Macro Comp - January 2004

Question 3 (Haltiwanger)

$$V(A, L_{-1}) = \text{Max } R(A,L,H) - w(L,H) - C(L_{-1},L) + \beta E V(A', L) \quad (1)$$

s.t. $R(A,L,H) = A(LH)^{\alpha}$ (2)

$w(L,H) = L(w_0 + w_1 H^{\varepsilon})$ (3)

$C(L_{-1},L) = F(I) + (v/2)((L-L_{-1})/L_{-1})^2$ (4)

A: profit shock, L: employment, H: hours per worker, L_{-1} is the inherited stock of workers, I indicator variable ($I=0$ if $L=L_{-1}$)
$\alpha, \varepsilon, v, F, w_0, w_1$ are positive constants.

a) Suppose that $F = v = 0 \Rightarrow C(L_{-1},L) = 0$ (no adj. cost)

Problem can be solved period-by-period:

Max $R(A,L,H) - w(L,H)$ s.t. $R(A,L,H) = A(LH)^{\alpha}$ (5)
$w(L,H) = L(w_0 + w_1 H^{\varepsilon})$

$$\text{Max } A(LH)^{\alpha} - L(w_0 + w_1 H^{\varepsilon}) \quad (6)$$

FOC:

$L: \quad \alpha A H^{\alpha} L^{\alpha-1} = w_0 + w_1 H^{\varepsilon} \quad (7)$

$H: \quad \alpha A L^{\alpha} H^{\alpha-1} = L \varepsilon w_1 H^{\varepsilon-1} \quad (8)$

Divide (7) by (8)

$$\frac{\alpha A H^{\alpha} L^{\alpha-1}}{\alpha A L^{\alpha} H^{\alpha-1}} = \frac{w_0 + w_1 H^{\varepsilon}}{L \varepsilon w_1 H^{\varepsilon-1}} \therefore \frac{H}{L} = \frac{w_0 + w_1 H^{\varepsilon}}{L \varepsilon w_1 H^{\varepsilon-1}} \quad (9)$$

Solve (9) for H:

$$H \varepsilon w_1 H^{\varepsilon-1} = w_0 + w_1 H^{\varepsilon} \therefore \varepsilon w_1 H^{\varepsilon} = w_0 + w_1 H^{\varepsilon} \therefore$$

$$H^{\varepsilon} w_1 (\varepsilon - 1) = w_0 \therefore H^{\varepsilon} = \frac{w_0}{w_1(\varepsilon-1)} \therefore H^* = \left(\frac{w_0}{w_1(\varepsilon-1)}\right)^{\frac{1}{\varepsilon}} \quad (10)$$

You can use H^* from (10) in either (7) or (8) to obtain L^*. Note that convexity of $w(L,H)$ in H is sufficient to assume that $\varepsilon > 1$ $\left[\frac{\partial^2 w(L,H)}{\partial H^2} = L\varepsilon(\varepsilon-1) w_1 H^{\varepsilon-2} > 0\right]$ and thus $H^* > 0$.

The optimum has the property that the firm adjusts L^* instantaneously in response to any profit shock A. Also note that H^* is independent of A, so it does not fluctuate at all over time. All adjustment occurs through L.

As long $\varepsilon > 1$, $\uparrow \varepsilon \Rightarrow \downarrow H^*$

b) Now we use dynamic prog. to solve the problem (note we can no longer solve it period-by-period because L_{-1} is a relevant state variable, but at the same time the problem is still a nice concave one and we can solve analytically)

$$V(A,L_{-1}) = \text{Max}\left\{A(LH)^{\alpha} - L(\omega_0+\omega_1 H^{\varepsilon}) - (\upsilon/2)((L-L_{-1})/L_{-1})^2 + \beta E V(A',L)\right\} \quad (11)$$

FOCs:

(H): $\alpha A L^{\alpha} H^{\alpha-1} - \varepsilon L \omega_1 H^{\varepsilon-1} = 0 \quad (12)$

