implies that $\epsilon_d < 0$. An increase in $\sigma$ implies a decrease in the productive of the marginal job and therefore the surplus generated by it becomes negative. Therefore $\epsilon_d$ increases.

Business Cycles

An anticipated change in p causes $\epsilon_d$ and $\frac{v}{u}$ to jump on impact. "In contrast, unemployment is a sticky variable, since it changes according to the laws governing the matching technology. The differential equation describing the evolution of unemployment for a given price p

$\dot{u} = (1-u)\lambda F(\epsilon_d) - um(\frac{v}{u}, 1)$

... When aggregate productivity falls, some marginal jobs are immediately destroyed and some vacancies close down. In contrast, when aggregate productivity increases new vacancies are opened up but nothing happens to employment on impact. This asymmetry will turn out to have an important cyclical implication for the behavior of the job creation and job destruction rate." (Mortensen&Pissarides, 1994)

" After a positive shock to aggregate productivity, firms open up more job vacancies and hold on to more jobs after unfavorable job-specific shocks. Thus the job creation rate, $vq(v/u)$, increases and the job destruction rate decreases, inducing a fall in employment. Since neither $v/u$ nor $\epsilon_d$ have dynamics of its own at given $p$, the decrease in unemployment induces a fall in job creation (to maintain $v/u$ constant $v$ has to fall when $u$ falls) and an increase in job destruction, until there is convergence to a new steady state, or until there is a new cyclical shock.

When the aggregate productivity falls, the dynamics of job creation follow a pattern similar to that after the price increase: $v/u$ falls once for all, job creation falls on impact but again increases as unemployment begins to rise. The dynamics of job destruction, however, are different, because the rise in the reservation productivity leads to an immediate destruction of all the jobs with idiosyncratic components between the two reservation productivity. Job destruction also rises for reasons similar to the ones that led to its decrease when the aggregate productivity increased, since with higher reservation productivity firms are more likely to destroy jobs as they are hit by job-specific shocks. But the increase in job destruction immediately after the cyclical downturn has no counterpart in the behavior of the job destruction rate when price increases, or in the behavior of the job creation rate. This imparts a cyclical asymmetry in the job destruction rate and in the dynamic behavior of unemployment. The short-run cyclicality of the job destruction rate increases, the job destruction rate leads the job creation rate as a cause of the rise in unemployment and the speed of change of unemployment at the start of recession is faster than its speed of change at the start of the boom" (Mortensen&Pissarides, 1994)

Part C

Strong assumptions made

Matching function and Nash-bargaining assumption are black boxes.

Bargaining power exogenously given.

Wages after the shock do not depend on wages before the shock.

Firms and workers homogeneity. All workers and firms have the same intrinsic productivity. All workers face the same probability of getting a job or being fired.

"Shimer(2004) studies the model quantitatively, pointing out that it predicts relatively small fluctuations in $m(u,v)/u$ in response to reasonably calibrated shocks to $y$ or $\lambda$. Costain-Reiter (2003) argue that if one calibrates to generate realistic business cycle fluctuations in the job-finding rate, the model predicts unrealistically large impact of unemployment insurance in the job-finding rate." (Rogerson, Shimer and Wright, 2004)