

4. Consider the following model of the relationship between firms and workers that combines features from both search and matching theory as well as contract theory. Firms maximize profits where the current period revenue function is given by  $a\varepsilon(eh)^\alpha$ , where  $a$  is the aggregate (profitability) shock,  $\varepsilon$  is the producer-specific shock and total labor input is the product of the number of workers,  $e$ , and hours per worker,  $h$ . Assume that  $\alpha < 1$ . Firms pay workers compensation  $\omega(s)$  which in principle may vary with the ex post state  $s$ . Firms hire workers through posting vacancies, are subject to exogenous quits and also may layoff workers. There is a CRS matching function given by  $m(\bar{v}, \bar{u})$  where  $\bar{u}$  is the aggregate unemployment rate and  $\bar{v}$  is the aggregate vacancy rate. The job filling rate  $H(\theta) = m(\bar{v}, \bar{u})/\bar{v}$  for firms is a decreasing function of labor market tightness  $\theta = \theta(\bar{v}/\bar{u})$ . The job finding rate  $\phi = \phi(\theta)$  for workers is an increasing function of labor market tightness.

Workers are in one of two states, employed or unemployed. If unemployed, the workers enjoy leisure time and/or the fruits of home production,  $U(b(a))$ , which is allowed to depend on the level of aggregate productivity,  $a$ . Employed workers have a contract for the current period which governs their state-contingent compensation and hours worked. Workers do not save in equilibrium so that compensation,  $\omega$ , and consumption,  $c$ , are identical. Instantaneous utility of consumption and hours worked,  $h$ , is given by  $U(\omega - g(h))$  where  $U$  is strictly concave and  $g$  is strictly convex.

There are two stages in the producer's problem. First, given the aggregate state, the producer determines employment (by choosing vacancies and layoffs) and contracts with its workers. Second, *ex post*, the producer-specific shock is realized, and state-contingent hours are determined given the contract.

In the state contingent contract,  $\omega(s)$  is compensation and  $h(s)$  is hours worked. The contract allows compensation and hours to be fully state contingent. In terms of timing, the contract is determined given  $(a, e)$  but prior to the determination of  $\varepsilon$ . All workers with a given producer get the same contract since they are identical and have the same

outside option of unemployment. For the contracting process, assume producers make a take-it-or-leave-it offer to workers so workers obtain  $U(b(a))$  each period.

The evolution of employment is given by

$$e = e_{-1}(1 - \bar{q} - f) + H(\theta)v \quad (1)$$

where  $v$  is the number of vacancies posted by the firm,  $\bar{q}$  is the exogenous quit rate and  $f$  is the endogenous rate of fires/layoffs. Both  $v$  and  $f$  are determined in the first stage (knowing the current period aggregate shock but not the producer-specific shock).

There are costs of posting vacancies and making layoffs. The cost of posting vacancies includes a fixed cost component,  $F^+$ , and a variable cost per vacancy posted,  $c^+$ . Similarly there are fixed and linear costs of firing workers with parameters  $F^-$  and  $c^-$ , respectively.

Firms and workers both discount the future at rate  $\beta$ .

Please answer the following questions:

- Specify and discuss the value functions for the firm in determining employment and the worker (including the value of being employed and being unemployed for the worker).
- Characterize the ex post contract that determines hours and wages.
  - What is the optimality condition for hours determination?
  - Do wages vary with the producer-specific shock  $\epsilon$ ?
- Without solving the problem analytically, discuss and describe the likely form of the decision rules for vacancy postings and fires/layoffs.
  - Will a firm ever simultaneously post vacancies and make layoffs?
  - What role will the fixed and linear vacancy posting and firing costs play?
  - What role will the elasticity of  $b(a)$  with respect to  $a$  play in the responsiveness of employment to  $a$ ?