Macroeconomic Comprehensive Theory Examination (January 2002)

Sketch of the Solution – Haltiwanger's Question (#2)

Part (i)

Consider the following definitions:

$V_E^S \equiv$ Expected lifetime utility of an employed shirker
$V_E^N \equiv$ Expected lifetime utility of an employed non-shirker
$V_U \equiv$ Expected lifetime utility of an unemployed individual

Asset equation for a shirker:

$$rV_E^S = w + (\mathbf{d} + q)(V_U - V_E^S) \qquad (1)$$

For a non-shirker:

$$rV_E^N = w - e + \mathbf{d}(V_U - V_E^N) \qquad (2)$$

Solving for V_E^S in equation (1) and V_E^N in equation (2):

$$V_E^S = \frac{w + (\mathbf{d} + q)V_U}{(r + \mathbf{d} + q)} \qquad (4)$$

$$V_E^N = \frac{w - e + \mathbf{d}V_U}{(r + \mathbf{d})} \qquad (5)$$

Incentive compatibility constraint (non shirking condition):

$$V_E^N \geq V_E^S \quad \Leftrightarrow \quad \frac{w - e + \mathbf{d}V_U}{(r + \mathbf{d})} \geq \frac{w + (\mathbf{d} + q)V_U}{(r + \mathbf{d} + q)} \qquad (6)$$

Solving for w:

$$w \geq rV_U + \frac{e}{q}(r + \mathbf{d} + q) \equiv \hat{w} \quad \text{(NSC)} \qquad (7)$$

Part (ii)

The labor market equilibrium is given by the combination of wage and employment level at which firms have no incentives to change their behavior. It is determined by the intersection of the NSC and the firm's labor demand.

Before obtaining such equilibrium, we take advantage of another equation present in the model.

Asset equation for unemployed individual (assuming $V_E^N = V_E^S = V_E$):

$$rV_U = y + a(V_E - V_U) \qquad (8)$$

Solving for V_U:

$$V_U = \frac{y + aV_E}{(r+a)} \qquad (8')$$

Plug V_E from (5) into (8'):

$$V_U = \frac{y + a\dfrac{w - e + \mathbf{d}V_U}{(r+\mathbf{d})}}{(r+a)} = \frac{(r+\mathbf{d})y + a(w - e + \mathbf{d}V_U)}{(r+\mathbf{d})(r+a)} \qquad (9)$$

Solve for rV_U (not V_U):

$$rV_U = \frac{(r+\mathbf{d})y + a(w-e)}{(r+a+\mathbf{d})} \qquad (10)$$

Plug (10) back into (7):

$$w \geq \frac{(r+\mathbf{d})y + a(w-e)}{(r+a+\mathbf{d})} + \frac{e}{q}(r+\mathbf{d}+q) \qquad (11)$$

The following steps solve the above equation for *w*, detailing some additional algebra work:

$$w - \frac{a}{(r+\mathbf{d}+a)}w \geq \frac{(r+\mathbf{d})}{(r+a+\mathbf{d})}y - \frac{a}{(r+a+\mathbf{d})}e + \frac{e}{q}(r+\mathbf{d}+q) \quad \therefore$$

$$w\left(\frac{r+\mathbf{d}}{r+\mathbf{d}+a}\right) \geq \frac{(r+\mathbf{d})}{(r+a+\mathbf{d})}y - \frac{a}{(r+a+\mathbf{d})}e + \frac{e}{q}(r+\mathbf{d}+q) \quad \therefore$$

$$w \geq y - \frac{a}{(r+\mathbf{d})}e + \frac{e}{q}\frac{(r+\mathbf{d}+q)(r+\mathbf{d}+a)}{(r+\mathbf{d})} = y + e\left(\frac{(r+\mathbf{d}+q)(r+\mathbf{d}+a) - aq}{q(r+\mathbf{d})}\right) \quad \therefore$$

$$w \geq y + e\left(\frac{r^2 + r\mathbf{d} + ra + \mathbf{d}r + \mathbf{d}^2 + \mathbf{d}a + qr + q\mathbf{d} + qa - aq}{q(r+\mathbf{d})}\right) \quad \therefore$$

$$w \geq y + e\left(\frac{r(r+\mathbf{d}+a) + \mathbf{d}(r+\mathbf{d}+a) + q(r+\mathbf{d})}{q(r+\mathbf{d})}\right) = y + e\left(\frac{(r+\mathbf{d})(r+\mathbf{d}+a) + q(r+\mathbf{d})}{q(r+\mathbf{d})}\right)$$

