Solution to Haltiwanger's January 2007 Comp Question
(Sketch of Solutions)

Part i

The ex-post constraint on the take it or leave it wage offers implies

$$w_t = b + g(h_t) \tag{1}$$

The state variables in the ex-post problem are a_t, ε_t, e_t. Note that the ex-post problem has no dynamic implications. Then, the ex-post problem of the firm is

$$\pi(e_t, a_t, \varepsilon_t) \equiv \max_{h_t} a_t \varepsilon_t \left(e_t h_t\right)^{\alpha} - w_t e_t h_t$$

$$\pi(e_t, a_t, \varepsilon_t) \equiv \max_{h_t} a_t \varepsilon_t \left(e_t h_t\right)^{\alpha} - \left(b + g(h_t)\right) e_t h_t \tag{2}$$

FOC

$h_t \rfloor$

$$\alpha a_t \varepsilon_t \left(e_t h_t\right)^{\alpha-1} e_t - g'(h_t) e_t h_t - \left(b + g(h_t)\right) e_t = 0$$

$$\alpha a_t \varepsilon_t \left(e_t h_t\right)^{\alpha-1} - g'(h_t) h_t - b - g(h_t) = 0 \tag{3}$$

which implicitly defines h_t as function of the shocks and the model's parameters.

To see how ex post hours respond to shocks, take a total derivative of (3) (we first assume that $da_t = 0$)

$$\alpha a_t d\varepsilon_t \left(e_t h_t\right)^{\alpha-1} + \alpha\left(\alpha-1\right) a_t \varepsilon_t \left(e_t h_t\right)^{\alpha-2} e_t dh_t - g''(h_t) h_t dh_t - g'(h_t) dh_t - g'(h_t) dh_t = 0$$

$$\alpha a_t d\varepsilon_t \left(e_t h_t\right)^{\alpha-1} + \left[\alpha\left(\alpha-1\right) a_t \varepsilon_t \left(e_t h_t\right)^{\alpha-2} e_t - g''(h_t) h_t - 2g'(h_t)\right] dh_t = 0$$

$$\frac{dh_t}{d\varepsilon_t} = \frac{\alpha a_t \left(e_t h_t\right)^{\alpha-1}}{\left[\alpha\left(1-\alpha\right) a_t \varepsilon_t \left(e_t h_t\right)^{\alpha-2} e_t + g''(h_t) h_t + 2g'(h_t)\right]} > 0$$

Similarly,

$$\frac{dh_t}{da_t} = \frac{\alpha \varepsilon_t \left(e_t h_t\right)^{\alpha-1}}{\left[\alpha\left(1-\alpha\right) a_t \varepsilon_t \left(e_t h_t\right)^{\alpha-2} e_t + g''(h_t) h_t + 2g'(h_t)\right]} > 0$$

Hours workers is then increasing in both shocks. Given that $g(\cdot)$ is an increasing function of h_t, equation (1) implies that wages and hours are positively correlated.

Part ii

$$V(e_{t-1}, a_t, \varepsilon_t) = \max_{v_t, s_t} \left\{ \pi(e_t, a_t, \varepsilon_t) - cv_t - ks_t + \beta E_t \left[V(e_t, a_{t+1}, \varepsilon_{t+1}) \left| a_t, \varepsilon_t \right. \right] \right\} \tag{4}$$

subject to

$$e_t = (1 - q - s_t) e_{t-1} + f(u_t^a, v_t^a) v_t \tag{5}$$

FOC

$v_t \rfloor$

$$\frac{\partial \pi(\cdot)}{\partial e_t} f(u_t^a, v_t^a) - c + \beta f(u_t^a, v_t^a) \frac{\partial E_t \left[V(\cdot) \left| a_t, \varepsilon_t \right. \right]}{\partial e_t} = 0 \tag{6}$$

$s_t \rfloor$

$$-\frac{\partial \pi(\cdot)}{\partial e_t} e_{t-1} - k - \beta e_{t-1} \frac{\partial E_t \left[V(\cdot) \left| a_t, \varepsilon_t \right. \right]}{\partial e_t} = 0 \tag{7}$$

Envelope Conditions

$$\frac{\partial \pi(\cdot)}{\partial e_t} = \alpha a_t \varepsilon_t \left(e_t h_t\right)^{\alpha-1} e_t - w_t h_t \tag{8}$$

$$\frac{\partial V(e_{t-1}, a_t, \varepsilon_t)}{\partial e_{t-1}} = (1 - q - s_t) \left[\frac{\partial \pi(\cdot)}{\partial e_t} + \beta \frac{\partial E_t \left[V(\cdot) \left| a_t, \varepsilon_t \right. \right]}{\partial e_t} \right] \tag{9}$$

