Solutions to Haltiwanger's August 2006 Comp Question

Question 1

Households maximize $\sum_{t=0}^{\infty} \beta^t U(C_t)$

Firms production function $Y_{it} = A_i(n_{it} - f)^\gamma \qquad \gamma < 1$

$\pi_{it} = A_i(1-\tau_i)(n_{it} - f)^\gamma - w_t n_{it}$

Ex-ante joint productivity and distortion distribution is given by $G(A, \tau)$

Entry fee c_e. Exit probability λ.

$\beta(1+R) = 1$

Part i

Assuming that overhead labor is fixed, the optimal employment problem for the firms already in the market is given by

$$\max_{n_{it}} \{A_i(1-\tau_i)(n_{it} - f)^\gamma - w_t n_{it}\}$$

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$$\gamma A_i(1-\tau_i)(n_{it} - f)^{\gamma-1} - w_t = 0$$
$$(n_{it} - f)^{\gamma-1} = \frac{w_t}{\gamma A_i(1-\tau_i)}$$
$$n_{it} = \left(\frac{\gamma A_i(1-\tau_i)}{w_t}\right)^{1/1-\gamma} + f$$

In a non-distorted economy $\tau_i = 0$ and at an optimum $\gamma A_i(n_{it} - f)^{\gamma-1} = w_t$. That is, firms hire an extra units of labor until the marginal product of labor (MPL) equals the marginal cost of hiring that extra unit of labor. In the case of the distorted economy $MPL \lesseqgtr w_t$. depending on whether $\tau_i \gtreqless 0$. This implies that whenever a positive (negative) distortion is present firms optimal employment will be lower (higher) than optimal employment in the case without distortions. In what follows we assume that $\tau_i > 0$. Most of the statements get reverted in the case of $\tau_i < 0$.

To look at the role of overhead labor, note that if we replace optimal n_{it} into the production function we get

$$Y_{it} = A_i \left(\frac{\gamma A_i(1-\tau_i)}{w_t}\right)^{\gamma/1-\gamma}$$

Total output is indedent of f and so is the marginal productivity of labor. The average productivity of labor is $\frac{Y_{it}}{n_{it}}$

$$ALP_i = \frac{A_i(1-\tau_i)\left(\frac{\gamma A_i(1-\tau_i)}{w_t}\right)^{\gamma/1-\gamma}}{\left(\frac{\gamma A_i(1-\tau_i)}{w_t}\right)^{1/1-\gamma}+f}$$

$$ALP_i = \frac{(A_i(1-\tau_i))^{1/1-\gamma}\left(\frac{\gamma}{w_t}\right)^{\gamma/1-\gamma}}{(A_i(1-\tau_i))^{1/1-\gamma}\left(\frac{\gamma}{w_t}\right)^{1/1-\gamma}+f}$$

$$ALP_i = \frac{\left(\frac{\gamma}{w_t}\right)^{\gamma/1-\gamma}}{\left(\frac{\gamma}{w_t}\right)^{1/1-\gamma}+f(A_i(1-\tau_i))^{1/\gamma-1}}$$

Then,

$$\frac{\partial ALP_i}{\partial f} = -\frac{\left(\frac{\gamma}{w_t}\right)^{\gamma/1-\gamma}(A_i(1-\tau_i))^{1/\gamma-1}}{\left[\left(\frac{\gamma}{w_t}\right)^{1/1-\gamma}+f(A_i(1-\tau_i))^{1/\gamma-1}\right]^2} < 0$$

<u>Part ii</u>

Let's first derive the firm's profit once in the market. To do that plug equilibrium n_{it} into the profit function

$$\pi_{it} = A_i(1-\tau_i)\left(\frac{\gamma A_i(1-\tau_i)}{w_t}\right)^{\gamma/1-\gamma} - w_t\left(\frac{\gamma A_i(1-\tau_i)}{w_t}\right)^{1/1-\gamma} - w_t f$$

$$\pi_{it} = [A_i(1-\tau_i)]^{1/1-\gamma}\left(\frac{\gamma}{w_t}\right)^{\gamma/1-\gamma} - w_t\left(\frac{\gamma}{w_t}\right)^{1/1-\gamma}(A_i(1-\tau_i))^{1/1-\gamma} - w_t f$$

$$\pi_{it} = \left[\left(\frac{\gamma}{w_t}\right)^{\gamma/1-\gamma} - \gamma\frac{w_t}{\gamma}\left(\frac{\gamma}{w_t}\right)^{1/1-\gamma}\right][A_i(1-\tau_i)]^{1/1-\gamma} - w_t f$$

