

Consider the following model of the optimal choice of retirement age. People live for T periods. In each period they have a time endowment of 1 unit, which can be split between working and leisure. Their preferences are given by

$$\sum_{t=0}^T [u(c_t) + v(1-h_t)],$$

where c_t is consumption in period t ; h_t is hours worked in period t , bounded between 0 and 1, so that $1-h_t$ is leisure time; and note that for convenience I assume no discounting. Assume that preferences are CRRA in consumption and log in leisure:

$$u(c) = (1/1-\sigma) c^{1-\sigma} \quad v(1-h) = \log(1-h)$$

Suppose that agents can choose to retire at some date R between 0 and T . Labor markets work the following way: prior to retirement (for $t < R$), agents must work L hours, where $0 < L < 1$, for an hourly wage w . Assume that L is a fixed parameter over which the agent has no choice; intuitively, L can be thought of as a "standard workweek". After retirement (for $t \geq R$), agents work zero hours. Therefore, labor income prior to retirement is wL per period, while labor income is zero after retirement. Assume that retirement is a one-time irrevocable decision; once the agent retires at date R , he cannot re-enter the labor market thereafter. Notice that under these assumptions the retirement date is the only labor supply choice that the agent makes.

Finally, assume that the agent can save or borrow at rate r , which for simplicity is set equal to zero, subject to solvency. The agent's budget constraints are thus

$$A_0 \text{ given; } A_{T+1} = 0$$

$$A_{t+1} = A_t + wL - c_t \quad [\text{for } t < R]$$

$$A_{t+1} = A_t - c_t \quad [\text{for } t \geq R]$$

Notice that there is no uncertainty in this economy.

(1) Solve for the optimal retirement date R as a closed-form function of: lifespan T , initial wealth A_0 ; wages w ; the workweek parameter L ; and the CRRA parameter σ .

[HINT: start by solving the optimal consumption decision taking R as given]

(2) Over the last 50 years, we observe that the retirement age has fallen in the US, while lifespan and wages have risen (and workweeks have remained constant). Using your solution from (1), (and setting initial wealth A_0 to zero), what assumptions must we make on the CRRA parameter σ in order to explain the declining retirement age in the face of rising lifespan, rising wages and constant workweeks? Explain intuitively.