Finally

$$w \geq y + e\left(\frac{(r+\mathbf{d}+a)+q}{q}\right) = y + e + \frac{e}{q}(r+\mathbf{d}+a) \qquad (12)$$

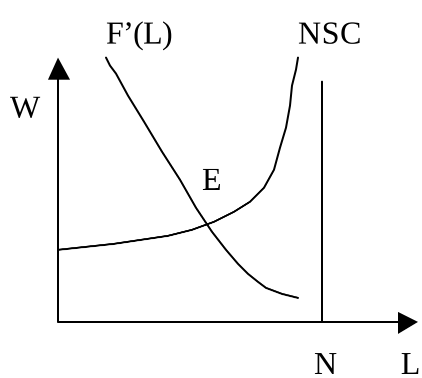
Equation (12) can be also expressed in terms of employment levels by exploiting the fact that in steady state the flow out of the unemployment pool must be equal to the flow into it. Then:

$$\mathbf{d}L = a(N-L) \quad \Leftrightarrow \quad a = \frac{\mathbf{d}L}{(N-L)} \qquad (13)$$

Plug (13) into (12):

$$w \geq y + \frac{e}{q}\left(r + \mathbf{d} + \frac{\mathbf{d}L}{N-L}\right) = y + \frac{e}{q}\left(r + \frac{N\mathbf{d}}{N-L}\right) \qquad \text{(NSC)} \quad (14)$$

Graphically:



The equilibrium in the labor market occurs at point E (intersection of NSC and labor demand).

Are wages rigid?

They are slow to adjust. In response to aggregate fluctuations, the equilibrium wage changes less than a Walrassian labor market would imply. Such phenomenon takes place because wages in this model cannot exclusively perform the task of attaining input allocational efficiency. In particular, wages are also used to alleviate informational problems by granting rents to employed workers and avoid shirking. Because of this dual role, shocks to the labor demand curve will be partially accommodated by changes in the employment level (even though the labor supply is inelastic).

Is there involuntary, inefficient unemployment?

In this model, even when the labor market is in equilibrium, jobs are not available to unemployed individuals willing to work for less than the market rate. So, there is involuntary equilibrium. The very existence of an unemployment pool is a result of the strategy adopted by firms to minimize the effects of their informational problems. Such informational problems also imply that the equilibrium of the model is not Pareto optimal.

Does this model exhibit persistence in response to a shock?

The unemployment level is partially determined by the severity of the informational problems in the model, and thus it presents persistency in response to aggregate shocks. Indeed, given its discipline device role, unemployment will not fully react to labor demand fluctuations.

Part (iii)

First case

Let's first deal with the case when the workers do not know their own current probabilities of being caught (that is, workers do not have private information about their detection likelihood). In this situation, we exploit the assumption of risk neutrality by writing the asset equation for a shirker as:

$$rV_E^S = w + \boldsymbol{d}(V_U - V_E^S) + \int_0^Q (V_U - V_E^S) q h(q) dq \qquad (15)$$

Or simply:

$$rV_E^S = w + (\boldsymbol{d} + \overline{q})(V_U - V_E^S), \quad \overline{q} = \int_0^Q q h(q) dq \qquad (16)$$

Under this specification, the mean of the *h(.)* distribution simply replaces the constant q of the original model and the solution of the model remains the same as before.