$$\pi_{it} = \left[\left(\frac{\gamma}{w_t}\right)^{\gamma/1-\gamma} - \gamma\left(\frac{\gamma}{w_t}\right)^{\gamma/1-\gamma}\right][A_i(1-\tau_i)]^{1/1-\gamma} - w_t f$$

$$\pi_{it} = [1-\gamma]\left(\frac{\gamma}{w_t}\right)^{\gamma/1-\gamma}[A_i(1-\tau_i)]^{1/1-\gamma} - w_t f$$

$$\pi_{it} = (1-\gamma)\left[A_i(1-\tau_i)\left(\frac{\gamma}{w_t}\right)^{\gamma}\right]^{1/1-\gamma} - w_t f \equiv \Pi_t(A,\tau)$$

Now, note that in period $t+1$ the firm can either still be producing or it can exit the market. The probability that the firm will still be producing is $(1-\lambda)$. The profit if producing is given by $\Pi_{t+1}(A,\tau)$. The firm exists the market with probability λ and the profits in that case are 0. So, expected profits in period $t+1$ conditional on producing in period t and being a type (A_i,τ_i) firm is $(1-\lambda)\,\Pi_{t+1}(A,\tau)$.

For period $t+2$ the probability of being producing is $(1-\lambda)^2$ (probability of not exiting between period t and $t+1$ times the probability of not exiting between period $t+1$ and $t+2$). Expected profits in period $t+2$ conditional on producing in period t and being a type (A_i, τ_i) firm is $(1-\lambda)^2 \Pi_{t+2}(A, \tau)$.

If one keep iterating and takes the present discounted value of expected future profits conditional on being a type (A_i, τ_i) firm is $\sum_{j=1}^{\infty} \beta^j (1-\lambda)^j \Pi_{t+2}(A, \tau)$

We still need to look at period t profits. If the firm decides to enter, period t profits are given by

$$\pi_{it} = [1-\gamma] \left(\frac{\gamma}{w_t}\right)^{\gamma/1-\gamma} [A_i(1-\tau_i)]^{1/1-\gamma} - w_t f - c_e = \Pi_t(A, \tau) - c_e$$

Then expected stream of profit conditional on being a type (A_i, τ_i) firm is $\sum_{j=0}^{\infty} \beta^j (1-\lambda)^j \Pi_t(A, \tau) - c_e \equiv W(A, \tau) - c_e$

Then, the free entry condition is given by $\int \int W(A, \tau) g(A, \tau) dA d\tau - c_e = 0$ where $g(A, \tau)$ is the joint density function implied by $G(A, \tau)$. That is, firms enter the market until the point when the expected benefits from entry equal the cost of entry.

Assumptions: The set of firms has measure m. Firms can either be active or inactive (not producing). The measure of inactive firms at a point in time is given by I_t. There's also infinitely many agents in the economy. The set of agents has measure 1.

To analyze the effects of a change in the exit rate note that at a steady state the measure of active firms in the market $(m-I)$ is fixed. For this to happen the measure of firms entering has to equal to number of firms exiting, that is $E = \lambda(m-I)$, where E_t is the number of firms entering the market in a given period. A higher exit rate has two effects in the steady state. On the one hand, for a fixed level of active firms and real wages, a higher exit rate increases the measure of firms exiting the market in steady state. This has a positive effect on the measure of firms entering the market in the steady state. On the other hand, for a fixed level of real wages, a higher exit rate decreases benefits from entry which in turn reduces the measure of active firms and the measure of firms in the market. The overall effect on the new steady state equilibrium is then ambiguous. Note that in our argument we are keeping real wages fixed. Given that we are working with a general equilibrium model real wages are not fixed. We cannot calculate the exact general equilibrium effect because we do not have a functional form for the joint distribution $G(A, \tau)$.

Also note that π_{it} is concave in A_i and τ_i. This implies that expected profit valuated at the mean values of A_i and τ_i is lower that profits valuated at the mean values. Because of this reduction there is fewer firms in the market.

Part iii

We have two markets in this economy: the labor market and the goods market. The labor market equilibrium condition is given by

$$(m - I_t) \int \int \left[\left(\frac{\gamma A_i (1-\tau_i)}{w_t} \right)^{1/1-\gamma} + f \right] g(A, \tau) dA d\tau = 1$$

Note that we are taking advantage of the law of large numbers and use $g(A, \tau)$ to aggregate up.

