

Part II - Question 1 (Comp Jan 2004)

(1)

$$\text{Max } u(c_1) + E_1 u(c_2) \text{ s.t. } c_2 = (w_1 - c_1) + k + k\varepsilon \quad (1)$$

$$\text{Max } u(c_1) + E_1 u((w_1 - c_1) + k + k\varepsilon) \quad (2)$$

Given $u(\cdot)$

$$\text{Max } (-1/a) \exp(-ac_1) + E_1 [(-1/a) \exp(-ac_2)] \quad (3)$$

Because $E(x) = \exp(\mu_x + s_x^2/2)$ (if $\log(x) \sim N(\mu_x, s_x^2)$)

$$\Rightarrow E_1 [(-1/a) \exp(-ac_2)] = (-1/a) E_1 [\exp(-ac_2)] =$$

$$(-1/a) \exp(E(-ac_2) + \text{Var}(-ac_2)/2) =$$

$$(-1/a) \exp(-a((w_1 - c_1) + k) + a^2 k^2 \sigma^2 / 2) \quad (4)$$

So the optimization problem is now:

$$\text{Max}_{c_1, k} (-1/a) \exp(-ac_1) + (-1/a) \exp(-a((w_1 - c_1) + k) + a^2 k^2 \sigma^2 / 2) \quad (5)$$

$$\text{FOC: } (c_1): \exp(-ac_1) = \exp(-a((w_1 - c_1) + k) + a^2 k^2 \sigma^2 / 2) \quad (6)$$

$$(k): (-1/a) \exp(-\cdot) \cdot [-a + 2ka^2 \sigma^2 / 2] = 0$$

The higher the risk-aversion (a) and σ^2 are, the less risk the agent assumes (that is, he chooses a lower k)

$$\hookrightarrow ka^2 \sigma^2 = a$$

$$\boxed{k = 1/a\sigma^2} \quad (7)$$

FROM (6):

$$-ac_1 = -a(w_1 - c_1 + k) + a^2 k^2 \sigma^2 / 2 \quad \therefore$$

$$ac_1 = a(w_1 - c_1 + k) - a^2 k^2 \sigma^2 / 2 \quad \therefore$$

$$2ac_1 = a(w_1 + k) - a^2 k^2 \sigma^2 / 2 \quad \therefore$$

$$c_1 = (w_1 + k) / 2 - ak^2 \sigma^2 / 4 \quad (8)$$

To get precautionary savings, first assume that agent would get avg income in 2nd period (which is k since $E(\varepsilon) = 0$) with certainty.

$$s^{CE} = w_1 - c_1^{CE} = w_1 - (w_1 + k) / 2 = (w_1 - k) / 2 \quad (9)$$

Actual savings from (8) is:

$$s^* = w_1 - c_1^* = w_1 - (w_1 + k) / 2 + ak^2 \sigma^2 / 4 = (w_1 - k) / 2 + ak^2 \sigma^2 / 4 \quad (10)$$

Precautionary saving is $s^* - s^{CE} = ak^2 \sigma^2 / 4$

↳ Notice, however, that k is a choice variable and we can use (7) to write:

$$PS = s^* - s^{CE} = ak^2 \sigma^2 / 4 = \frac{a\sigma^2}{4a^2(\sigma^2)^2} = \frac{1}{4a\sigma^2} \Rightarrow \boxed{PS = 1/4a\sigma^2} \quad (11)$$

High risk aversion (a) reduces Precautionary saving. This is because the agent can choose how risky 2nd period income will be, so $\uparrow a \Rightarrow \downarrow k$ and there will not be a strong need for PS