Macro Theory Prelim Question 1, Summer 2002

THE INTERGENERATIONAL CORRELATION OF WEALTH

Many studies have examined the intergenerational correlation of income and have shown this to be positive. That is, if a father has a higher-than-average income for his generation, his sons are likely to have a higher-than-average income for their generation as well. Some recent work has looked at the intergenerational correlation of consumption, saving and wealth. This problem will ask you to look at the intergenerational correlation of wealth; some basic review facts about correlations and covariances are presented in the hint at the end of the problem.

Suppose there are two generations in the economy, composed of heterogeneous one parent-one child families indexed by i. Parents are alive in periods 1 and 2. Parent i works and receives labor income in period 1 only, out of which she must finance consumption C in periods 1 and 2 as well as a transfer T to child i. Parent i's utility is given by:

$$\log C_1(i,p) + \beta(i,p) \log C_2(i,p) + \gamma \log T(i)$$

where $\beta(i,p)$ is the parent's discount factor, which in general will vary across i. Assume that interest rates are zero so that the budget constraint for parents is given by $C_1(i,p) + C_2(i,p) + T(i) = Y(i,p)$, where $Y(i,p)$ is first-period labor income, which in general will vary across i. Credit markets are perfect; there is free borrowing and lending at interest rate zero, although parents cannot leave debts to their children or vice-versa.

Children are alive in periods 2 and 3. Child i works and receives labor income $Y(i,c)$ in period 2 only; she also receives the transfer $T(i)$ from her parents. Out of these resources she finances consumption in periods 2 and 3; children are assumed to make no transfers to their parents or their own children. Assume that children receive the transfer at the start of period 3, after their parents have died; however, there is no uncertainty so children have perfect foresight about their future transfers. Child i's utility is given by

$$\log C_2(i,c) + \beta(i,c) \log C_3(i,c)$$

and her budget constraint is given by $C_2(i,c) + C_3(i,c) = Y(i,c) + T(i)$.

(a) [WARM-UP QUESTION] Solve for the optimal consumption choices of parent and child, as well as the optimal transfer. Write down an expression for the parent's wealth holdings $W(i,p)$ at the end of period 1, and the child's wealth holdings $W(i,c)$ at the end of period 2, noting that by definition and by the fact that the transfer is not paid until the start of period 3, we have $W(i,p) = Y(i,p) - C_1(i,p)$, while $W(i,c) = Y(i,c) - C_2(i,c)$.

Now suppose $\gamma = 0$, so that there are no transfers. Define $w(i,p)$ and $w(i,c)$ as the logs of parental and child wealth for family i; define $y(i,p)$ and $y(i,c)$ as the logs of labor income; define $b(i,p)$ as the log of $\beta(i,p)/(1+\beta(i,p))$, and define $b(i,c)$ similarly.

Assume that y(i,p) and y(i,c) are random variables with mean zero, variance $\sigma^2(y)$ and correlation $\rho(y)$; if high income parents tend to have high income children, due to positively correlated ability perhaps, then $\rho(y)$ will be positive. Similarly, assume that b(i,p) and b(i,c) are random variables with mean zero, variance $\sigma^2(b)$ and correlation $\rho(b)$; if patient parents tend to have patient children then $\rho(b)$ will be positive. Assume that y(i,p) is uncorrelated with both b(i,p) and b(i,c); and similarly for y(i,c).

(b) Write down a formula for the intergenerational correlation of log wealth (that is, the correlation between w(i,p) and w(i,c)). Prove that the intergenerational correlation of wealth is larger than the intergenerational correlation of income if and only if $\rho(b) > \rho(y)$, meaning that preferences are more strongly correlated across generations than income. [HINT: remember that the ρ's are correlations, not covariances; use the formula below to go back and forth between correlations and covariances when necessary].

(c) Now suppose that preferences are the same across i and across generations, so that everyone has the same discount factor β. Consider now the intergenerational COVARIANCE of the level (not the log) of wealth, W(i,p) and W(i,c). Show that this covariance will be lower if there are bequests than if there are no bequests (Hint: compare the case in which $\gamma = 0$ to the case in which γ is slightly greater than zero; in the latter case, for convenience, assume that $\beta/(1+\beta)$ is roughly equal to $(\beta + \gamma)/(1+\beta+\gamma)$). Explain intuitively why bequests should reduce the intergenerational covariance of wealth, under our assumption that parental and child wealth are measured prior to the bequest taking place.

HINT: here are relevant facts about covariances and correlations. Let W, X, Y and Z be random variables, and let A and B be scalar constants. Then

$$Cov(W+X,Y+Z) = Cov(W,Y) + Cov(W,Z) + Cov(X,Y) + Cov(X,Z)$$

$$Cov(AX,BY) = AB\, Cov(X,Y)$$

$$Corr(X,Y) = Cov(X,Y) / [\sigma(X)\, \sigma(Y)]$$

$$Var(X+Y) = Var(X) + Var(Y) + 2\, Cov(X,Y)$$

where Cov stands for covariance, Corr stands for correlation, Var stands for Variance, and $\sigma(Y)$ is the standard deviation (square root of the variance) of Y.