c)

Euler: $U_c(C_t, N_t) = \beta(1+r)\, U_c(C_{t+1}, N_{t+1})$ $\therefore$

$$C_t^{-\gamma} \cdot \frac{(T-N_t)^{1-\varphi}}{\cancel{1-\varphi}} = \beta(1+r) \cdot C_{t+1}^{-\gamma} \cdot \frac{(T-N_{t+1})^{1-\varphi}}{\cancel{1-\varphi}}$$

$$-\gamma \ln C_t + (1-\varphi)\ln(T-N_t) = \ln\beta(1+r) - \gamma \ln C_{t+1} + (1-\varphi)\ln(T-N_{t+1}) \quad \therefore$$

$$\gamma \ln C_{t+1} - \gamma \ln C_t = \ln\beta(1+r) + (1-\varphi)\ln(T-N_{t+1}) - (1-\varphi)\ln(T-N_t)$$

$$\gamma \Delta \ln C_t = \ln\beta(1+r) + (1-\varphi)\Delta\ln(T-N_t)$$

$\therefore$

$$\Delta \ln C_t = \frac{\ln\beta(1+r)}{\gamma} + \frac{(1-\varphi)}{\gamma}\Delta\ln(T-N_t)$$

$\uparrow N_t \Rightarrow \uparrow C_t$ only if $\varphi > 1$

a) $u'(c) \cdot w = v'(m)$ holds at any t

Fact: m is $\approx$ constant $\Rightarrow v'(m) \approx cte$

Fact: w and c have steadily risen at the same rate

For both facts hold $u'(c) = \frac{1}{c}$

Indeed if $u(c)$ is $\ln c$ $\Rightarrow u'(c) \cdot w = v'(m)$ $\therefore$

$$\frac{w}{c} = v'(m)$$

Both facts are consistent with $\ln c$.

b) $$U(C,N) = C^{1-\gamma}/(1-\gamma) * (T-N)^{1-\varphi}/(1-\varphi)$$

$$U_c(C,N) = C^{-\gamma} \cdot (T-N)^{1-\varphi}/(1-\varphi)$$

$$U_N(C,N) = C^{1-\gamma}/(1-\gamma) * (T-N)^{-\varphi}$$

Optimality condition:

$$U_c(C,N) \cdot w = U_N(C,N) \therefore$$

$$C^{-\gamma}\left[(T-N)^{1-\varphi}/(1-\varphi)\right] w = \left[C^{1-\gamma}/(1-\gamma)\right] * (T-N)^{-\varphi}$$

$$\frac{w}{c} = \frac{(1-\varphi)}{(1-\gamma)} \cdot \frac{1}{(T-N)}$$ $\Rightarrow$ consistent with both facts