

1. An old idea in macroeconomics is that good news about the future (such as scientific discoveries that will eventually increase total factor productivity) can create a boom today. A recent paper by Jaimovic and Rebelo (2006), however, shows that it is difficult for standard equilibrium business cycle models to generate booms in response to good news that display realistic positive comovements among output, consumption, investment and hours worked.

First consider the following basic model. Preferences in period 0 are given by

$$(1) \quad E_0 \sum_{t=0}^{\infty} (\beta^t) u(C_t, N_t)$$

where C is consumption, N is hours worked, $0 < \beta < 1$ and where

$$(2) \quad u(C_t, N_t) = \log(C_t) - BN_t^\theta$$

where $B > 0$ and $\theta > 1$. Output depends on total factor productivity A , on capital K and on hours worked, as follows:

$$(3) \quad Y_t = A_t N_t^\alpha K_t^{1-\alpha}$$

where $0 < \alpha < 1$. The capital stock evolves as follows:

$$(4) \quad K_{t+1} = (1-\delta) K_t + I_t,$$

where I stands for investment, and where

$$(5) \quad I_t = Y_t - C_t.$$

(a) Write down the Bellman's equation characterizing the social planner's problem (and also the competitive equilibrium, in this case), using the minimum number of state and control variables sufficient to characterize the problem. Derive a static first order condition describing optimal current labor supply as a function of current control and state variables, and an Euler Equation relating consumption today to state and control variables tomorrow.

Now suppose in period 1 that the economy is in a steady state, with constant A . Then, in period 2, the economy learns that there will be a permanent increase in A , starting in period 4. There are no other shocks that hit the economy prior to or after period 4. Figure 3 (see below), taken from the Jaimovic-Rebelo paper, shows how C , N , I and Y in the basic economy will respond to this good news over time. The horizontal axis plots the time period, and the vertical axis plots each variables' percentage deviation from the initial steady state. (These figures are computed assuming particular, noncontroversial values for the parameters B , α , δ , θ , and β , which are unimportant for the purposes of this problem.)

(b) Explain intuitively why consumption jumps in period 2. Then, taking the path of consumption as given, explain the behavior of the other variables, using both intuition and the equilibrium conditions derived in part (a), as well as the constraints (3)-(5). In particular, explain (i) why hours, investment and output fall in period 2, and (ii) why hours, investment and output fall further in period 3.

Now consider the following extensions to the basic model. Preferences still obey (1), but now let the period utility function take the form (note the position of the parentheses!)

$$(2') \quad u(C_t, N_t) = \log(C_t - BN_t^{\theta})$$

Suppose now that output depends on capital utilization u , as well as technology, hours worked and capital.

$$(3') \quad Y_t = A_t N_t^{\alpha} (u_t K_t)^{1-\alpha}$$

Intuitively, u could represent the workweek of capital; the more hours that a given set of machines is utilized, the more productive it will be in the short run. The capital stock now evolves as

$$(4') \quad K_{t+1} = [1-d(u_t)] K_t + I_t$$

where I still equals $Y - C$ (as in equation 5), but where d is now a function relating depreciation of capital to utilization. Assume that $d(u)$ is between 0 and 1, and that $d'(u) > 0$ and $d''(u) > 0$. Thus, utilizing capital more intensely causes it to depreciate faster.

(c) Write down the Bellman's Equation for the social planner's problem consisting of (1), (2'), (3'), (4') and (5), using the minimum number of state and control variables. Derive two static first order conditions relating optimal labor supply and optimal utilization to current state and control variables, and an Euler Equation for consumption.

(d) Again consider the scenario in which we learn in period 2 that technology will increase permanently in period 4. Prove that optimal labor supply and utilization will not change in period 2. Describe what will happen to consumption, output and investment in period 2.

Figure 3: Response to TFP News Shock, Benchmark RBC with KPR Preferences

Percentage Deviations from Steady State

