

Solution to Haltiwanger's January 2008 Comp Question  
(Sketch of Solutions)

Part i

$$Max_{c_i, e_i} L = c_i^\lambda (T - e_i)^\theta \quad (1)$$

subject to

$$c_i = A_i e_i^\alpha \underline{e}^\beta \quad (2)$$

Replacing **(2)** into **(1)**

$$Max_{e_i} L = (A_i \underline{e}^\beta)^\lambda e_i^{\alpha\lambda} (T - e_i)^\theta$$

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$e_i$ ]

$$\alpha\lambda (A_i \underline{e}^\beta)^\lambda e_i^{\alpha\lambda-1} e_i^{\alpha-1} (T - e_i)^\theta - \theta (A_i \underline{e}^\beta)^\lambda e_i^{\alpha\lambda} (T - e_i)^{\theta-1} = 0$$

$$\alpha\lambda e_i^{\alpha\lambda-1} e_i^{\alpha-1} (T - e_i)^\theta = \theta e_i^{\alpha\lambda} (T - e_i)^{\theta-1}$$

$$\lambda\alpha e_i^{\alpha-1} (T - e_i) = \theta e_i^\alpha$$

$$\lambda\alpha(T - e_i) = \theta e_i$$

$$e_i = \frac{\lambda\alpha T}{\lambda\alpha + \theta} \quad (3)$$

Replacing equation **(3)** into **(2)**

$$c_i = A_i \underline{e}^\beta \left( \frac{\lambda\alpha T}{\lambda\alpha + \theta} \right)^\alpha \quad (4)$$

Note that agent's  $i$  effort is independent of other's people effort but consumption depends on other people's effort. There are externalities present in the model but there is no complementarity in effort.

Part ii

Due to the lack of complementarity in agent's action there can be at most one equilibrium. In the rational expectations equilibrium all agents will be exerting

effort equal to  $\frac{\lambda\alpha T}{\lambda\alpha + \theta}$ . To calculate consumption in equilibrium use the definition of  $\underline{e}$

$$\underline{e} \equiv \sum_{j=1}^I \frac{e_j}{I} \quad (5)$$

Plug **(3)** into **(5)**

$$\underline{e} = \frac{\lambda\alpha T}{\lambda\alpha + \theta} \quad (6)$$

and replace **(6)** into **(4)**

$$c_i = A_i \left( \frac{\lambda\alpha T}{\lambda\alpha + \theta} \right)^{\alpha + \beta} \quad (7)$$

### Part iii

Given that effort is independent of the productivity shock, there is persistence or amplification mechanism.

### Part iv

The unique equilibrium is locally stable.

### Part v

Because expectations don't matter for effort, having a mix some agents using naïve expectations won't change the equilibrium.

### Part vi

Because productivity doesn't matter for effort, people won't acquire information and the equilibrium that will emerge will be the same.