

1. Equilibrium Interest Rates in a Heterogeneous Agent Economy

This question asks you to set up and solve analytically for the equilibrium real interest rate in an incomplete markets economy in which agents are subject to idiosyncratic as well as aggregate income shocks and therefore engage in non-trivial borrowing and lending.

Make the following assumptions:

(1) PREFERENCES: there are two periods. There are N households with identical preference functions. For each household, lifetime expected utility depends on consumption in both periods. Utility is quadratic over the single consumption good and the discount factor is β .

(2) INCOME: households receive a stochastic flow of endowment income each period. Household i 's income at time t is denoted Y_{it} and is the sum of two components: an aggregate component, e_{at} , common to all households, and an idiosyncratic component, e_{it} , which is specific to household i . Assume that both e_{at} and e_{it} are mean-zero and i.i.d. across periods. In addition, assume that e_{it} is i.i.d. across households. Households choose consumption and saving in period 1 after period 1 income is realized, but prior to the realization of period 2 income. The endowment is perishable and there is no capital, so in equilibrium, aggregate consumption must equal aggregate income.

$$c_{2t} = y_{2t} + R(y_{1t} - c_{1t}).$$

(3) MARKETS: there are no securities markets for diversifying idiosyncratic risk. However, households can borrow and lend freely using one-period loans at a risk-free gross interest rate R . (I omit the time subscript because there are only two periods, so R is the gross return on saving from period 1 to period 2). Household i 's flow budget constraint in period two is thus;

$$c_{2t} = y_{2t} + R(y_{1t} - c_{1t}).$$

The interest rate R will be determined in equilibrium in order to equate the aggregate supply and demand for loans, or equivalently, to equate aggregate consumption and aggregate income.

Set up a model incorporating these assumptions, using the notation given to you, and derive a formula for the gross interest rate R as a function of model parameters and aggregate income Y_1 , where Y_1 is defined as the sum of Y_{i1} across all N households. Your formula should have the feature that when $Y_1 = 0$ (in other words, when first period aggregate income equals its unconditional mean), $R\beta = 1$. Show that R is countercyclical in your model (in other words, R is higher when aggregate income is low) and explain why this makes sense intuitively.

[HINT: solve for individual behavior first, then aggregate across households]