Macroeconomic Comprehensive Theory Examination (January 2002)

Sketch of the Solution - Shea's Question (#1)

Part (a)

Optimization problem:

$$\underset{c_1, c_2}{Max} \quad u(c_1) + \beta u(c_2) \qquad (1)$$

Subject to $c_1 + \frac{c_2}{R} = y_1 + \frac{y_2}{R} \qquad (2)$

Euler equation:

$$u'(c_1) = \beta R u'(c_2) \qquad (3)$$

Simplifying assumptions:

$$u(c_t) = c_t - \frac{b}{2} c_t^2 \qquad (4)$$

$$R = \beta = 1 \qquad (5)$$

Optimal behavior under (5):

$$c^* = c_1^* = c_2^* = \frac{y_1 + y_2}{2} \qquad (6)$$

Now consider the following suboptimal behavior:

$$c_1^s = y_1 \qquad (7)$$

$$c_2^s = y_2 \qquad (8)$$

The utility loss caused by such suboptimal behavior can be defined as:

$$L = (u(c^*) + u(c^*)) - (u(c_1^s) + u(c_2^s)) \qquad (9)$$

Making use of equation (4), we get:

$$L = 2\left[\frac{y_1 + y_2}{2} - \frac{b}{2}\left(\frac{y_1 + y_2}{2}\right)^2\right] - \left[y_1 - \frac{b}{2}y_1^2 + y_2 - \frac{b}{2}y_2^2\right] \quad (10)$$

Some algebra…

$$L = (y_1 + y_2) - (y_1 + y_2) - \frac{b}{4}(y_1 + y_2)^2 + \frac{b}{2}y_1^2 + \frac{b}{2}y_2^2 \quad \therefore$$

$$L = \frac{2by_1^2 + 2by_2^2 - b(y_1 + y_2)^2}{4} \quad \therefore$$

$$L = \frac{2by_1^2 + 2by_2^2 - by_1^2 - 2by_1y_2 - by_2^2}{4} \quad \therefore$$

$$L = \frac{by_1^2 - 2by_1y_2 + by_2^2}{4} \quad \therefore$$

And finally

$$L = \frac{b(y_1 - y_2)^2}{4} \quad (11)$$

It is clear from (11) that the utility loss from suboptimal behavior increases with the size of income changes and also with the parameter b, which captures the curvature of the quadratic utility function.

Thus, utility losses are likely to be big whenever the combination of a large income change and a highly curved utility function occurs. Intuitively, a high curvature of the utility function indicates that the agent has a strong consumption-smoothing motive. If, on top of that, the income changes a lot between periods, rational behavior implies intense reshuffling of resources inter-temporally. Adopting the suggested suboptimal behavior impedes such intense reshuffling, causing high utility losses.

Part (b)

Calculate the income change between periods for each agent. Assuming identical *b's* and identical costs of computing optimal behavior, if near-rationality is to blame for the failure of LCH then there should be a cutoff amount of income change below which the rule of thumb behavior is observed (LCH fails) AND above which optimal smoothing behavior is observed (LCH holds). That is, the LCH should fail only for the group of agents experiencing sufficiently low changes in income.