

1. Precautionary Saving and Occupational Choice

Models with precautionary savings typically imply that, holding income volatility and all else constant, more risk averse agents will save more. Consider the following problem in which households choose both their consumption and their labor income risk. There are two periods. Assume that first period income w_1 is known as of period 1, while second period income w_2 equals $k + k*\epsilon$, where $k > 0$ is chosen by the household as of period 1, and where ϵ is a mean-zero normally distributed random variable with variance σ^2 . Households choosing a high k have a high mean income and a high variance of income. Households choosing a low k have a low mean income and a low variance of income. In period one, the household chooses c_1 and k to maximize

$$U(c_1) + E_1U(c_2), \quad \text{s.t. } c_2 = (w_1 - c_1) + k + k*\epsilon.$$

where utility is CARA:

$$U(c) = (-1/a) \exp(-ac)$$

and where recall that if $\log(X)$ is a normal random variable with mean μ and variance s^2 , then $E(X) = \exp(\mu + s^2/2)$.

Calculate optimal precautionary saving in period 1 as a function of underlying parameters. How does precautionary saving vary with risk aversion? Explain this relationship intuitively.