

4. Consider a firm choosing optimal hours per worker and employment subject to the following conditions:
- The production function is given by $a_t \varepsilon_t (e_t h_t)^\alpha$ where a_t is an aggregate productivity shock, ε_t is an idiosyncratic productivity shock, e_t is employment, h_t is hours per worker and $\alpha < 1$. All firms produce the same good but they differ in their productivities.
 - The timing each period is as follows. At the beginning of the period the firm observes the aggregate shock and must choose its workforce by choosing and filling vacancies and making layoffs.
 - After the idiosyncratic shock is observed the firm can adjust its hours per worker.
 - Firms obtain all the surplus in the match and make take it or leave it offers to the worker. The implication is that the wage and hours combination in any state of the world ex post must satisfy $U(w_t - g(h_t)) = U(b)$ where w_t is the wage per hour, $g' > 0$, $g'' > 0$, $U' > 0$, $U'' < 0$ and $b > 0$ (and b is the opportunity cost of a worker's time).
 - Firms can hire workers via posting vacancies – where the number of posted vacancies at the firm level is v_t . The job filling rate for a posted vacancy is given by $f(u_t^a, v_t^a)$ where u_t^a is aggregate unemployment and v_t^a is aggregate vacancies. The cost of posting each vacancy is c .
 - Firms can layoff workers previously employed at the beginning of each period at an endogenous rate s_t . The adjustment cost for each layoff is given by k .
 - The evolution of employment each period is accordingly given by $e_t = (1 - q - s_t)e_{t-1} + f(u_t^a, v_t^a)v_t$ where q is an exogenous quit rate.
 - Firms discount the future at rate $\beta < 1$.

Answer the following questions:

- Write down the ex post maximization problem for a given number of contracted workers and solve for optimal hours. Discuss the nature of the variation in ex post hours and wages in response to shocks.
- Write down the value function for choosing vacancies and layoffs and discuss the nature of the firm's decision problems. In your discussion, compare this specification to that of Mortensen and Pissarides type search and matching models. Discuss the key differences and their likely importance.
- Without necessarily solving, characterize the likely dynamics of employment and hours in this model at the firm level discussing the role of k , c and f in the dynamics of employment and hours and the correlation of employment and hours. In addition, discuss the role of q in the model.
- The model as written down is partial equilibrium. Discuss how to close the model and imbed it in an industry or general equilibrium setting. What are the likely implications of moving from partial to general equilibrium? Discuss.