Question I. Hyperbolic Discounting and Consumption

Recent work by David Laibson has explored the implications of a special class of preferences exhibiting what is called hyperbolic discounting. Under hyperbolic discounting, the value function of an agent at time t is given by

(*) $$V_t = u(c_t) + E_t[\beta \sum_{k=1}^{T} \delta^t u(c_{t+k})]$$

where u(.) is a standard, concave flow utility function, δ is a standard discount factor between 0 and 1, E_t denotes expectations conditional on time t information, and β is a hyperbolic discounting factor, between 0 and 1. If β equals 1, then this is a standard value function with the usual exponential discounting. Under exponential discounting, the agent at time t applies the same discount factor (namely, δ) when comparing consumption at time t and time t+1 as he does when comparing consumption at time t+1 and t+2, or t+2 and t+3, and so on. However, if β is less than one, then the agent at time t applies a lower discount factor when comparing consumption at t and t+1 (namely, $\beta\delta$) than he does when comparing consumption at t+1 and t+2; to see this, note that under hyperbolic discounting, the agent at time t discounts consumption at t+1 by $\beta\delta$, but discounts consumption at t+2 by $\beta\delta^2$, so that from the point of view of the time t agent, consumption at time t+2 is discounted by only δ when compared to consumption at t+1. Hyperbolic discounting thus captures the idea that people are more impatient when comparing utility flows today and tomorrow, than they are when comparing utility flows between two distant dates.

Consider a three period model with the following simplifying assumptions. The agent is given an endowment Y prior to period one, and gets no more income flows. The gross interest rate is R and is the same in all periods; there is no uncertainty. Assume that agents can save at this rate but cannot borrow. Preferences are of the form (*), with the expectations sign omitted and T = 3; thus, the period-1 agent discounts period 2 by $\beta\delta$ and period 3 by $\beta\delta^2$, while once period 2 arrives, the agent discounts period 3 by $\beta\delta$.

(a) Prove that optimal behavior is not time consistent in this model. In other words, show that the allocation under full commitment—in which the period 1 agent makes binding choices for consumption in all 3 dates to maximize his period 1 value function—is different from the allocation that emerges under no commitment—in which the period 1 agent makes an optimal choice for first period consumption, knowing that in period 2 the agent will get to make his own choice of second and third period consumption. Show that time consistency holds if $\beta = 1$, so that discounting is exponential rather than hyperbolic. [Hint: under no commitment, the period-2 problem must be solved before the period one problem].

(b) Now suppose that the period 1 agent can use part or all of his savings to purchase an annuity. An annuity is a financial instrument that yields a constant flow of income for the rest of one's life. Assume that the annuity begins paying out in period 2, and that the annuity is actuarially fair, so that if the agent spends X on an annuity in period 1, the present discounted value of the income flows he receives in periods 2 and 3 will sum to X. Show that the period 1 agent can use an annuity to implement his preferred, full-commitment allocation, and solve for the optimal allocation of savings between annuities and other assets in period 1.

(c) Would your answer to part (b) change if agents could borrow against future income?