(L): $A\alpha L^{\alpha-1} H^{\alpha} - (\omega_0+\omega_1 H^{\varepsilon}) - \upsilon((L-L_{-1})/L_{-1}) \cdot 1/L_{-1} + \beta E V_2(A',L) = 0 \quad (13)$

BS:

(L_{-1}): $V_2(A,L_{-1}) = -\upsilon((L-L_{-1})/L_{-1}) \cdot \frac{L}{L_{-1}^2} \cdot (-1) \quad (14)$

FROM (12): $\alpha A L^{\alpha} H^{\alpha-1} = \varepsilon L \omega_1 H^{\varepsilon-1} \therefore \frac{\alpha A H^{\alpha-1-\varepsilon+1}}{\varepsilon} = L^{1-\alpha} \therefore$

$\frac{\alpha A}{\varepsilon} H^{\alpha-\varepsilon} = L^{1-\alpha} \therefore \frac{\alpha A}{\varepsilon L^{1-\alpha}} = H^{\varepsilon-\alpha}$

$$\Rightarrow H^* = \left(\frac{\alpha A}{\varepsilon L^{1-\alpha}}\right)^{\frac{1}{\varepsilon-\alpha}} \quad (15)$$

H^* now depends on L and it is not insulated from A shocks. H^* and L^* are negatively related. It is still the case that $\uparrow \varepsilon \Rightarrow \downarrow H^*$ (all else constant)

Now we need to get the Euler for employment dynamics (recall that the dynamic behavior of L affects H via (15), something that did not happen in part a)

Update (14):

$$V_2(A', L) = \upsilon((L'-L)/L) \cdot L'/L^2 \quad (16)$$

Combine (16) and (13):

$$A\alpha L^{\alpha-1} H^{\alpha} - (w_0 + w_1 H^{\varepsilon}) - \upsilon(L - L_{-1})/L_{-1}^2 = -\beta E\, \upsilon((L'-L)) \cdot L'/L^3 \quad (17)$$

One can use (15) to express (17) only in terms of parameters, exogenous state (A), and Employment in t and t-1. This is the Euler equation that fully describes the behavior of employment. The model features only smoothing incentives and if adjusting employment is very expensive, hours will fluctuate more in response ~~the~~ to profitability shocks A.

c) If $\upsilon = 0$ and $F > 0$, then adjustment costs are purely non-convex. There will periods without any adjustment in the ~~labor~~ employment level (during which hours will absorbing A shocks) and discrete ~~periods~~ adjustment episodes of L.

the Form of the solution will be something like

$$V(A,L_{-1}) = \text{Max}\{\underset{I=1}{V^A}, \underset{I=0}{V^{NA}}\} \quad (18)$$

where $V^A(A,L_{-1}) = \underset{L,H}{\text{Max}}\{A(LH)^{\alpha} - L(w_0 + w_1 H^{\varepsilon}) - \overset{\neq 0}{F(1)} + \beta EV(A',L)\}$

$$V^{NA}(A,L_{-1}) = \underset{H}{\text{Max}}\{A(L_{-1}H)^{\alpha} - L_{-1}(w_0 + w_1 H_{-1}^{\varepsilon}) - \overset{F(0)}{0} + \beta EV(A',L)\}$$

when $L = L_{-1}$

c) if $\upsilon > 0$ and $F > 0$ there is a mix of smoothing and non-convex adjustment in the model. It is hard to say which one will dominate or it depends on the parameters, but one can expect that our conclusions on parts b and a are going to be present.

Form of the solution

$$V(A,L_{-1}) = \text{Max}\{\underset{I=1}{V^A}, \underset{I=0}{V^{NA}}\} \quad (19)$$

where $V^A(A,L_{-1}) = \underset{L,H}{\text{Max}}\{A(LH)^{\alpha} - L(w_0 + w_1 H^{\varepsilon}) - F(1) - (\upsilon/2)((L-L_{-1})/L_{-1})^2 + \beta EV(A',L)\}$

$$V^{NA}(A,L_{-1}) = \underset{H}{\text{Max}}\{A(L_{-1}H)^{\alpha} - L_{-1}(w_0 + w_1^{\varepsilon}) - 0 + \beta EV(A',L)\}$$