ANSWERS TO AUGUST 2005 MACRO COMP QUESTIONS: John Shea

SHORT QUESTION 3: Interest rates should be countercyclical in an endowment economy with iid aggregate shocks, not procyclical. Intuitively, during booms, everyone has unusually high incomes that are not expected to persist. People would like to save this good shock, so the demand for bonds rises, raising their price and lowering the interest rate. (Alternatively, everyone tries to make loans, pushing the price of loans or the interest rate on loans downward). Mathematically, the interest rate satisfies the following Euler equation:

$$u'[c(t)] = \beta (1+r(t)) E(t)u'[c(t+1)]$$

Rewriting and recognizing that for an endowment economy we must have $c(t) = y(t)$ in equilibrium, we have

$$(1+r(t)) = u'[y(t)] / \beta E(t)u'[y(t+1)]$$

Since shocks are iid the denominator of this expression is a constant, and the numerator declines with y since utility is concave.

Either correct intuition or correct math was sufficient to pass this question.

LONG QUESTION

(a) This is a life cycle model with uncertain labor income, assuming quadratic utility with an infinite horizon, no risky assets and a time discount factor equal to the inverse of the constant riskless interest rate. I worked this case out in detail in class, so it should have been straightforward for you to set up and solve the Bellman's equation, to arrive at the correct Euler Equation

$$c(t) = E(t)c(t+1) = E(t)c(t+s) \text{ for all } s>0$$

and to combine this with an intertemporal budget constraint to arrive at the standard closed-form solution for optimal consumption in terms of financial wealth and the expected present discounted value of labor income:

$$c(t) = (r/1+r) (a(t) + h(t))$$

I asked you to solve the impact of unemployment on consumption--more precisely, the difference in consumption at time t between someone who learns that he is employed at t, and someone with the same financial wealth who learns that he is unemployed at t. Since a(t) is the same for both people, it is clear that

$$\text{Impact on } c(t) = (r/1+r) * (\text{impact on } h(t))$$

where the impact on h(t) equals the difference in the expected PDV of labor income between someone employed at t and someone not employed at t.

There are probably alternative ways of solving for the impact on h(t) involving manipulations of Markov chains, or involving figuring out an iterative pattern that I could evaluate using infinite sums, but the way I solved this problem was to set up the following recursive system: let PDVU equal the human wealth of someone currently unemployed and let PDVE equal the human wealth of someone currently employed. Since this is a time invariant environment (including, importantly, an infinite horizon) and since the transition matrix is first-order Markov, we know that PDVU and PDVE are also time invariant. The following are true:

$$PDVE = w + (1/1+r)^* [\alpha\ PDVE + (1-\alpha)\ PDVU]$$
$$PDVU = (1/1+r)^*[\lambda\ PDVU + (1-\lambda)PDVE]$$

This is just two equations in two unknowns. Solving this system yields closed-form expressions for PDVE and PDVU in terms of w, r, α and λ. The impact of unemployment on consumption is just $(r/1+r)^*(PDVE-PDVU)$. What you will find when you solve this is that this impact increases in both α and λ.

(b) The solution to part (a) implies that the drop in consumption upon unemployment should be larger for people in education-occupation groups with higher duration of unemployment (a higher λ) and with lower incidence of unemployment, or alternatively with longer duration of employment (a higher α). Even if you couldn't get the math from part (a) you might have been able to intuitively explain the impact of duration and incidence on the drop in consumption. The intuition for duration is that if unemployment is more persistent, then a job loss is a bigger drop in expected future income for the household, since the unemployed are likely to stay unemployed for a long time. The intuition for incidence is that if job loss is unlikely, then losing one's job is a larger shock to one's expected future income; the employed expect to stay employed for a long time since jobs are highly durable, so losing a job is the loss of something very valuable.

Correctly re-deriving the closed form solution for optimal consumption in this case, plus showing substantial insight on the intuition for part (b) OR making some substantial progress on solving for the impact of unemployment on h(t) in part (a) was sufficient to pass this question.