Solution to Haltiwanger's August 2007 Comp Question
(Sketch of Solutions)

Part i

Firm Value Function

$$V(e_{t-1}, a_t, \varepsilon_{t-1}) = \max_{v_t, f_t} \{E_t [\pi(e_t, a_t, \varepsilon_t)] - V(v_t) - F(f_t) + \beta E_t [V(e_t, a_{t+1}, \varepsilon_t)]\} \tag{1}$$

subject to

$$e_t = (1 - \bar{q} - f_t)e_{t-1} + H(\theta)v_t \tag{2}$$

$$\pi(e_t, a_t, \varepsilon_t) = \arg\max_{w(\cdot),h(\cdot)} \left\{ \begin{array}{l} \int [a_t\varepsilon_t (e_t h(e_t, a_t, \varepsilon))^\alpha - \omega(e_t, a_t, \varepsilon)e_t] f(\varepsilon)d\varepsilon \\ \text{subject to worker's participation constraint.} \end{array} \right\} \tag{3}$$

$$V(v_t) = \begin{cases} F^+ - c^+ v_t & \text{if } v_t > 0 \\ 0 & \text{if } v_t = 0 \end{cases} \tag{4}$$

$$F(f_t) = \begin{cases} F^- - c^- f_t & \text{if } f_t > 0 \\ 0 & \text{if } f_t = 0 \end{cases} \tag{5}$$

Searcher' Value Function

$$S(a) = U(b(a)) + \beta(1 - \phi(\theta))E_t [S(a_{t+1})] + \beta\phi(\theta)E_t [W(w(\cdot), h(\cdot))] \tag{6}$$

Worker Value Function

$$\begin{aligned} W(w(\cdot), h(\cdot)) \quad = \quad & \int U [w(\cdot) - g(h(\cdot))] f(\varepsilon)d\varepsilon + \beta\bar{q}(1 - \phi(\theta))E_t [S(a_{t+1})] + \\ & \beta [(1 - \bar{q}) + \bar{q}\phi(\theta)] E_t [W(w(\cdot), h(\cdot))] \end{aligned} \tag{7}$$

Part ii

The participation constraint is given by

$$\int U\left[\omega(e_t, a_t, \varepsilon) - g(h(e_t, a_t, \varepsilon))\right] f(\varepsilon) d\varepsilon = U(b(a)) \tag{8}$$

Note that the ex-post problem has no dynamic implications. Then, the ex-post problem of the firm is to maximize expected profits

$$\int \pi(e_t, a_t, \varepsilon) f(\varepsilon) d\varepsilon \equiv \int \left[a_t \varepsilon_t \left(e_t h(e_t, a_t, \varepsilon)\right)^{\alpha} - \omega(e_t, a_t, \varepsilon) e_t\right] f(\varepsilon) d\varepsilon$$

choosing wages and hours worked subject to (8).

FOC

$\omega(\cdot)\rfloor$

$$-e_t + \lambda U'(\cdot) = 0 \tag{9}$$

$h(\cdot)\rfloor$

$$\alpha a_t \varepsilon_t \left(e_t h(e_t, a_t, \varepsilon)\right)^{\alpha-1} e_t - \lambda U'(\cdot) g'(\cdot) = 0 \tag{10}$$

Given that e_t is a constant in the ex-post problem, $\omega(e_t, a_t, \varepsilon) - g(h(e_t, a_t, \varepsilon)$ is independent of ε. To see this rewrite equation (9) as

$$\omega(e_t, a_t, \varepsilon) - g(h(e_t, a_t, \varepsilon) = U'^{-1}\left(\frac{e_t}{\lambda}\right) \tag{11}$$

This last equation also implies that compensation and hours worked are positively correlated.

To see how wages vary with ε, combine equations (9) and (10) we get

$$\alpha a_t \varepsilon_t \left(e_t h(e_t, a_t, \varepsilon)\right)^{\alpha-1} e_t - e_t g'(\cdot) = 0$$

$$\alpha a_t \varepsilon_t \left(e_t h(e_t, a_t, \varepsilon)\right)^{\alpha-1} = g'\left(h(e_t, a_t, \varepsilon)\right) \tag{12}$$

Note that the left hand side of equation (12) above is strictly decreasing in $h(e_t, a_t, \varepsilon)$ while the right hand side is increasing in $h(e_t, a_t, \varepsilon)$. Then, higher ε_t requires higher $h(\cdot)$ for equation (12) to hold. Given that compensation and hours worked are positively correlated, higher ε_t also requires higher compensation.

Part iii

- The firm will never post vacancies and fire workers simultaneously given fix costs of hiring and firing.

- Fix costs of firing and posting vacancies generate ranges of inaction. There is no smoothing incentive given linearity.

- The higher the elasticity of b(a), better contracts (in terms of higher compensation or fewer hours) need to offered. This makes employment less responsive to aggregate shocks.