1. Borrowing constraints and precautionary saving

Suppose there is a consumer who lives for three periods. His flow utility function each period is quadratic:

$$U(c) = Ac - 1/2c^2$$

where A is a large positive number, so that utility will increase in c in the economically relevant region of this problem. Assume for simplicity that the time discount factor β and the gross interest rate on saving R are both 1. Assume that income in the first period and the third period are both equal to 1 with certainty. In the second period, income is either equal to (1-d) or (1+d), where $0 < d < 1$ represents the amount of income variation in percentage terms. Suppose the agent can save freely (at a rate R = 1) and can borrow freely at R = 1 in period 1, but that in period 2 the agent cannot borrow. The agent dies after the third period and there are no bequests.

(a) Suppose that the agent is uncertain as of period 1 as to what his income will be in period 2, with both states equally likely. Compute the amount of precautionary saving, defined here as the difference between optimal period 1 saving in this problem, and optimal period 1 saving in an otherwise identical model in which d = 0 so that the period 1 agent knows period 2 income with certainty. [HINT: solving the model with d = 0 is trivial. For the model with d > 0: first, solve the problem faced by the period 2 agent after the uncertainty is revealed. Then, using these results, solve for the first period's optimal saving. Prove that the liquidity constraint will bind in period 2 if income is low but not if income is high, and use this result in solving the problem].

(b) Would there be precautionary saving in period 1 if there was no borrowing constraint in period 2?

(c) Now suppose that income is not uncertain, but that there is heterogeneity: at the beginning of period 1, half of the agents (the "lucky" ones) learn that their period 2 income will be (1+d) with certainty, while the other half (the "unlucky" ones) learns that their period 2 income will be (1-d) with certainty. Agents are still unable to borrow in period 2, but may save in period 2 and may save or borrow in period 1. Compute optimal first period saving for both lucky and unlucky agents, and compute the amount of aggregate saving or borrowing in period 1, defined as the average of the saving (or borrowing) of the lucky and unlucky agents.

(d) Comparing your answers from (a) or (b), how much does uncertainty per se, as opposed to heterogeneity, contribute to period 1 saving?