<u>Second case</u>

When a worker does have private information about his current detection likelihood, there is adverse selection in the labor market. In that event, it is not possible to talk about a single NSC anymore because there is a different critical wage associated with each draw q_i. The critical wage w_i, which determines the decision of each individual worker i regarding his shirk behavior, will be given by:

$$w_i \geq y + \frac{e}{q_i}\left(r + \frac{N\boldsymbol{d}}{N - L} \right) \qquad (17)$$

Note that to completely eliminate shirking behavior, the firm would have to set an infinite wage. Indeed, this is the only way to prevent shirking from agents with zero detection probability. Additionally, note that if (17) holds for a certain $\overline{q}_i$ then it must also hold for all q_i greater than $\overline{q}_i$.

Thus, when a firm chooses its wage rate and the employment level, it is also implicitly determining those NSC that will be met. Since the firm does not know the detection likelihood of each worker, it cannot fire (or not hire) low detection types. In general, and differently from the original formulation, there is going to be partial shirking under this specification.

The labor market equilibrium will entail involuntary unemployment as before, but in contrast to the original formulation, there may be shirking at any period. The equilibrium is going to be particularly sensitive to the shape of the *h(.)* distribution. This happens because, for example, a right-skewed *h(.)* distribution requires a higher wage (and thus higher unemployment pool) to induce the same number of non-shirkers that a left-skewed distribution would.

Part (iv) - Assume that "Phi" below denotes the bond B

In order to incorporate the possibility of workers posting bonds as suggested in the question, the asset equation for a shirker can be modified as:

$$rV_E^S = w + (\boldsymbol{d} + q)(V_U - V_E^S) - q\boldsymbol{f} \qquad (18)$$

Solving for V_E^S:

$$V_E^S = \frac{w + (\boldsymbol{d} + q)V_U - q\boldsymbol{f}}{(\boldsymbol{d} + q + r)} \qquad (19)$$

Equation (2) does not change and the incentive compatibility constraint is given by:

$$V_E^N \geq V_E^S \quad \Leftrightarrow \quad \frac{w - e + \boldsymbol{d}V_U}{(r+\boldsymbol{d})} \geq \frac{w + (\boldsymbol{d}+q)V_U - \boldsymbol{f}q}{(r+\boldsymbol{d}+q)} \qquad (20)$$

Solving for w:

$$w \geq rV_U + \frac{e}{q}(r+\boldsymbol{d}+q) - (r+\boldsymbol{d})\boldsymbol{f} \equiv \hat{w} \quad \text{(NSC)} \qquad (21)$$

It is already possible to see from the equation above that the performance bonds will reduce the critical wage.

Note that rV_U would still be given by (10):

$$rV_U = \frac{(r+\boldsymbol{d})y + a(w-e)}{(r+a+\boldsymbol{d})} \qquad (10)$$

Plug (10) into (21):

$$w \geq \frac{(r+\boldsymbol{d})y + a(w-e)}{(r+a+\boldsymbol{d})} + \frac{e}{q}(r+\boldsymbol{d}+q) - (r+\boldsymbol{d})\boldsymbol{f} \qquad (22)$$

In the same spirit of what was done in part (ii), we solve the above equation for w, detailing some additional algebra work:

$$w - \frac{a}{(r+\boldsymbol{d}+a)}w \geq \frac{(r+\boldsymbol{d})}{(r+a+\boldsymbol{d})}y - \frac{a}{(r+a+\boldsymbol{d})}e + \frac{e}{q}(r+\boldsymbol{d}+q) - (r+\boldsymbol{d})\boldsymbol{f} \quad \therefore$$

$$w\left(\frac{r+\boldsymbol{d}}{r+\boldsymbol{d}+a}\right) \geq \frac{(r+\boldsymbol{d})}{(r+a+\boldsymbol{d})}y - \frac{a}{(r+a+\boldsymbol{d})}e + \frac{e}{q}(r+\boldsymbol{d}+q) - (r+\boldsymbol{d})\boldsymbol{f} \quad \therefore$$

$$w \geq y - \frac{a}{(r+\boldsymbol{d})}e + \frac{e}{q}\frac{(r+\boldsymbol{d}+q)(r+\boldsymbol{d}+a)}{(r+\boldsymbol{d})} - (r+\boldsymbol{d}+a)\boldsymbol{f} \quad \therefore$$

