Macro Comp Question 4

Housing and the equity premium puzzle (Piazzesi, Schneider and Tuzel (2006))

Suppose that households maximize a standard intertemporal preference function:

(2) $$E\sum_{t=0}^{\infty}\beta^{t}U(C_{t})$$

where

(2) $$\mathrm{U(C)} = (\sigma/\sigma - 1)C^{(\sigma-1)/\sigma}$$

In the usual specification, C in equations (1) and (2) refers to nondurables and services consumption. For this problem, however, assume that C refers to a CES aggregate of nondurables and services consumption (n) and housing services consumption (h):

(3) $$C = [n^{(\varepsilon-1)/\varepsilon} + \omega h^{(\varepsilon-1)/\varepsilon}]^{\varepsilon/(\varepsilon-1)}$$

where $\varepsilon > 0$ is the intratemporal elasticity of substitution between nondurables and housing, and $\omega > 0$ indexes the weight of housing in the consumer's overall utility. For this problem, assume that there are no transactions costs involved with housing. Housing is sold in a frictionless rental market each period at a price p_t in terms of units of n, which is taken to be the numeraire.

(PART A): Show that if $\varepsilon=\sigma$, then (2) is separable in n and h (in other words, the marginal utility of n is independent of h). Show that if instead $\varepsilon>\sigma$, then n and h are Edgeworth-Pareto substitutes in utility, in that the marginal utility of n is decreasing in h.

(PART B): Suppose there is an asset (which we can think of as equities) whose risky rate of return between t and t+1, payable in units of the numeraire n, is given by RR_{t+1}. Show that the following Euler Equation must hold at any time t:

(4) $$E_t\,[M_{t+1}\,RR_{t+1}] = 1$$

where the pricing kernel M_{t+1} is given by

(5) $$M_{t+1} = \beta\,(n_{t+1}/n_t)^{-1/\sigma}(X_{t+1})^{(\sigma-\varepsilon)/\sigma(\sigma-\varepsilon)}$$

and

(6) $$X_{t+1} = [1 + \omega\,(h_{t+1}/n_{t+1})^{(\varepsilon-1)/\varepsilon}] / [1 + \omega(h_t/n_t)^{(\varepsilon-1)/\varepsilon}].$$

(PART C): Let α denote the share of household spending on nondurables and services:

(7) $$\alpha_t = n_t / (n_t + p_t h_t)$$

Solve for the household's optimal α and rewrite X_{t+1} in equation (5) as a function of α_{t+1} and α_t. Re-write the Euler equation (4), substituting (5) and your new expression for X.

(PART D): Under what conditions does the model with housing reduce to the standard C-CAPM? Define the equity premium puzzle in the C-CAPM.

(PART E): Assume that $\varepsilon>\sigma$. In postwar US data, the empirical analog to α_t is negatively correlated with equity returns; in booms, equity returns are high, while the composition of household spending shifts towards housing and other durable goods, while in recessions, equity returns are low, while the share of household spending on housing and other durable goods falls.

Given our preference assumption and this fact, can the model with housing help explain the equity premium puzzle? Explain and provide an intuition. Use the form of the Euler Equation you derived in part C to answer this question.