

3. Consider the maximizing decision of a firm with current period profits given by:

$$\pi_t = p_t S_t - w_t u_t L_t - H(Z_t) - \tau(1 - u_t)L_t$$

where p_t is the price set ex ante each period, S_t is sales, w_t is the wage, u_t is the fraction of the *contractual* workers that are employed ex post in the period, L_t is the number of workers contracted for ex ante in each period, Z_t is end of period inventories, H is a convex inventory holding cost function ($H' > 0$, $H'' > 0$), and τ is a turnover/layoff cost. The firm maximizes the present discounted value of profits with a discount factor δ by choosing L_t and p_t each period before the state of demand is realized and choosing u_t , w_t , and S_t ex post. The firm makes these choices subject to the following constraints:

$$u_t \leq 1$$

$$S_t \leq m(p_t) + \varepsilon_t$$

$$S_t \leq Z_{t-1} + g(u_t L_t)$$

where

$$Z_t = Z_{t-1} + g(u_t L_t) - S_t$$

$$\int w_t u_t f(\varepsilon) d\varepsilon + (1 - \int u_t f(\varepsilon) d\varepsilon) Y \geq V$$

Where $g(\cdot)$ is a strictly concave production function ($g' > 0$ and $g'' < 0$), $m(\cdot)$ is a demand function with $m' < 0$ and $m'' < 0$, and ε_t is a random demand shock with a mean of zero and pdf given by $f(\cdot)$, Y is the income equivalent of the additional leisure time a worker acquires when laid off and any government-financed unemployment benefits, and V is a market equilibrium contract value representing the expected income the firm must offer in order to attract contractual workers ($V > Y$). In each period, the firm chooses the ex ante variables before the realization of the demand shock and the ex post variables contingent on the demand shock.

Please answer the following questions:

- (i) Set up the maximization problem as a dynamic programming problem.
- (ii) Derive the necessary conditions for a maximum and provide intuition about the necessary conditions.
- (iii) In response to an adverse demand shock, will the firm accumulate inventories, make layoffs or both? Discuss.
- (iv) Discuss how the response to demand shocks in this model differs from the standard inventory model and from the standard implicit contract model.