$$w \geq y + e\left(\frac{(r+d+q)(r+d+a) - aq}{q(r+d)}\right) - (r+d+a)f \quad \therefore$$

$$w \geq y + e\left(\frac{r^2 + r\boldsymbol{d} + ra + \boldsymbol{d}r + \boldsymbol{d}^2 + \boldsymbol{d}a + qr + q\boldsymbol{d} + qa - aq}{q(r+\boldsymbol{d})}\right) - (r+\boldsymbol{d}+a)\boldsymbol{f} \quad \therefore$$

$$w \geq y + e\left(\frac{r(r+\boldsymbol{d}+a)+\boldsymbol{d}(r+\boldsymbol{d}+a)+q(r+\boldsymbol{d})}{q(r+\boldsymbol{d})}\right) - (r+\boldsymbol{d}+a)\boldsymbol{f} \quad \therefore$$

$$w \geq y + e\left(\frac{(r+\boldsymbol{d})(r+\boldsymbol{d}+a)+q(r+\boldsymbol{d})}{q(r+\boldsymbol{d})}\right) - (r+\boldsymbol{d}+a)\boldsymbol{f} \quad \therefore$$

$$w \geq y + e\left(\frac{(r+\boldsymbol{d}+a)+q}{q}\right) - (r+\boldsymbol{d}+a)\boldsymbol{f} = y + e + \frac{e}{q}(r+\boldsymbol{d}+a) - (r+\boldsymbol{d}+a)\boldsymbol{f} \quad \therefore$$

Finally,

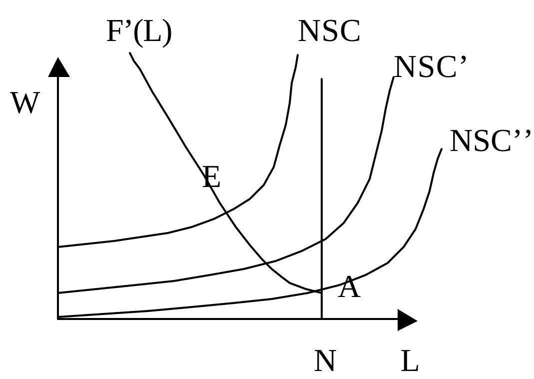
$$w \geq y + e + (r+\boldsymbol{d}+a)\left(\frac{e}{q} - \boldsymbol{f}\right) \qquad (23)$$

Now plug (13) into (23):

$$w \geq y + \left(\frac{e}{q} - \boldsymbol{f}\right)\left(r + \boldsymbol{d} + \frac{\boldsymbol{d}L}{N-L}\right) = y + \left(\frac{e}{q} - \boldsymbol{f}\right)\left(r + \frac{N\boldsymbol{d}}{N-L}\right) \qquad \text{(NSC)} \quad (24)$$

As the value of bond $\boldsymbol{f}$ increases, the non-shirking condition curve shifts to the right. Note that it is possible to eliminate all involuntary equilibrium and still prevent shirking from workers. Essentially, bond posting is an alternative compensation scheme that eliminates the need for unemployment as a discipline device.

Graphically:



The economy can now attain the efficient Walrassian equilibrium as long as $\boldsymbol{f}$ is set appropriately (that is, as long as it forces the equilibrium wage to occur at the point where supply and demand for labor meet).

Thus, the optimal bond is implied by the first-best solution in the labor market and, given that supply of labor is inelastic, the optimal f can be determined by first obtaining the walrassian equilibrium wage as follows:

$$ef'(eN) = w^* \qquad (25)$$

Where N is the total supply of labor.

Then, the optimal f^* should be such that:

$$w^* = y + \left(\frac{e}{q} - f^*\right)\left(r + \frac{Nd}{N - L}\right) \qquad \text{(NSC)} \quad (26)$$

Even though bond posting works nicely in theory, market incompleteness prevent it from being a common alternative compensation scheme. Nevertheless, one may point to indirect ways of dealing with the problem. For example, a worker with a mortgage on his home may find it optimal not to shirk, since being fired would imply a big loss of utility to him. In this case, the firm would be certain that the worker would always exert effort, and there would be no need to use unemployment as a discipline device.