The good's market equilibrium condition establishes that aggregate output equals aggregate consumption plus resources spent on entry

$$(m - I_t) \int \int A_i (1 - \tau_i) \left(\frac{\gamma A_i (1-\tau_i)}{w_t} \right)^{\gamma/1-\gamma} g(A, \tau) dA d\tau = C_t + E_t c_e$$

The real wages is playing two roles: 1) real wages adjusts to clear the labor market. In this model the labor market is perfectly competitive. Given firm's labor demand, wages adjust so that the measure of employed agents equal one. 2) real wages affect aggregate output. Higher wages imply a higher wage. This reduces the profitability of producing, fewer firms then enter the market which affects overall production. To better understand the dynamic, think of a situation with too many active firms in the market. Given that there is no endogenous exit, this situation is going to persist for some periods. If we have to many firms demanding labor, real wages are originally high. High wages imply that firms will not enter the market. Because some firms are leaving, the amount of active firms start falling. As the amount of active firms fall, labor demand falls which reduces real wages. Real wages will go on falling until a point where firms inactive firms are indifferent between entering the market or not.

Part iv

Above we obtained that
$LP_i = \frac{\left(\frac{\gamma}{w_t}\right)^{\gamma/1-\gamma}}{\left(\frac{\gamma}{w_t}\right)^{1/1-\gamma} + f(A_i(1-\tau_i))^{1/\gamma-1}}$

For any given firm

$$\frac{\partial LP_i}{\tau_i} = \frac{A_i (A_i(1-\tau_i))^{\frac{2-\gamma}{\gamma-1}} \left(\frac{\gamma}{w_t}\right)^{\gamma/1-\gamma} f}{(\gamma-1)\left[\left(\frac{\gamma}{w_t}\right)^{1/1-\gamma} + f(A_i(1-\tau_i))^{1/\gamma-1}\right]^2} < 0 \ (> 0) \qquad \text{if } \tau_i > 0 \ (\tau_i < 0)$$

To see what happens at an aggregate level we just need to look at total output (remember that labour supply has been normalized to one)

$$(m-I_t)\int\int (A_i(1-\tau_i))^{1/1-\gamma}\left(\frac{\gamma}{w_t}\right)^{\gamma/1-\gamma} dAd\tau$$

An increase in τ_i has to effects. At a micro level, for a given real wage, it reduces the firm's labor demand and output. At a macro level, it reduces the number of active firms $(m-I_t)$. Then economies with higher τ_is have lower output.

To analyze welfare we have to see what happens to consumption. Remember that

$$C_t=(m-I_t)\int\int A_i(1-\tau_i)\left(\frac{\gamma A_i(1-\tau_i)}{w_t}\right)^{\gamma/1-\gamma} dAd\tau - E_t c_e$$

We know already that the first term on the right hand side falls when τ_i goes up. We need to analyze what happens with E both during the transition and in the new steady state in order to make welfare evaluations. Assume the economy was originally at a steady state. In the original steady state $E_t=\lambda(m-I_t)$. Then the steady state level of consumption was

$$C_t=(m-I_t)\left[\int\int A_i(1-\tau_i)\left(\frac{\gamma A_i(1-\tau_i)}{w_t}\right)^{\gamma/1-\gamma} dAd\tau-\lambda c_e\right]$$

After an increase in τ_i there are too many firms producing and the economy has to increase the number of inactive firms in order for the free entry condition to hold. During the transition both production and expenditure on entry go down, so consumption could potentially go up. In the new steady state we know that $(m-I_t)$ and $\int\int A_i(1-\tau_i)\left(\frac{\gamma A_i(1-\tau_i)}{w_t}\right)^{\gamma/1-\gamma} dAd\tau$ are lower and λc_e remains the same. So consumption unambiguously goes down in the steady state.

As stated before we would not expect an economy with a zero average distortion but positive dispersion on τ_i to behave in the same way as one with no distortions. This is due to the concavity of the profit function and how this affect the entry decision.

Part v

Suppose that now the firm faces costs of adjusting the labor input. Let's call this cost $C(n_{it},n_{it-1})$

The Bellman Equation for an active firm problem

$$V^A(n_{it-1})=\max_{n_{it}}\left\{A_i(n_{it}-f)^\gamma-w_t n_{it}-C(n_{it},n_{it-1})+\beta(1-\lambda)V^A(n_{it})+\beta\lambda V^I\right\}$$

The problem of an inactive firm is whether to become active or not.

$$V^I = \max\left\{EV^A(0) - c_e; 0\right\}$$

Note that compared to case of positive distortions ($\tau_i > 0$) for any level of employment, labor productivity is now higher. If there are no idiosyncratic or aggragate shocks, asymptotically firms would demand more labor and given the inelastic supply wages will be higher. If for some reason firms need to adjust labor input, one would now observe smoother adjustment than in a case with distortions because adjusting is now more costly.