PLEASE RETURN TO JOHN SHEA

MACRO COMP FALL 2007 Q1

(a) State variables: A_t, K_t

Choice variables: N_t, C_t (or K_{t+1})

$$V(A_t, K_t) = \max_{(N_t, C_t)} \log(C_t) - BN_t^{\theta} + E_t \beta V(A_{t+1}, K_{t+1})$$

where $K_{t+1} = (1-\delta)K_t + A_t N_t^{\alpha} K_t^{1-\alpha} - C_t$

FOCs: C_t: $\frac{1}{C_t} = u'(C_t) = \beta E_t V_K(K_{t+1})$ $\boxed{(1)}$

N_t: $\theta B N_t^{\theta-1} = \beta(\alpha A_t N_t^{\alpha-1} K_t^{1-\alpha}) E_t V_K(K_{t+1})$ $\boxed{(2)}$

Benveniste-Scheinkman condition

K_t: $V_K(K_t) = \beta[(1-\delta) + (1-\alpha) A_t N_t^{\alpha} K_t^{-\alpha}] E_t V_K(K_{t+1})$ (3)

$= [(1-\delta) + (1-\alpha) A_t N_t^{\alpha} K_t^{-\alpha}] u'(C_t)$ (4)

(Combining with (1))

Combine (1) and (2): static first order condition for labor supply:

$$\underbrace{\theta B N_t^{\theta-1}}_{\text{Marginal Disutility of work at } t} = \underbrace{\alpha A_t N_t^{\alpha-1} K_t^{1-\alpha}}_{\text{Marginal Product of labor at } t} \cdot \underbrace{\left[\frac{1}{C_t}\right]}_{\text{marginal utility of consumption at } t}$$

Combining (1) + (4): Euler Equation for C:

$$u'(C_t) = \frac{1}{C_t} = \beta E_t \underbrace{\left[(1-\delta) + (1-\alpha) A_{t+1} N_{t+1}^{\alpha} K_{t+1}^{-\alpha}\right]}_{\text{Marginal Product of Capital at t+1}} u'(C_{t+1})$$

(b) Jump in C_2 is just standard LCH/PIH reasoning; people respond to good news about future by wanting to consume more.

$\boxed{\text{Period 2, Other Variables}}$

from static FOC for labor supply: A jump in C_2 causes $u'(C_2)$ to fall. To maintain equality, either the MPL must rise and/or the marginal disutility of leisure must fall. Both require a decline in N_2. This is just a standard income effect on labor supply. Note that A_2 and K_2 are predetermined and can't respond.

From the production function:

$N_2 \downarrow$ plus A_2, K_2 unchanged $\rightarrow Y_2 \downarrow$

From Accumulation Equation: $Y_2 \downarrow, C_2 \uparrow \rightarrow I_2 \downarrow, K_3 \downarrow$

$\boxed{\text{Period 3 responses}}$

Look at the static FOC for labor supply, compared to period 2, we have: $A_3 = A_2$; $C_3 > C_2$ and $K_3 < K_2$. Thus, $u'(c)$ is lower and marginal product of labor will (holding N fixed) be lower. To restore equality, we need $N_3 < N_2$ in order to reduce the marginal disutility of leisure and increase the marginal product of labor. Intuitively, a lower capital stock in period 3 operates like a bad technology shock, in that it shifts labor demand to the left, reducing optimal N

From the production function: $N_3 < N_2$ and $K_3 < K_2$ combined with $A_3 = A_2 \Rightarrow Y_3 < Y_2$

From accumulation equation: $Y_3 < Y_2$ and $C_3 > C_2$ imply $I_3 < I_2$ and $K_4 < K_3$

(c) State variables: K_t, A_t

Control variables: N_t, u_t, C_t (or K_{t+1})

Bellman:

$$V(A_t, K_t) = \max_{C_t, N_t, u_t} \log(C_t - BN_t^{\theta}) + \beta E_t V(A_{t+1}, K_{t+1})$$

$$\text{s.t. } K_{t+1} = [1-d(u_t)]K_t + A_t N_t^{\alpha}(u_t K_t)^{1-\alpha} - C_t$$

FOC

C_t: $$\frac{1}{C_t - BN_t^{\theta}} = u'(C_t) = \beta E_t V_K(A_{t+1}, K_{t+1})$$

N_t: $$\frac{\theta B N_t^{\theta-1}}{C_t - BN_t^{\theta}} = \theta B N_t^{\theta-1} u'(C_t) = \alpha \beta A_t N_t^{\alpha-1}(u_t K_t)^{1-\alpha} E_t V_K(A_{t+1}, K_{t+1})$$

→ Combine to form static FOC for labor supply:

$$\theta B N_t^{\theta-1} u'(C_t) = \alpha A_t N_t^{\alpha-1}(u_t K_t)^{1-\alpha} \cdot u'(C_t)$$

(Note that because of the functional form, $u'(C_t)$ drops out

$$(*) \quad \theta B N_t^{\theta-1} = \alpha A_t N_t^{\alpha-1}(u_t K_t)^{1-\alpha}$$

So that labor supply is independent of consumption.

FOC for u_t:

$$0 = \beta\left[-d'(u_t)K_t + (1-\theta)A_t N_t^{\theta} K_t^{1-\theta} u_t^{-\theta}\right] E_t V_K(A_{t+1}, K_{t+1})$$

$$\Rightarrow \quad u_t^{\theta}\, d'(u_t) = (1-\theta)A_t N_t^{\theta} K_t^{-\theta} \qquad (**)$$

Again, note that C_t is absent.

The Euler Equation derivation is similar to part (a), except that the marginal product of K_{t+1} is now

$$\left[1 - d(u_{t+1})\right] + (1-\theta) A_t N_t^{\theta} u_t^{1-\theta} K_t^{-\theta}$$

while the marginal utility of consumption is $1/C_t - BN_t^{\theta}$.

(d) Note that $(*)$ and $(**)$ are two equations relating u_t and N_t to A_t, K_t and various parameters.

For a given pair of state variables (A, K) there is a unique pair (u^*, N^*) satisfying $(*)$ and $(**)$.

Now note that in period 2, beliefs about the future and thus C_2 have changed, but A_2 and K_2 have not. Thus, u_2 and N_2 will not change.

Intuitively, C_2 will jump for the same reason as before, since $(K_2 = K_1, u_2 = u_1, N_2 = N_1, A_2 = A_1)$, Y_2 will equal Y_1. From the accumulation equation, $Y_1 = Y_2$ plus $C_2 > C_1 \Rightarrow I_2 < I_1$,