Solution to Haltiwanger's question in January 2005 Comp

PART i

Value of a vacancy

$$rV = -c + \frac{m(u,v)}{v}[J(1) - V] \tag{1}$$

where $J(x_1)$ is the value for the firm of a filled job with an idiosyncratic component x_1. We wrote J(1) in the expression above because at the time of creation of a match x=1 by assumption.

Value of a filled job

$$rJ(x) = px - w(x) + \delta(1-\beta)\int\{\max[S(z),0] - S(x)\}dz \tag{2}$$

where S(z) is the surplus of a match with idiosyncratic component z.

Given the Nash bargaining assumption,

$$w(x) = \beta px + (1-\beta)rU \tag{3}$$

where U is the value of being unemployed and searching. Then, we can write the value of a filled job as

$$rJ(x) = (1-\beta)px - (1-\beta)rU + \delta(1-\beta)\int\{\max[S(z),0] - S(x)\}dz \tag{4}$$

Value of being unemployed and searching

$$rU = b + \frac{m(u,v)}{u}[W(1) - U] \tag{5}$$

where W(x) is the value to workers of a job with idiosyncratic component x and it is given by

$$rW(x) = \beta px + (1-\beta)rU + \delta\beta\int\{\max[S(z),0] - S(x)\}dz \tag{6}$$

The Nash Bargaining assumption implies that $W(1) - U = \beta S(1)$, so (5) can be rewritten as

$$rU = b + \frac{m(u,v)}{u}\beta S(1) \tag{7}$$

Part ii

Adding up (4) and (6) and subtracting (7) we get

$$rS(x) = px - b + \delta\int\{\max[S(z),0] - S(x)\}dz - \beta\frac{m(u,v)}{u}S(1)$$

This last expression can be rewritten as

$$(r+\delta)S(x) = px - b + \delta\int\max[S(z),0]dz - \beta\frac{m(u,v)}{u}S(1) \tag{8}$$

The firm will keep a worker only if the surplus of the match is greater or equal to zero. This condition determine the job destruction equation, which is given by

$$px_d = b + \frac{\beta c}{1-\beta}\frac{v}{u} - \frac{\delta p}{r+\delta}\int_{x_d}^{1}(1-F(z))dz \tag{9}$$

where x_d is the value of the idiosyncratic component for which the match surplus is zero. The firm will keep posting vacancies until the value of a vacancy is zero. At that point V=0 and therefore (1) implies

$$c = \frac{m(u,v)}{v}J(1)$$

From Nash bargaining assumption we know that J(1)=(1-β)S(1). Therefore,

$$c = \frac{m(u,v)}{v}(1-\beta)S(1) \tag{10}$$

Now, evaluating equation (8) at x=1 and x=x_d and subtracting them we get

$$(r+\delta)[S(1) - S(x_d)] = p(1-x_d)$$

The definition of x_d implies that $S(x_d)=0$. This fact and the previous equation imply that

$$S(1) = \frac{p(1-x_d)}{(r+\delta)} \tag{11}$$

Replacing (11) in (10) we get

$$c = \frac{m(u,v)}{v}(1-\beta)\frac{p(1-x_d)}{(r+\delta)}$$

Rearranging we get the job creation equation

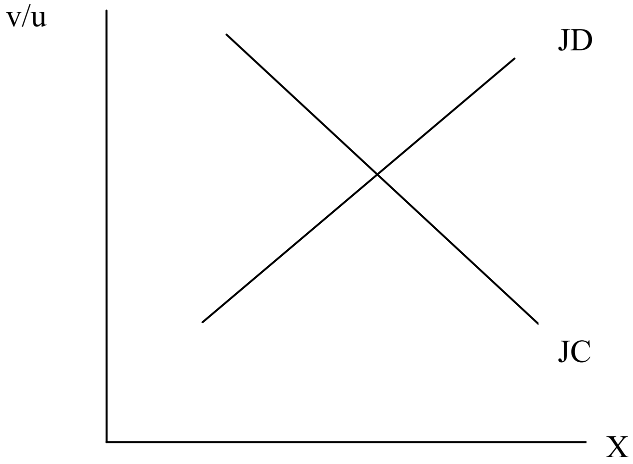
$$\frac{m}{v} = \frac{c}{1-\beta}\frac{r+\delta}{p(1-x_d)} \tag{12}$$

In a steady state the flow out from unemployment, which is given by m(1,v/u)u equals the flow into unemployment, $\delta F(x_d)$ (1-u). Then,