Replacing equation (9) into equation (6)

$$\frac{\partial \pi(\cdot)}{\partial e_t} f(u_t^a, v_t^a) - c + f(u_t^a, v_t^a) \left[(1 - q - s_t)^{-1} \frac{\partial V(e_{t-1}, a_t, \varepsilon_t)}{\partial e_t} - \frac{\partial \pi(\cdot)}{\partial e_t} \right] = 0$$

$$\frac{\partial \pi(\cdot)}{\partial e_t} f(u_t^a, v_t^a) - c + f(u_t^a, v_t^a) (1 - q - s_t)^{-1} \frac{\partial V(e_{t-1}, a_t, \varepsilon_t)}{\partial e_t} - f(u_t^a, v_t^a) \frac{\partial \pi(\cdot)}{\partial e_t} = 0$$

$$-c + f(u_t^a, v_t^a)(1-q-s_t)^{-1}\frac{\partial V(e_{t-1}, a_t, \varepsilon_t)}{\partial e_t} = 0$$

$$\frac{\partial V(e_{t-1}, a_t, \varepsilon_t)}{\partial e_t} = \frac{c(1-q-s_t)}{f(u_t^a, v_t^a)} \tag{10}$$

Similarly, replacing equation (9) into equation (7)

$$-\frac{\partial \pi(\cdot)}{\partial e_t}e_{t-1} - k + e_{t-1}(1-q-s_t)^{-1}\frac{\partial V(e_{t-1}, a_t, \varepsilon_t)}{\partial e_t} + e_{t-1}\frac{\partial \pi(\cdot)}{\partial e_t} = 0$$

$$\frac{\partial V(e_{t-1}, a_t, \varepsilon_t)}{\partial e_t} = \frac{k(1-q-s_t)}{e_{t-1}} \tag{11}$$

Replacing (10) into (6) and (11) into (7)

$$\frac{\partial \pi(\cdot)}{\partial e_t} f(u_t^a, v_t^a) - c + \beta f(u_t^a, v_t^a) E_t\left[\frac{c(1-q-s_{t+1})}{f(u_{t+1}^a, v_{t+1}^a)}\right] = 0 \tag{12}$$

$$-\frac{\partial \pi(\cdot)}{\partial e_t}e_{t-1} - k - \beta e_{t-1} E_t\left[\frac{k(1-q-s_{t+1})}{e_t}\right] = 0 \tag{13}$$

The main differences between this model and the Mortensen & Pissarides (M&P) model seen in class are that in the M&P model firms are one-worker firms, wages are set through Nash-bargaining and there's no hours margin.

Some implications of the fact that firm have multiple workers is that, on average, firms facing good shocks are bigger in size that firms facing bad shocks and that the number of workers hired by firms is going to be positively related to the change in the idiosyncractic component.

The take it or leave wage offer makes the value of having a hired worker bigger than that in the case of Nash-bargaining (for k low enough). Therefore, a take it or leave it wage offer increases the size of the firm compared to a setting with Nash bargaining.

The hours margin also increases the value of having a hired worker compared to a situation where such margin does not exist.

Part iii

In the presence of a positive shock, firms first increase hours worked but given equation (1), wages go up. To reduce wages paid, the firm hires more workers (note that the production function depends on $e_t h_t$ and not on e_t and h_t separately). Given that hiring new workers is costly, firms are going to rely on both margins (higher hours and higher employment) if the cost of posting a vacancy is not too big. Then hours and employment are going to be positively correlated. The higher k and c are, the more the firm relies on hours. The more difficult it is for firms to hire workers (that is, the lower $f(u_t^a, v_t^a)$ is), the more the firm relies on hours.

With a positive q, there's always unemployed workers in the economy. q also reduces the value for the firm to have an employed worker.

Part iv

To imbed this model in a general equilibrium model, we need to make extra assumptions on the way the goods markets and labor market operate and on the firms' entry/exit decision.

With respect to the goods market , the most usual assumption in the search literature is that workers consume all their wages and goods market are perfectly competitive. So aggregate demand equals aggregate labor income. Prices will adjust to clear the market.

With respect to the labor market, if there is competition among firms for the workers, firms will no longer be able to extract all the match surplus in a take it or leave it offer. Firms will have to offer higher wages now. This implies that we need to change the wage and hours combination constraint. The new constraint will be of the form

$$U(w_t - g(h_t)) = U(b) + A_t$$

where A_t is determined in equilibrium.

Finally, firms will enter and stay in the market until a point where the value of entering equals zero.