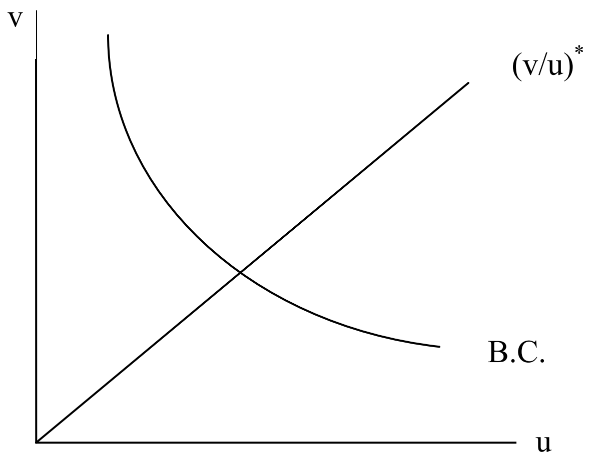
$$m(1,\frac{v}{u})u = \delta F(x_d)(1-u)$$

$$u = \frac{\delta F(x_d)}{\delta F(x_d) + m(1, v/u)} \qquad \text{Beveridge Curve}$$

In order to find the steady state, notice that the job destruction curve slopes up because for higher levels of v/u the opportunity cost of searching $\frac{\beta c}{1-\beta}\frac{v}{u}$ is higher and so x_d must be higher. Also, for higher values of x_d there is more job destruction and therefore less job creation. Graphically,



In order to get the steady state vacancies, we need to solve the system of equations formed by the job creation and job destruction flows and the Beveridge curve. The job creation and job destruction equations allow us to determine the equilibrium vacancy to unemployment ratio. Once we obtained that ratio, the Beveridge curve allows us to pin down the steady state level of vacancy and unemployment. Graphically,



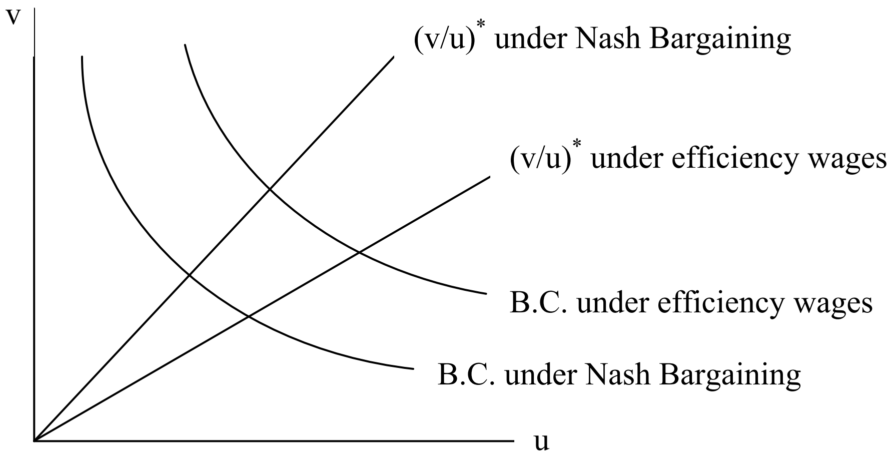
One important feature to notice in this model with search and matching frictions is that the average labor productivity in this economy is lower than the one that would be observed in a similar model without search and matching frictions. In a model with search and matching frictions the average productivity is going to be given by $E(px) = p\int_{Xd}^{1} xf(z)dz \leq$ p. The average productivity is going to be strictly less than p is the density function is not degenerate.

In a model without frictions, every time a negative idiosyncratic shock arrives, it would be optimal for the firm to terminate the job, reopen it and hire a new worker. Because we assumed no frictions, this adjustment is instantaneous and costless. Therefore the idiosyncratic component always equals 1, which means that the average productivity will be p.

Part iii

It is a feature of efficiency wages models that wages in equilibrium are higher that the ones that would be observed in the case of no monitoring cost. As a consequence of the higher wages one observes more unemployment in equilibrium. If there is more unemployment, one might think that more vacancies will be posted in equilibrium because it is easier for the firm to meet an unemployed worker (this is so if m(u,v) is increasing). But this may not be the case. The fact that wages are higher reduces the value of a filled job, which on impact reduces the value of a filled vacancy. Another thing to notice is that if wages are higher, the cut-off value of x must also be higher. This effect also reduces the value of a filled job. Given these effects one would expect the vacancy to unemployment ratio to go down. Given that the idiosyncratic component cutoff level increased and the vacancy-unemployment ratio decreased, the Beverage Curve will shift outward.

Graphically,



So, one would expect

- More unemployment.
- The effect on vacancies is ambiguous.
- Job destruction will be $\delta F(x_d)(1-u^*)$. The effect is ambiguous given that $F(x_d)$ went up and $(1-u^*)$ went down.
- Job creation will be $m(1,(v/u)^*)u^*$. Again, the effect is ambiguous given that $(v/u)^*$went down and u^* went up.
- Average productivity increases because the cut-off idiosyncratic component increases.